Directional Detection of Dark Matter Streams

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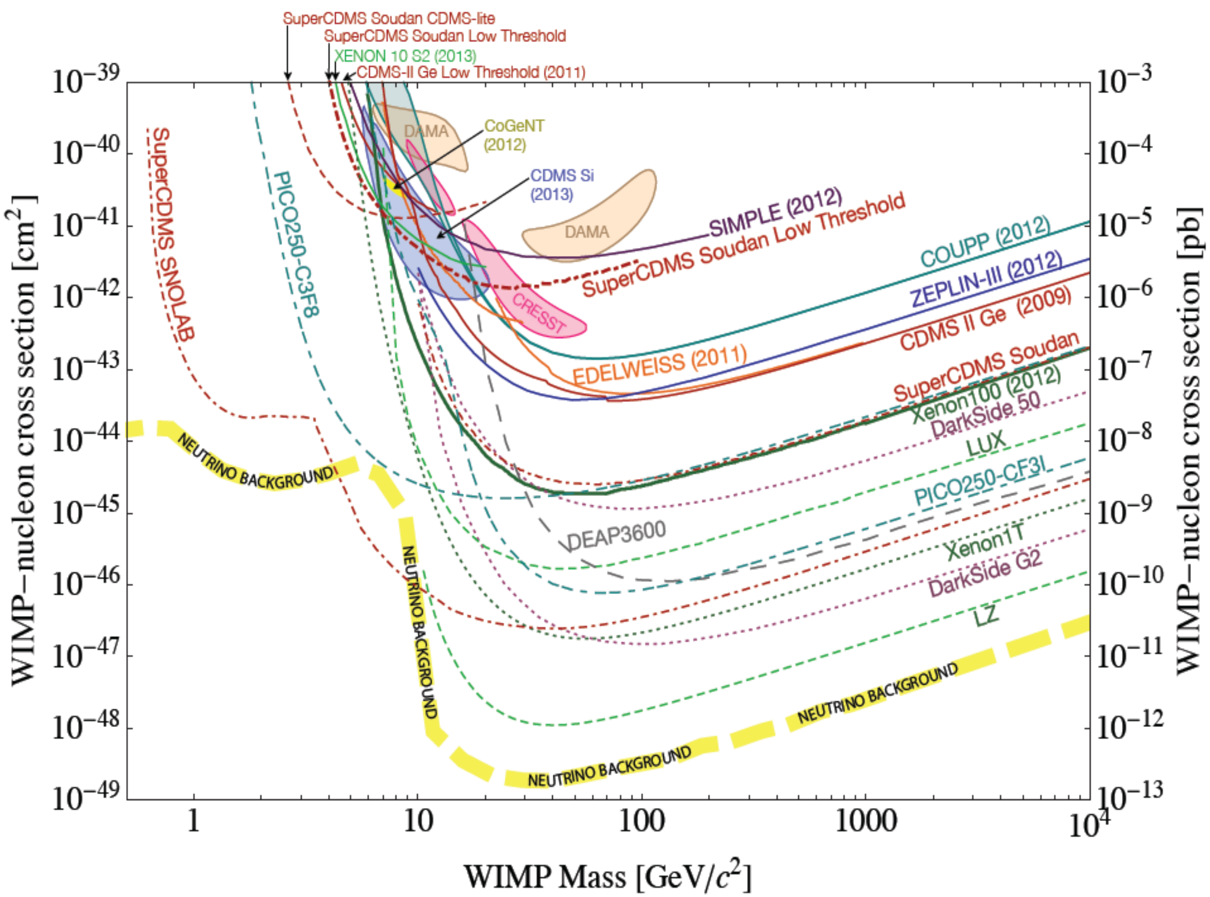
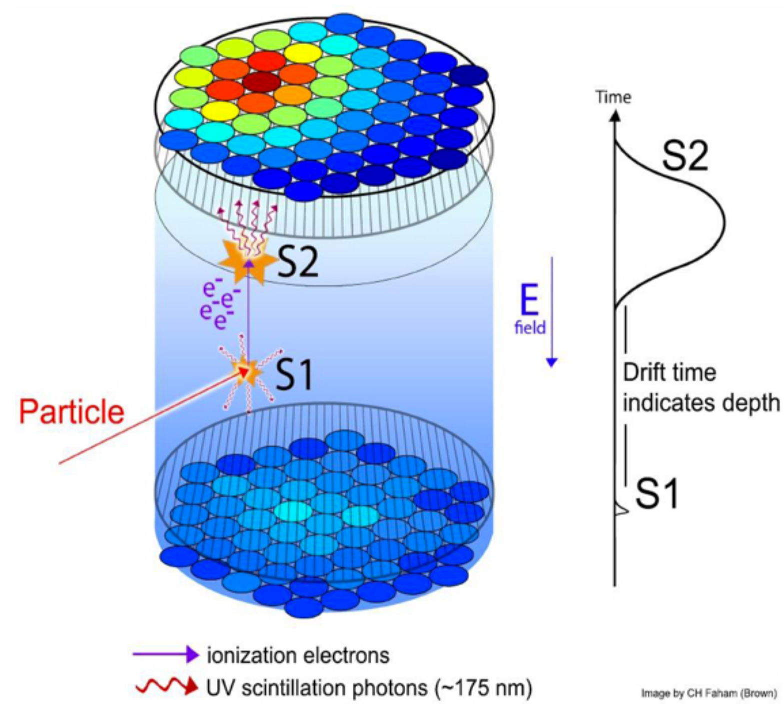
Overview

- Directional detection
- Streams
- Simulation
- Statistics
  - Non-parametric spherical statistics
  - Bayesian parameter estimation
  - Profile likelihood ratio

Direct Detection

- Look for recoiling nuclei struck by WIMPs in Milky Way halo
- Very hard – lots of backgrounds

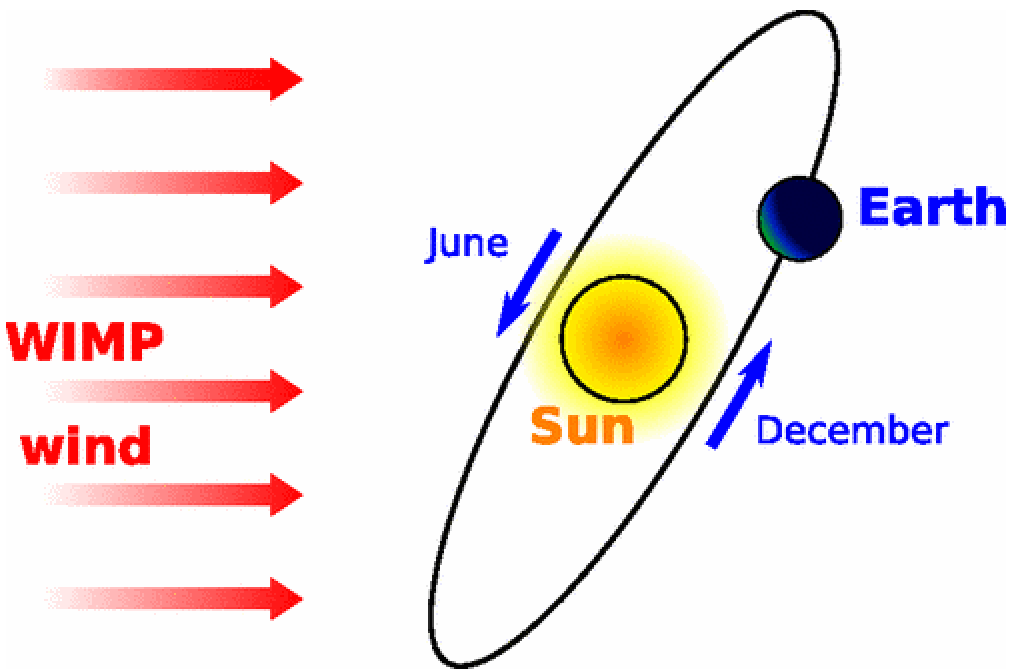




LUX Collaboration

“Smoking gun” signals

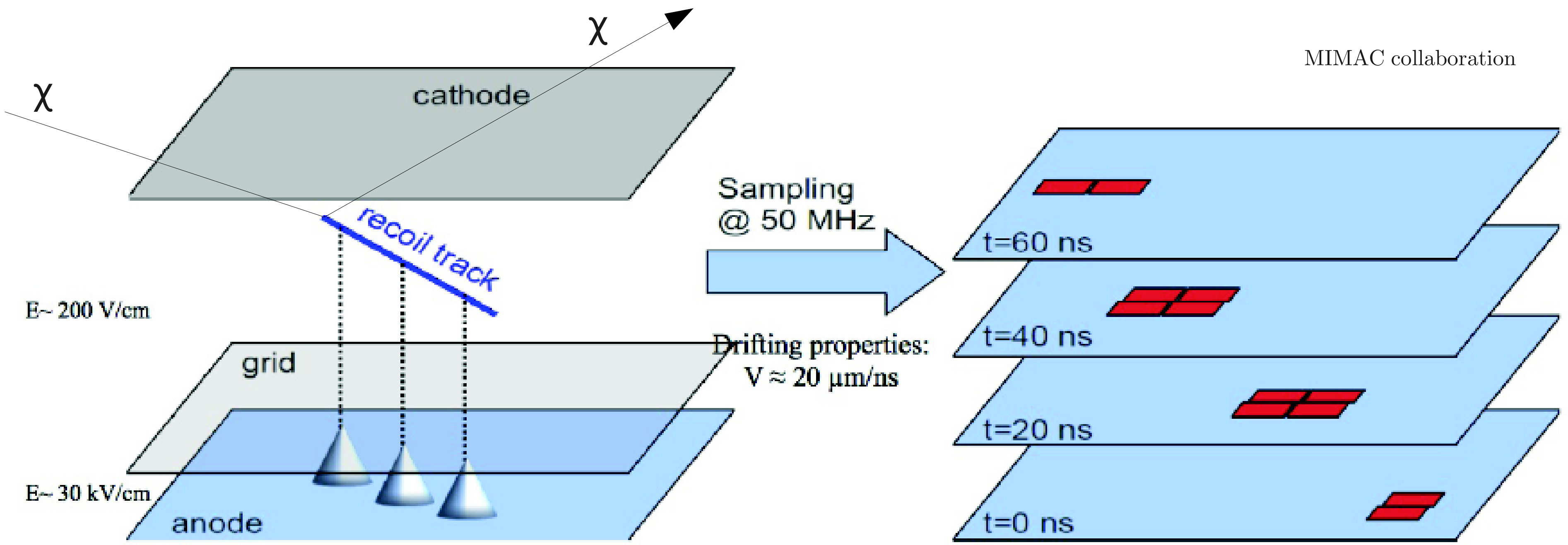
- Annual Modulation (*Drukier, Freese & Spergel 1986*)
- Direction dependence (*Spergel 1988*)



Directional Detection

- Measure **energy and direction** of nuclear recoils

MIMAC collaboration



- Advantages:
  - No known background that can mimic signal
  - Probe *velocity* distribution

Directional detection

- Double differential scattering rate:

$$\frac{\mathrm{d}^2 R}{\mathrm{d}E\mathrm{d}\Omega_q} = \frac{\rho_0 \sigma_N}{4\pi m_\chi \mu^2} F^2(E) \hat{f}(v_{\min}(E), \hat{\mathbf{q}})$$

WIMP:	m_χ	WIMP mass
	σ_N	WIMP-nucleus cross section
Galactic Halo:	ρ_0	Local WIMP density
	$\hat{f}(v_{\min}, \hat{\mathbf{q}})$	Radon transform of velocity dist.
Nuclear physics:	$F(E)$	Nuclear form factor
	m_N	Nucleus mass
	μ	$m_\chi m_N/(m_\chi + m_N)$

Directional detection

- Radon transform of WIMP velocity distribution,

$$\hat{f}(v_{\text{min}}, \hat{\mathbf{q}}) = \int \delta(\mathbf{v} \cdot \hat{\mathbf{q}} - v_{\text{min}}) f(\mathbf{v}) \mathrm{d}^3 v$$

- at smallest speed that can cause a recoil of energy, E

$$v_{\text{min}} = \frac{1}{\mu} \sqrt{\frac{m_N E}{2}}$$

- Lab frame distribution $=$ boost of Galactic frame distribution

$$f(\mathbf{v}) = f_{\text{gal}}(\mathbf{v} + \mathbf{v}_{\text{lab}})$$

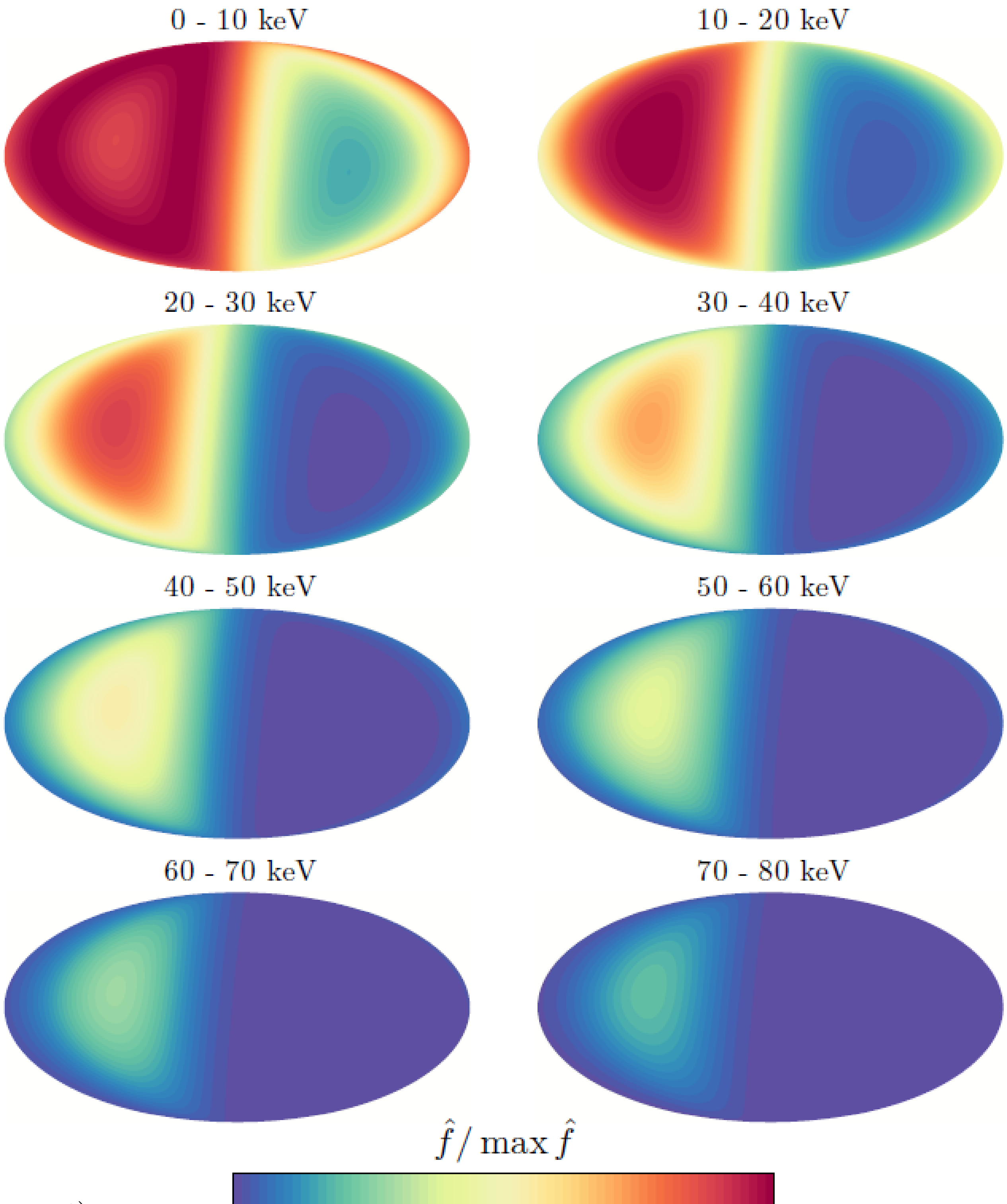
Velocity distribution

- Standard Halo Model used ubiquitously in current data analysis

$$f_{\mathrm{gal}}(\mathbf{v}) = \frac{1}{N_{\mathrm{esc}}(2\pi\sigma_v^2)^{3/2}} \exp\left(-\frac{|\mathbf{v}|^2}{2\sigma_v}\right)\theta(v_{\mathrm{esc}} - |\mathbf{v}|)$$

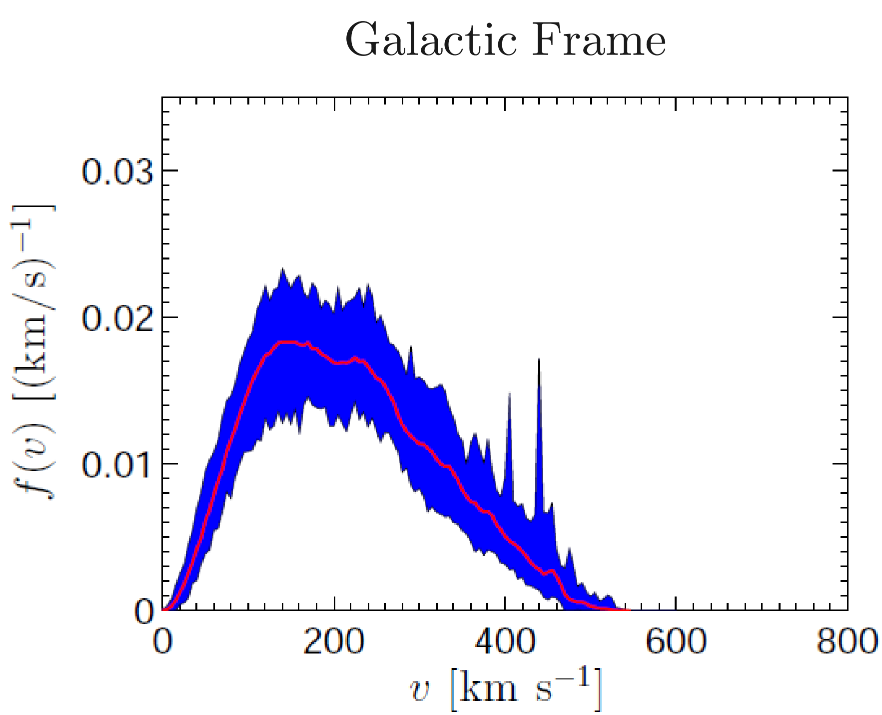
- Radon transform of Lab frame distribution

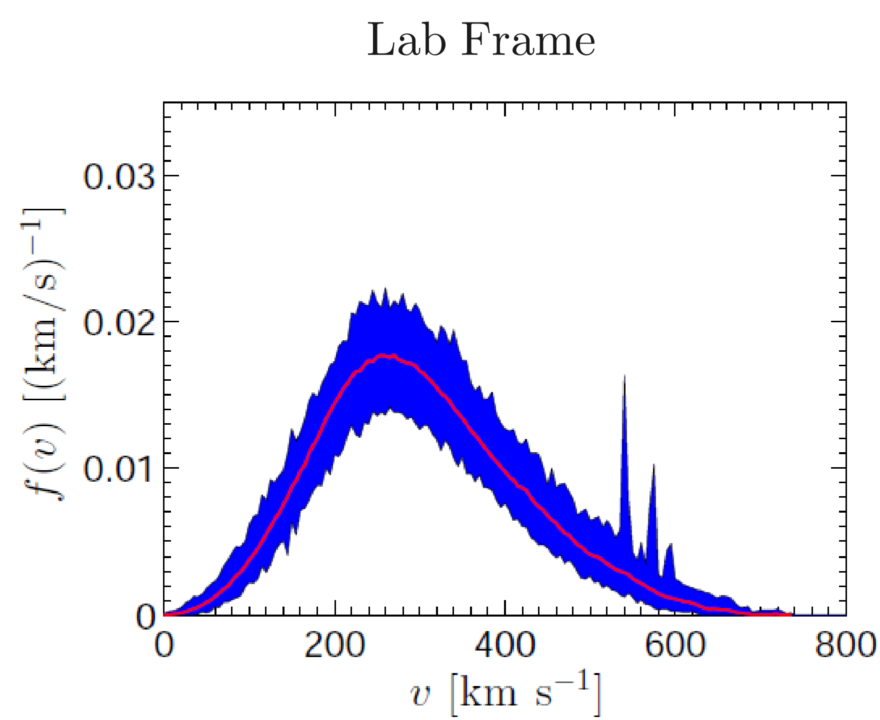
$$\hat{f}_{\mathrm{lab}}(v_{\mathrm{min}}(E), \hat{\mathbf{q}}) = \frac{1}{N_{\mathrm{esc}}(2\pi\sigma_v^2)^{1/2}}\left[\exp\left(-\frac{|v_{\mathrm{min}}(E) + \mathbf{v}_{\mathrm{lab}} \cdot \hat{\mathbf{q}}|^2}{2\sigma_v^2}\right) - \exp\left(-\frac{v_{\mathrm{esc}}^2}{2\sigma_v^2}\right)\right]$$


0 - 10 keV
10 - 20 keV
20 - 30 keV
30 - 40 keV
40 - 50 keV
50 - 60 keV
60 - 70 keV
70 - 80 keV
$\hat{f} / \max \hat{f}$

More realistic distributions

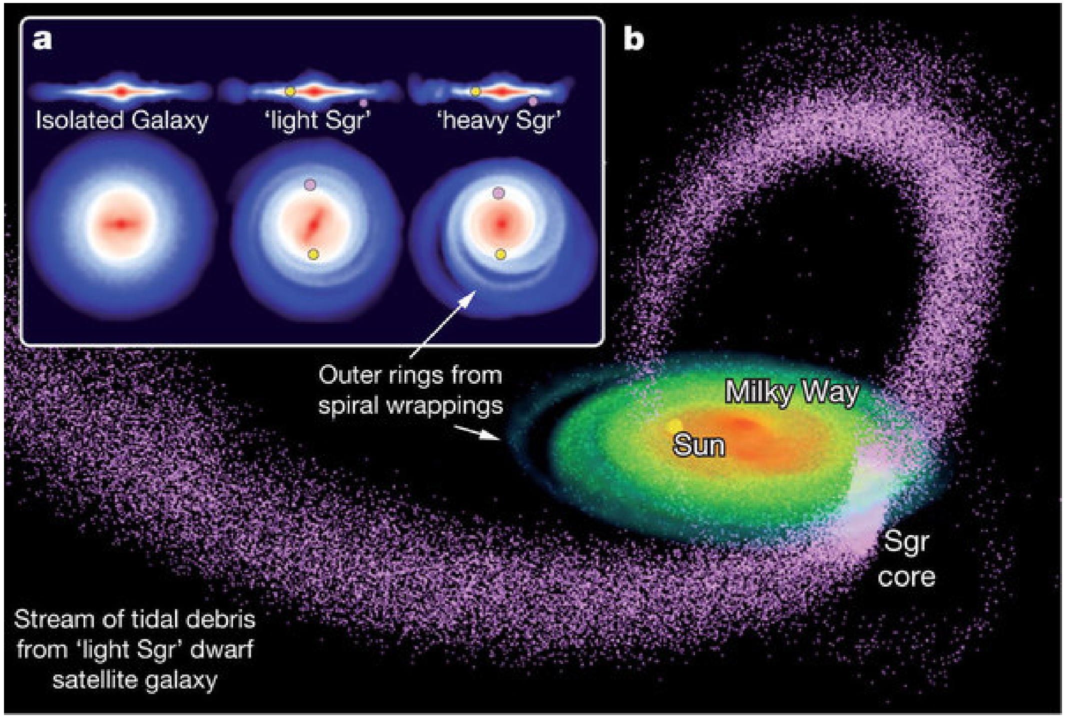
- Evidence from observation and simulation that distribution is *not* smooth or isotropic
- E.g. Via-Lactea2 Earth-like distributions:

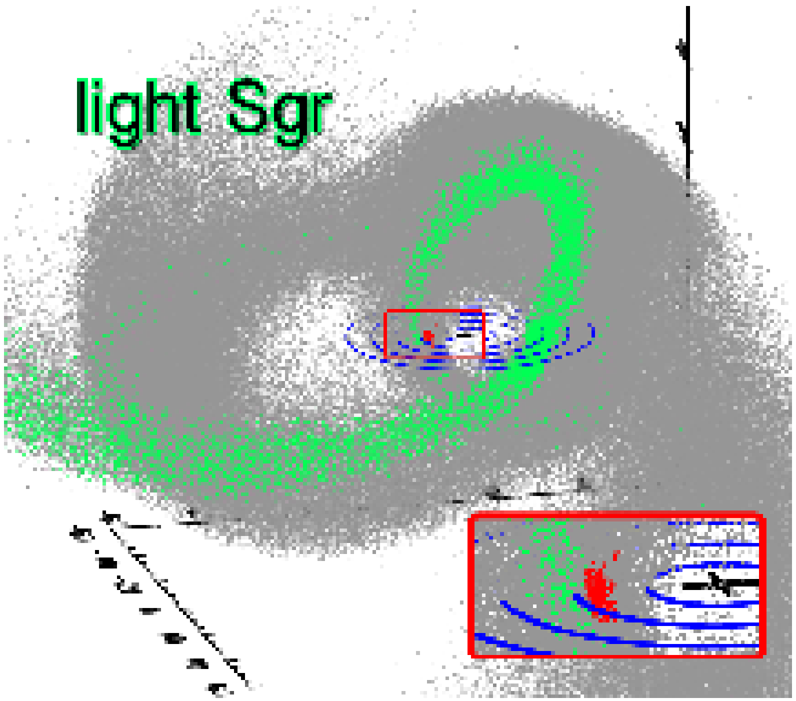




Streams

- Spatially and kinematically localised substructure
- Tidally stripped material from satellite galaxies
- Solar neighborhood stream (Sagittarius stream) motivated from simulation and observation





Purcell et al. (2011)

Stream model

- Stream takes a fixed fraction of the local DM density
- Full distribution = Background halo + Stream

Background halo dispersion ~ 200 km/s

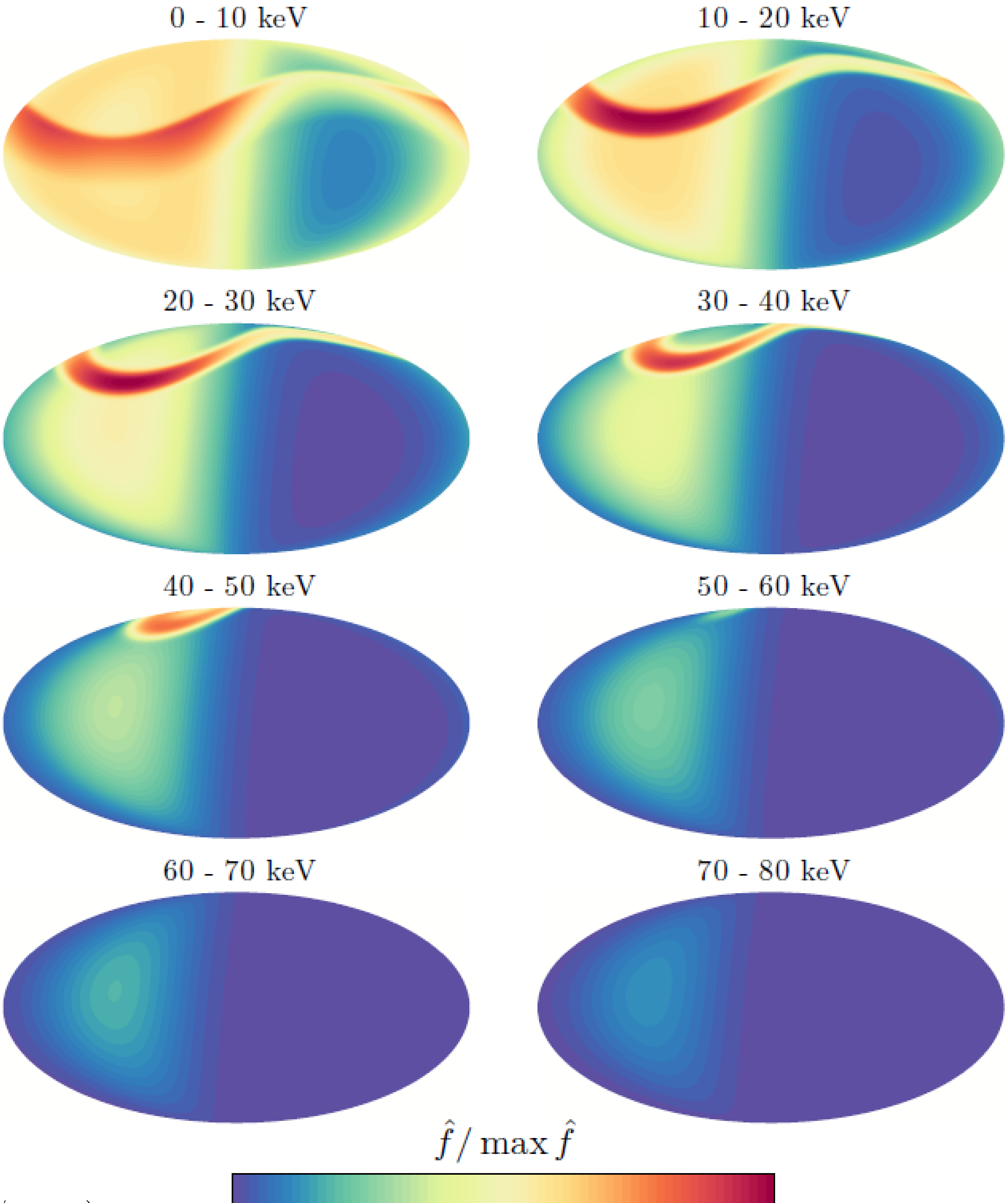
$$\hat{f}_{\rm lab}(v_{\rm min}, \hat{\mathbf{q}}) = (1-\xi)\hat{f}_{\rm gal}(v_{\rm min} + \mathbf{v}_{\rm lab}\cdot\hat{\mathbf{q}}, \hat{\mathbf{q}}; \sigma_v, v_{\rm esc})$$

$$+\xi\hat{f}_{\rm gal}(v_{\rm min} + (\mathbf{v}_{\rm lab} - \mathbf{v}_{\rm str})\cdot\hat{\mathbf{q}}, \hat{\mathbf{q}}; \sigma_{\rm str}, v_{\rm esc})$$

Stream fraction ~ 10%

Stream dispersion ~10 km/s

Galactic frame stream velocity


0 - 10 keV
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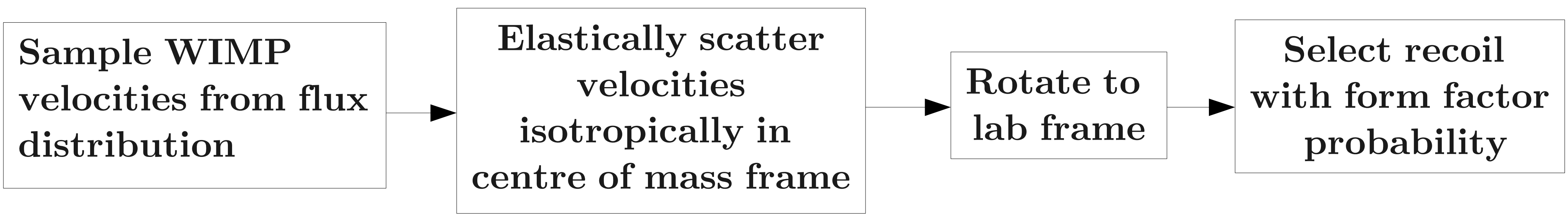
70 - 80 keV
$\hat{f}\,/\,\max\hat{f}$

Scattering simulation

- Simulate forecast of MIMAC experiment
- Target Fluorine-19 with spin-dependent scattering

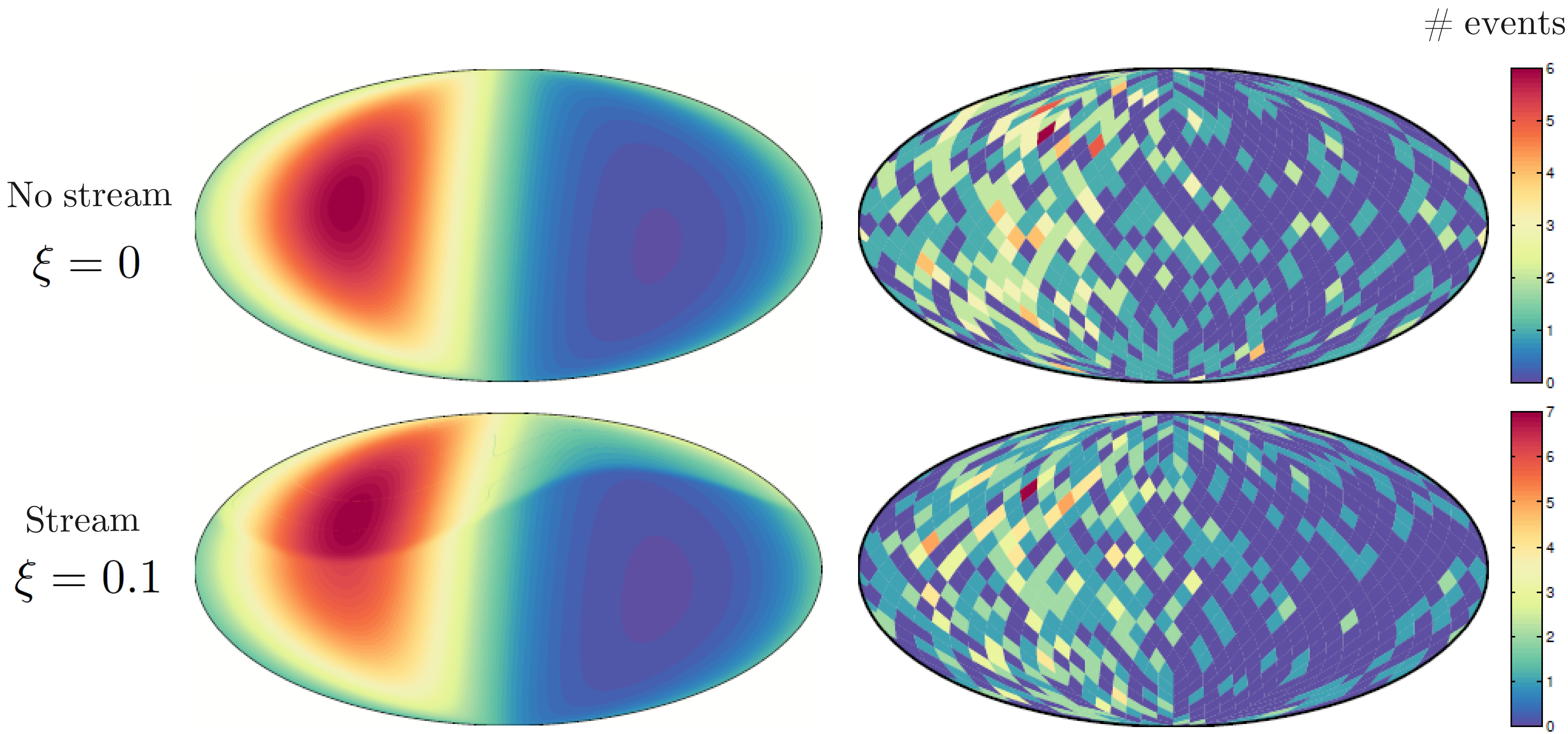


- Isotropic background model
- Vary energy window – threshold/maximum energy

Parameter values

WIMP:	m_χ	50 GeV
	σ_p (SD)	10^{-3} pb
Halo:	ρ_0	0.3 GeV cm^{-3}
	σ_v	$v_{\rm LSR}/\sqrt{2}$
	$v_{\rm esc}$	533 km s^{-1}
	$\mathbf{v}_{\rm lab}$	$(6.0, 230.6, 6.5)$ km s^{-1}
Experiment:	m_N	18.998 amu (F)
	$E_{\rm th}$	5, 20 keV
	$E_{\rm max}$	50, 100 keV
	$R_{\rm bg}$	10 kg^{-1} yr^{-1}
Sgt. stream:	$\mathbf{v}_{\rm str}$	$400 \times (0, 0.233, -0.970)$ km s^{-1}
	$\sigma_{\rm str}$	10 km s^{-1}
	$\xi_{\rm str}$	0.1

Detecting substructure



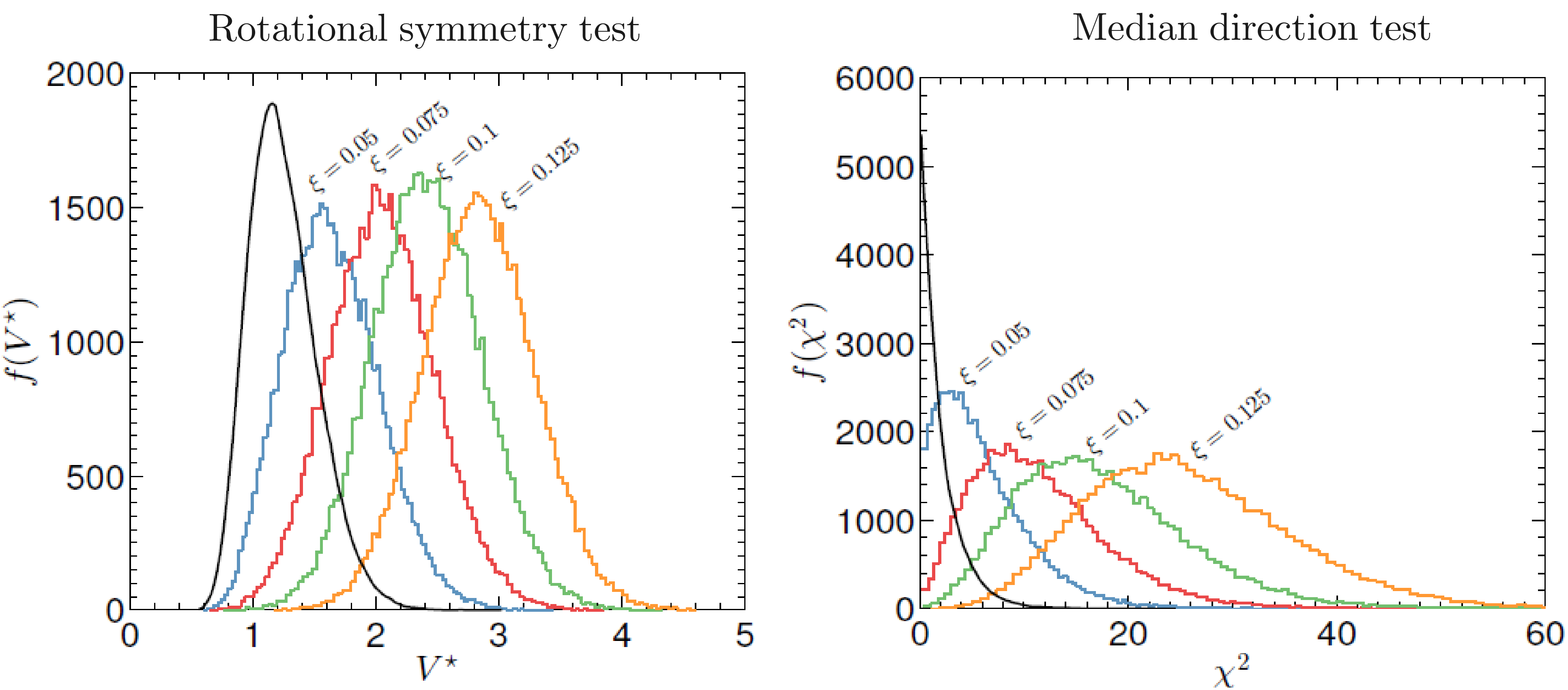
1. Discriminate between smooth halo and halo with substructure
2. Measure properties of stream (density, velocity, dispersion)

Tests

- No stream - expect signal to have:
  - Median direction opposite to that of solar motion
  - Rotational symmetry about this direction

- Frequentist tests on spherical data:
  - Median direction "chi squared" test
  - Modified Kuiper test (rotational symmetry)

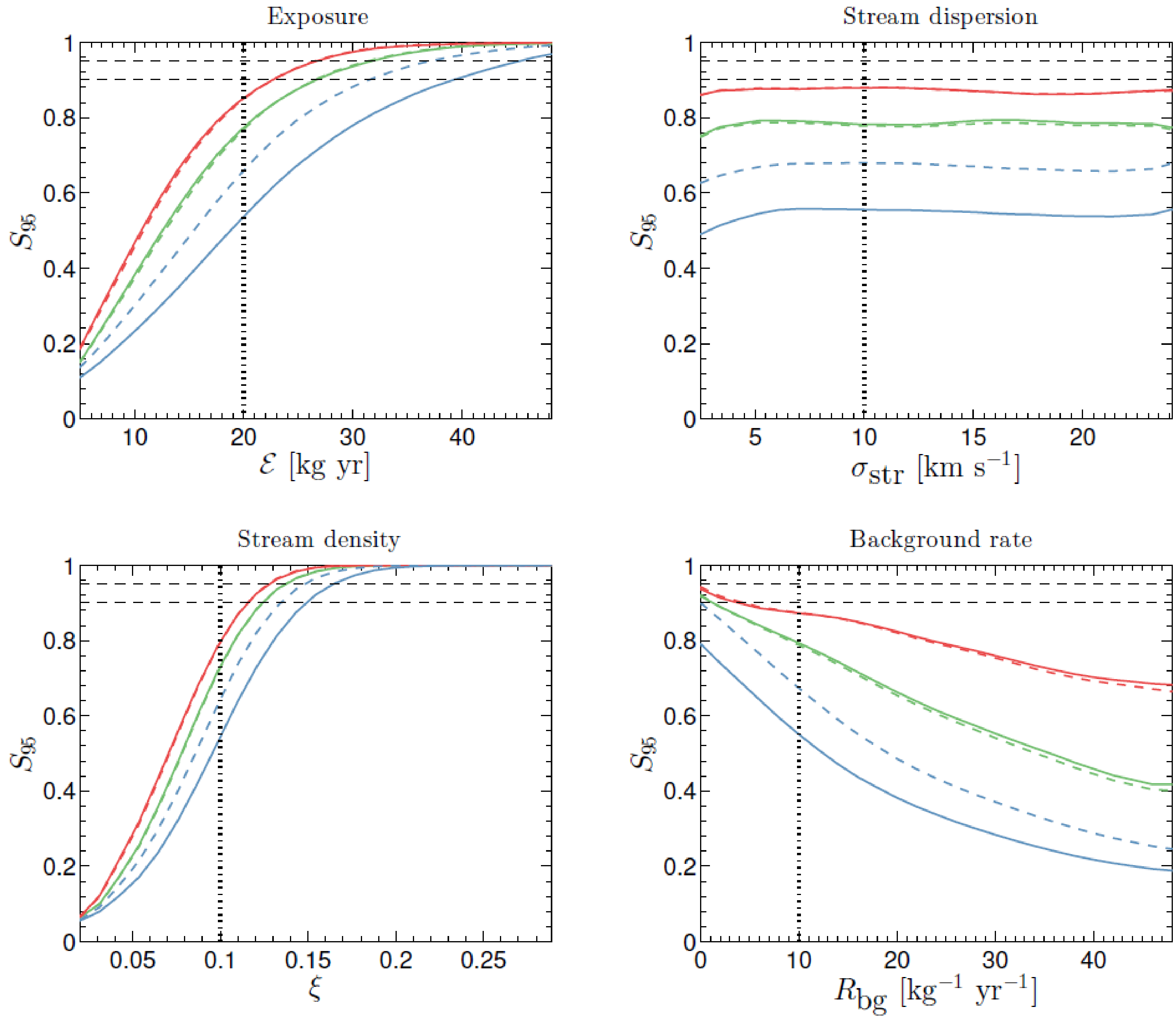
- Monte-Carlo simulate mock experiments and perform tests on each one



  - Calculate detection significance and power using these distributions

- Example: Sagittarius stream

S95: Detection significance achievable by 95% of hypothetical experiments

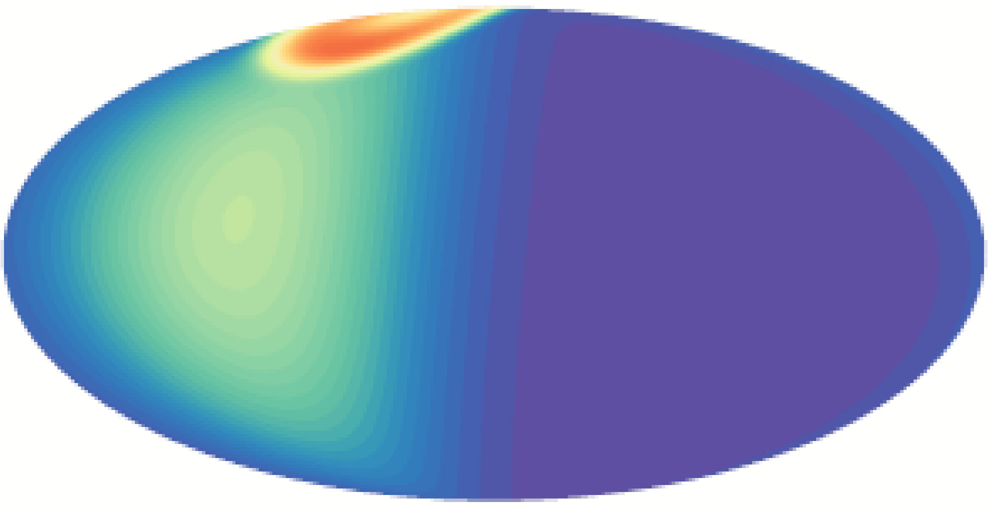


Energy window:

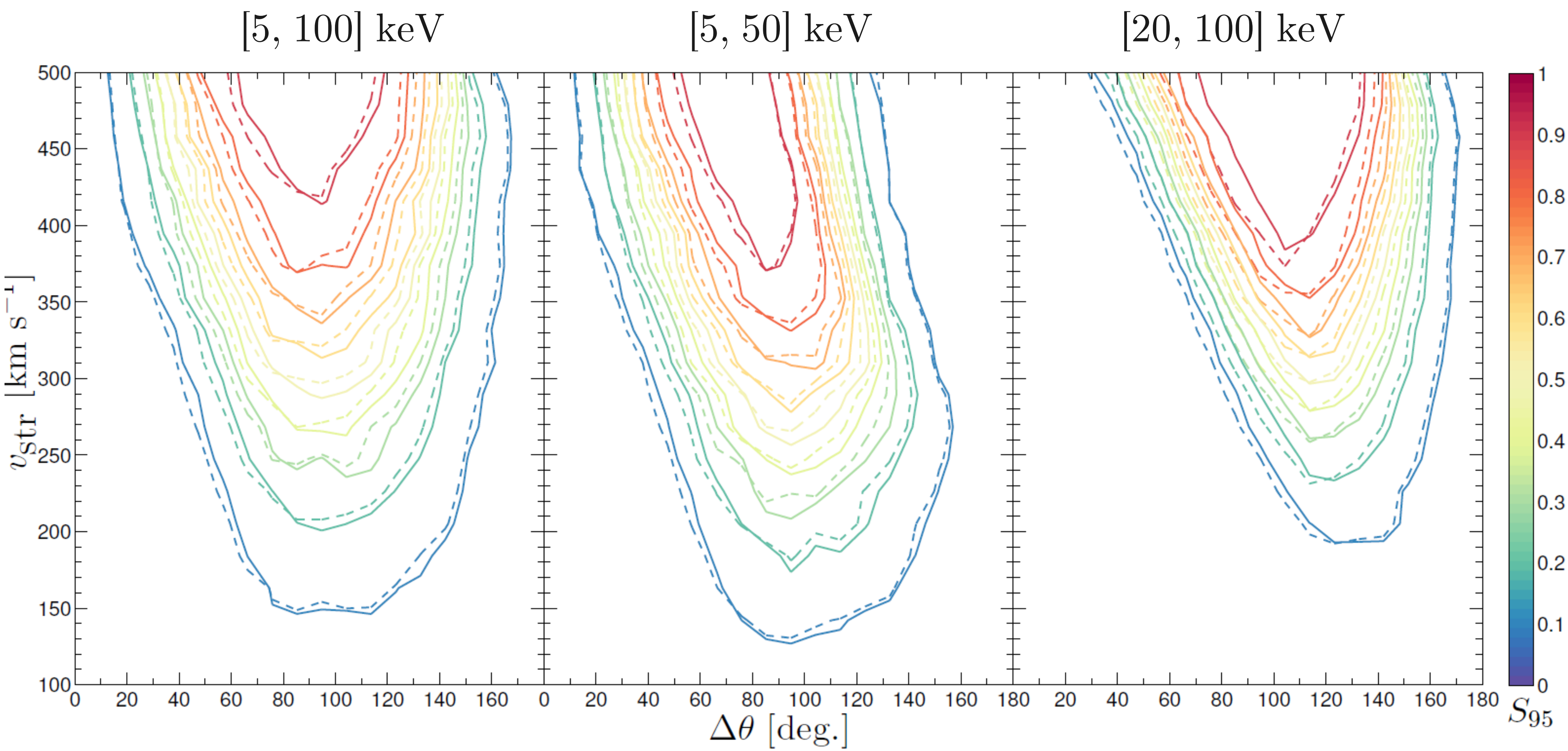
- [5,100] keV
- [5,50] keV
- [20,100] keV

Stream velocity

- Want to explore all possible stream velocities
- Can describe using 2 parameters
  - Stream speed, v_{str}
  - Angle between stream velocity and solar velocity $\Delta\theta = \cos^{-1}(\hat{\mathbf{v}}_{\mathrm{lab}} \cdot \hat{\mathbf{v}}_{\mathrm{str}})$



- Tests not powerful over full range of stream velocities
- Threshold/maximum energies limit detectability



Likelihood

- 11 unknown parameters,
- $\boldsymbol{\theta} = \{m_\chi, \sigma_p, \rho_0, \sigma_v, v_{\rm esc}, \sigma_{\rm str}, \mathbf{v}_{\rm str}, \xi, R_{\rm bg}\}$

$$\mathcal{L}(\boldsymbol{\theta}) = \frac{N_e(\boldsymbol{\theta})^{N_o}}{N_o!} e^{-N_e(\boldsymbol{\theta})} \prod_{i=1}^{N_o} \left[\lambda P_{\rm wimp}(E_i, \hat{\mathbf{q}}_i; \boldsymbol{\theta}) + (1-\lambda) P_{\rm bg} \right]$$

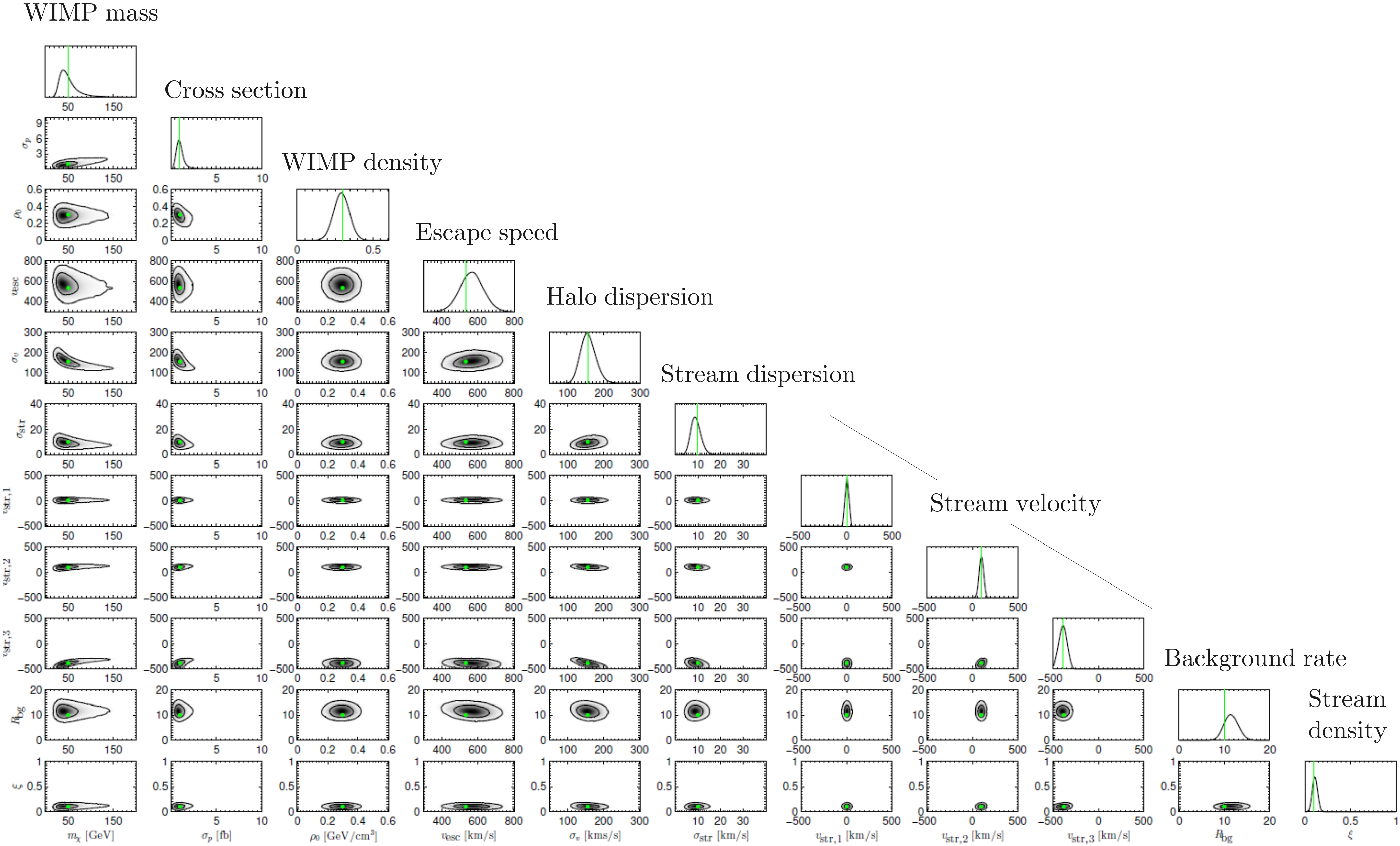
Signal:

$$P_{\rm wimp}(E_i, \hat{\mathbf{q}}_i; \boldsymbol{\theta}) = \frac{1}{R} \frac{{\rm d}^2 R}{{\rm d}E {\rm d}\Omega_q}\bigg|_{E_i, \hat{\mathbf{q}}_i; \boldsymbol{\theta}}$$

Background:

$$P_{\rm bg} = \frac{1}{4\pi(E_{\rm max} - E_{\rm th})}$$

Parameter estimation



Model comparison

- Null Hypothesis (no stream), H_0
- Alternative hypothesis, H_ξ

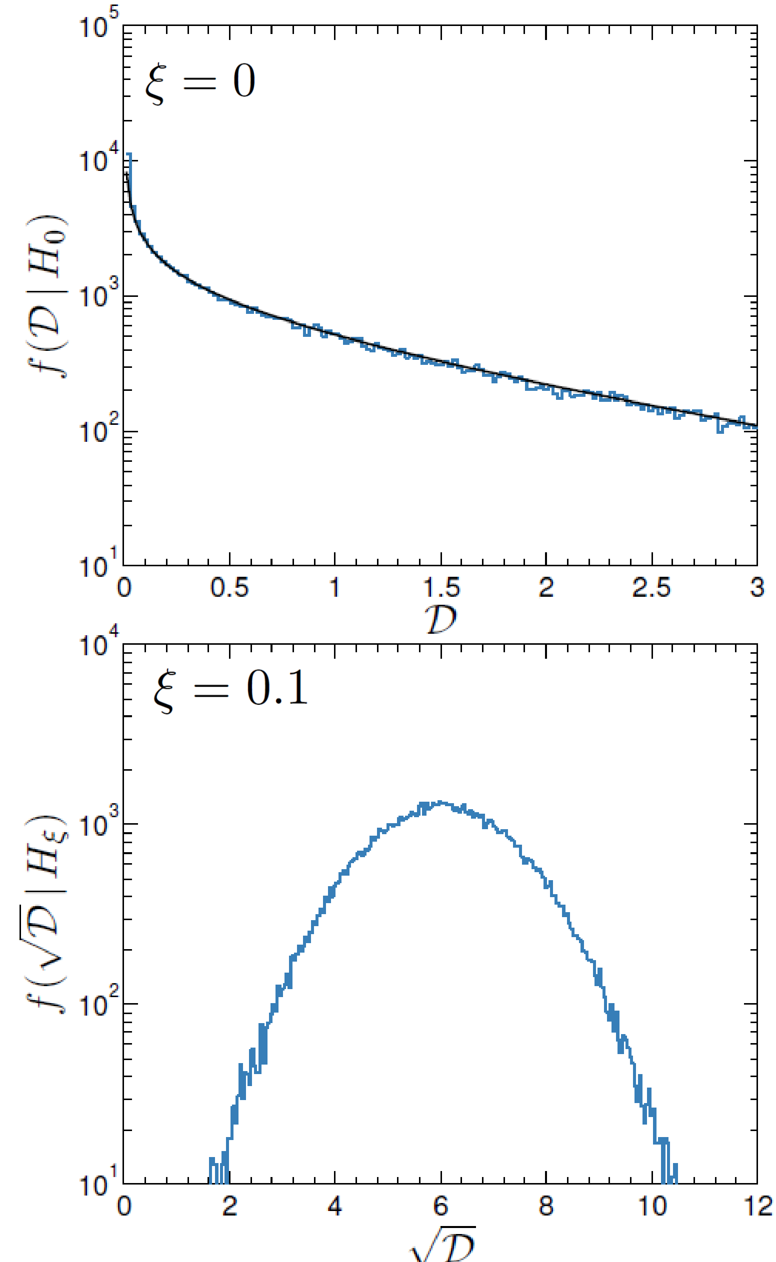
- Profile likelihood ratio test

Profile likelihood ratio: $$\Lambda = \frac{\mathcal{L}(\hat{\hat{\boldsymbol{\theta}}}|\hat{\hat{\xi}}=0)}{\mathcal{L}(\hat{\boldsymbol{\theta}})}$$

Test statistic: $$\mathcal{D} = \begin{cases} -2\ln\Lambda & 0 \le \hat{\xi} \le 1\,, \\ 0 & \hat{\xi} < 0,\ \hat{\xi} > 1 \end{cases}$$

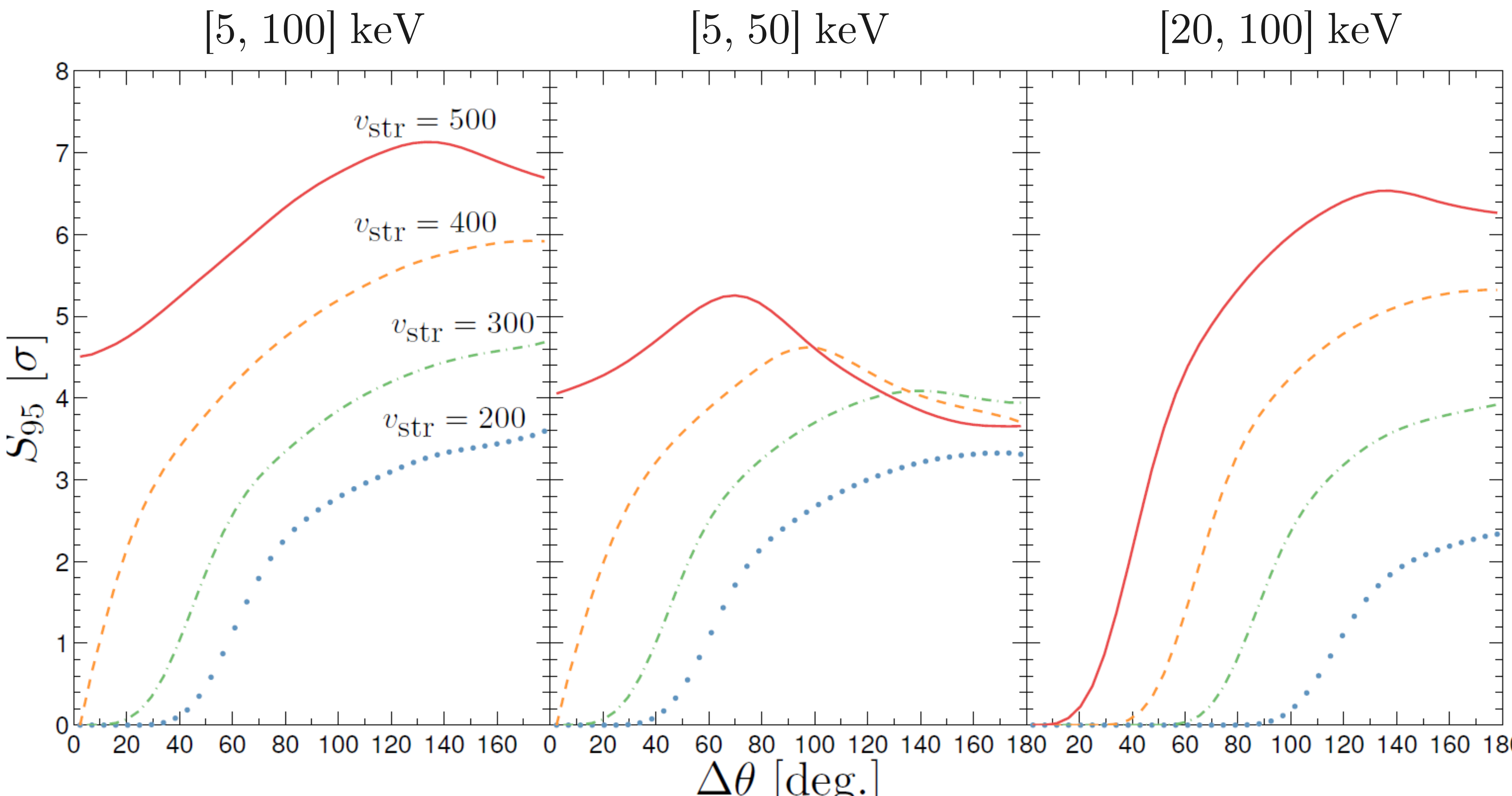
Significance: $$S = \int_0^{\mathcal{D}_{\rm obs}} f(\mathcal{D}|H_0)\,{\rm d}\mathcal{D} = \sqrt{\mathcal{D}_{\rm obs}}$$

Power: $$\int_0^{S_{95}} f(\sqrt{\mathcal{D}}){\rm d}\sqrt{\mathcal{D}} = 0.95$$



Results

- Only limiting factor is number of stream WIMPs
- Threshold/maximum energies still limit detectability



Summary

- Local Milky Way likely contains substructure such as tidal streams
- Directional detectors offer best prospect for detecting them
- Streams may be detected by the next generation of directional detectors with exposures >10 kg yr
- Directional tests not powerful over full range of stream velocities
- Parameter estimates very good for astrophysical parameters
- Maximum of 100 keV and threshold of 5 keV opens up much of the stream parameter space for detection.