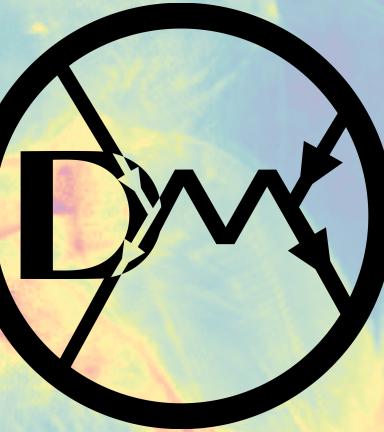




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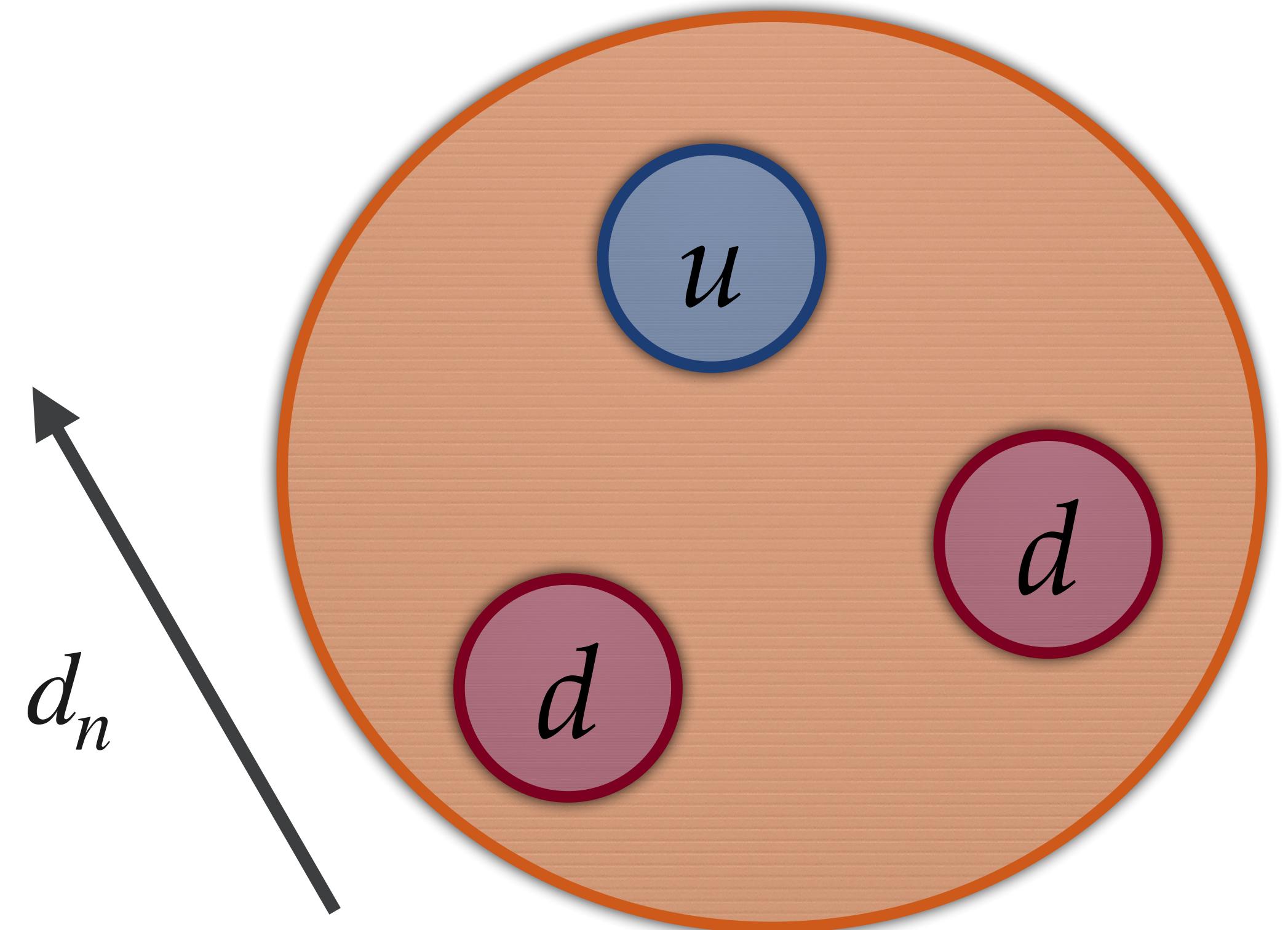
ARC CENTRE OF EXCELLENCE FOR

DARK  
MATTER



# Axions

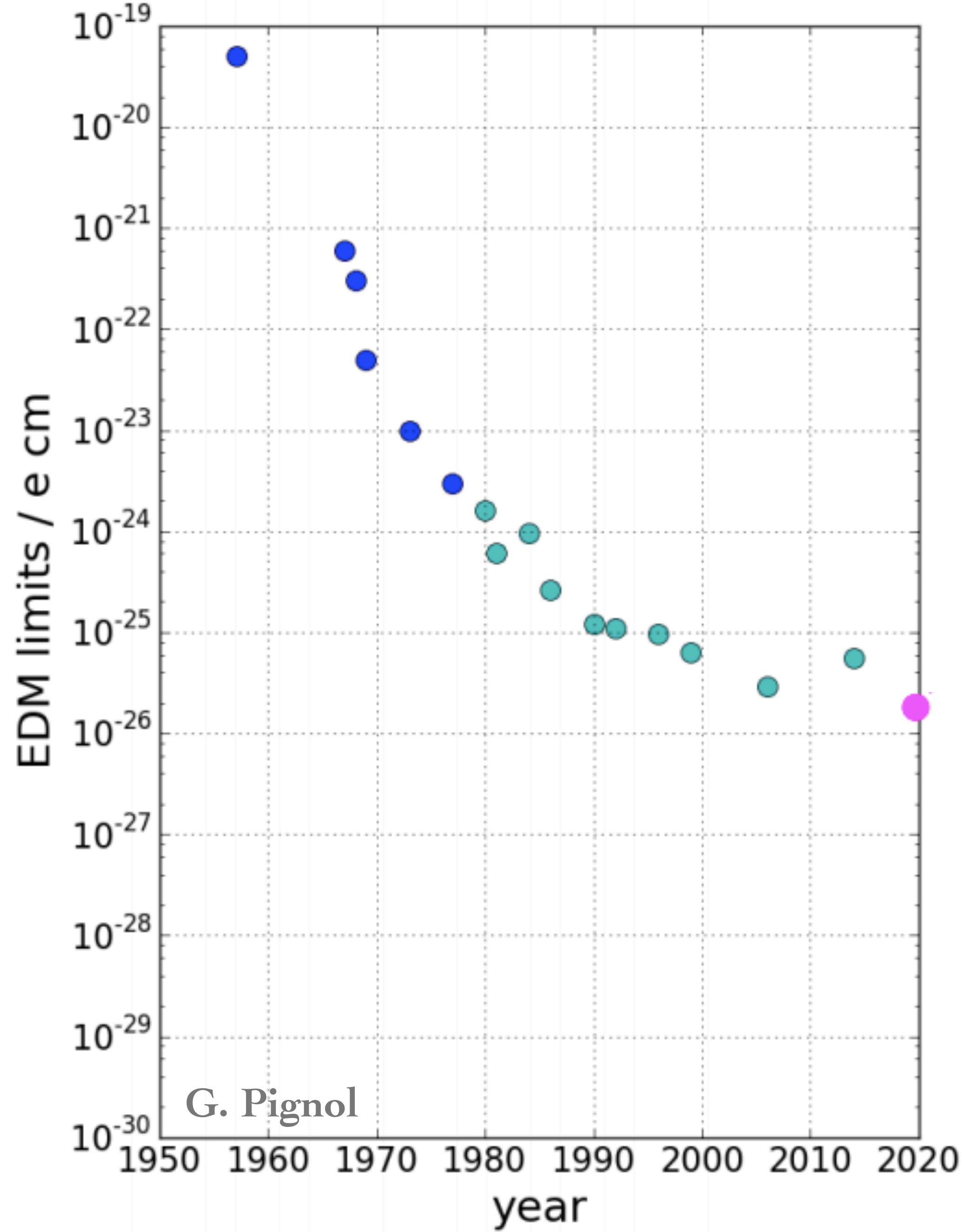
Ciaran O'Hare  
U. Sydney



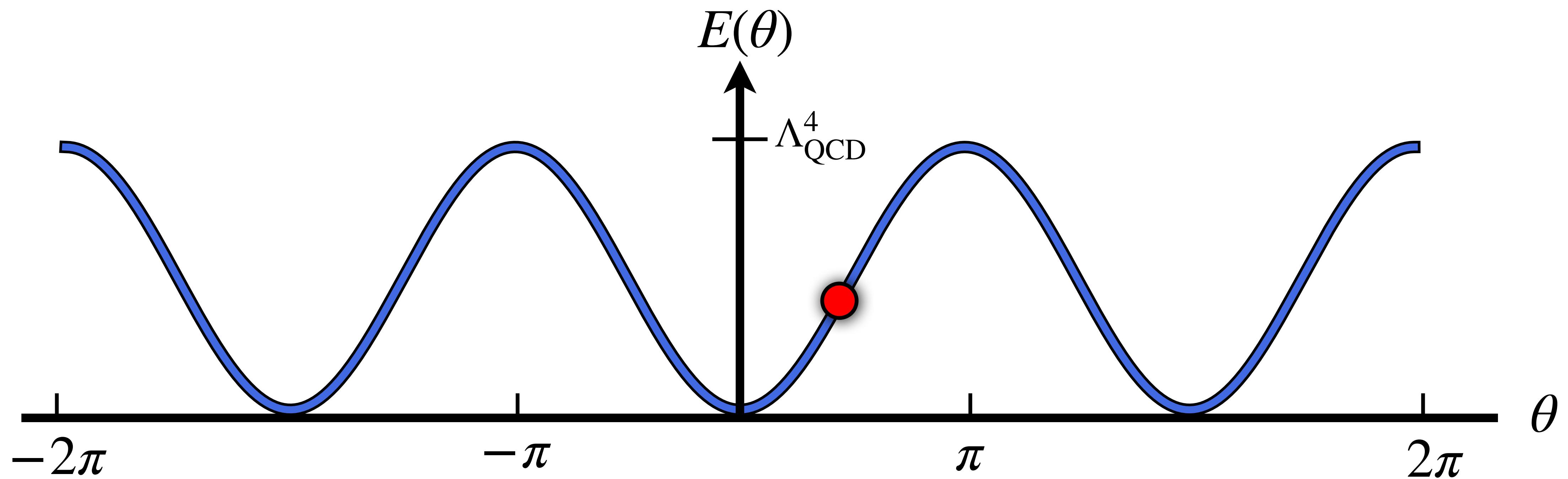
$d_n \approx e \cdot \text{fm} ?$

$$d_n = (2.4 \pm 1.0) \bar{\theta} \times 10^{-3}\, e\,\mathrm{fm}$$

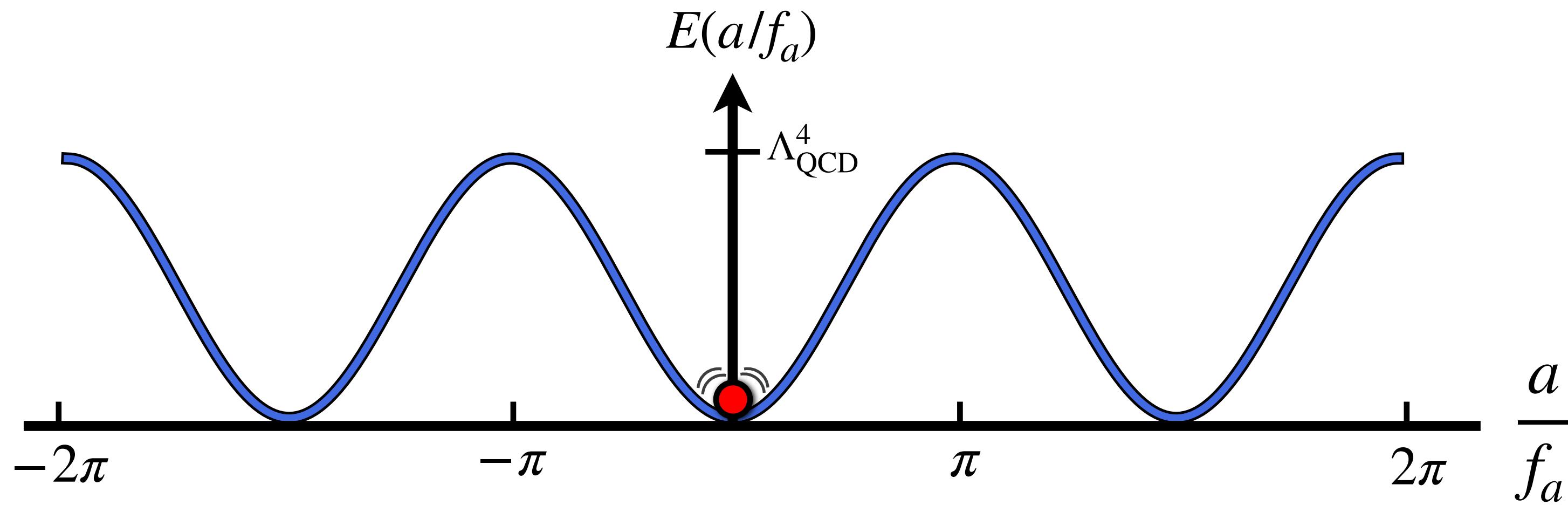
$$\bar{\theta}=\theta_{\rm QCD}+\theta_q$$



$$|d_n| < 1.8 \times 10^{-26} \text{ e cm} \quad (90\% \text{ CL})$$
$$\Rightarrow \theta \lesssim 10^{-10}$$

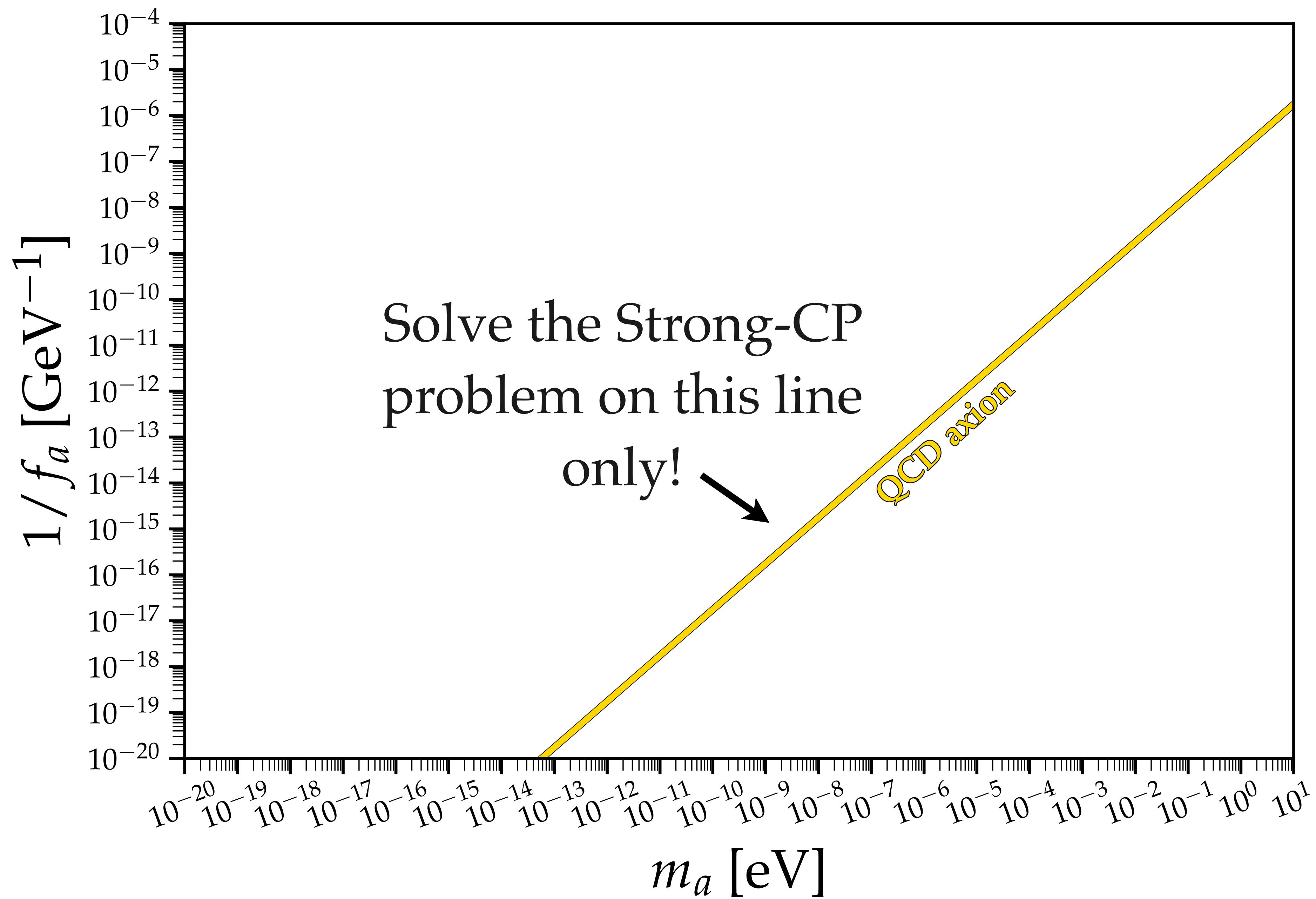


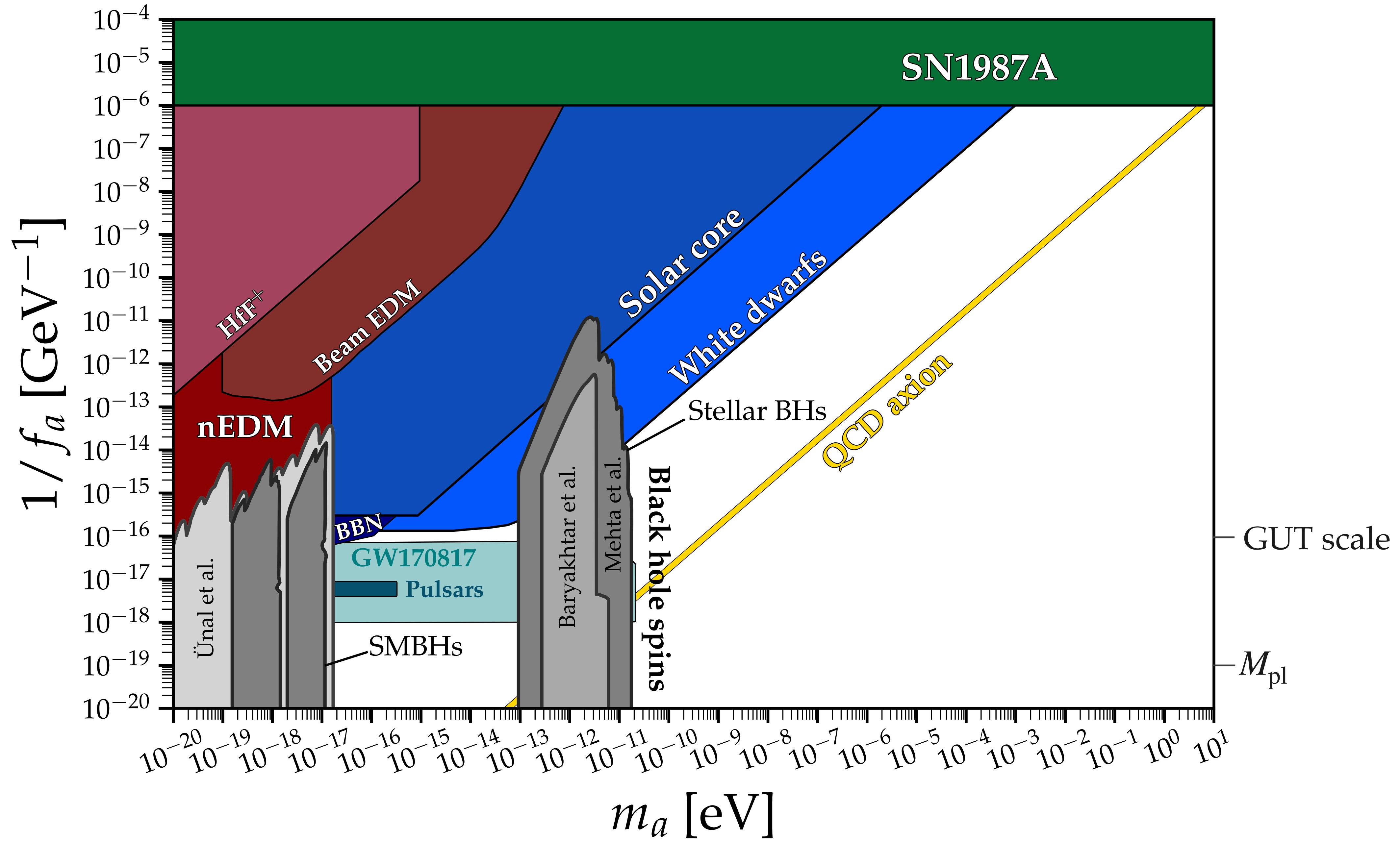
# The axion



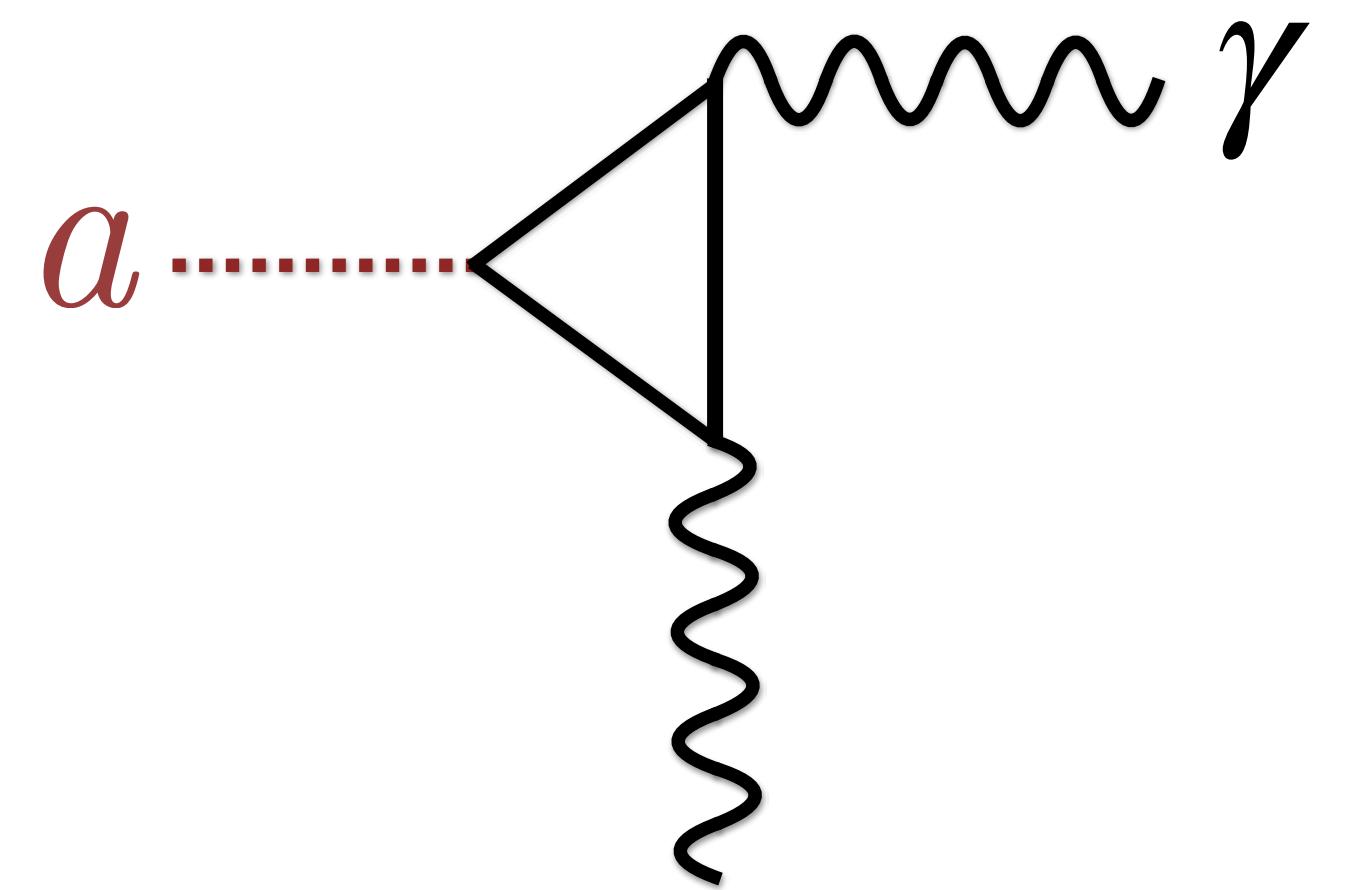
$$V(a) \approx \Lambda_{\text{QCD}}^4 \left[ 1 - \cos \left( \bar{\theta} + \frac{a}{f_a} \right) \right]$$

$$m_a \simeq \frac{\Lambda_{\text{QCD}}^2}{f_a}$$

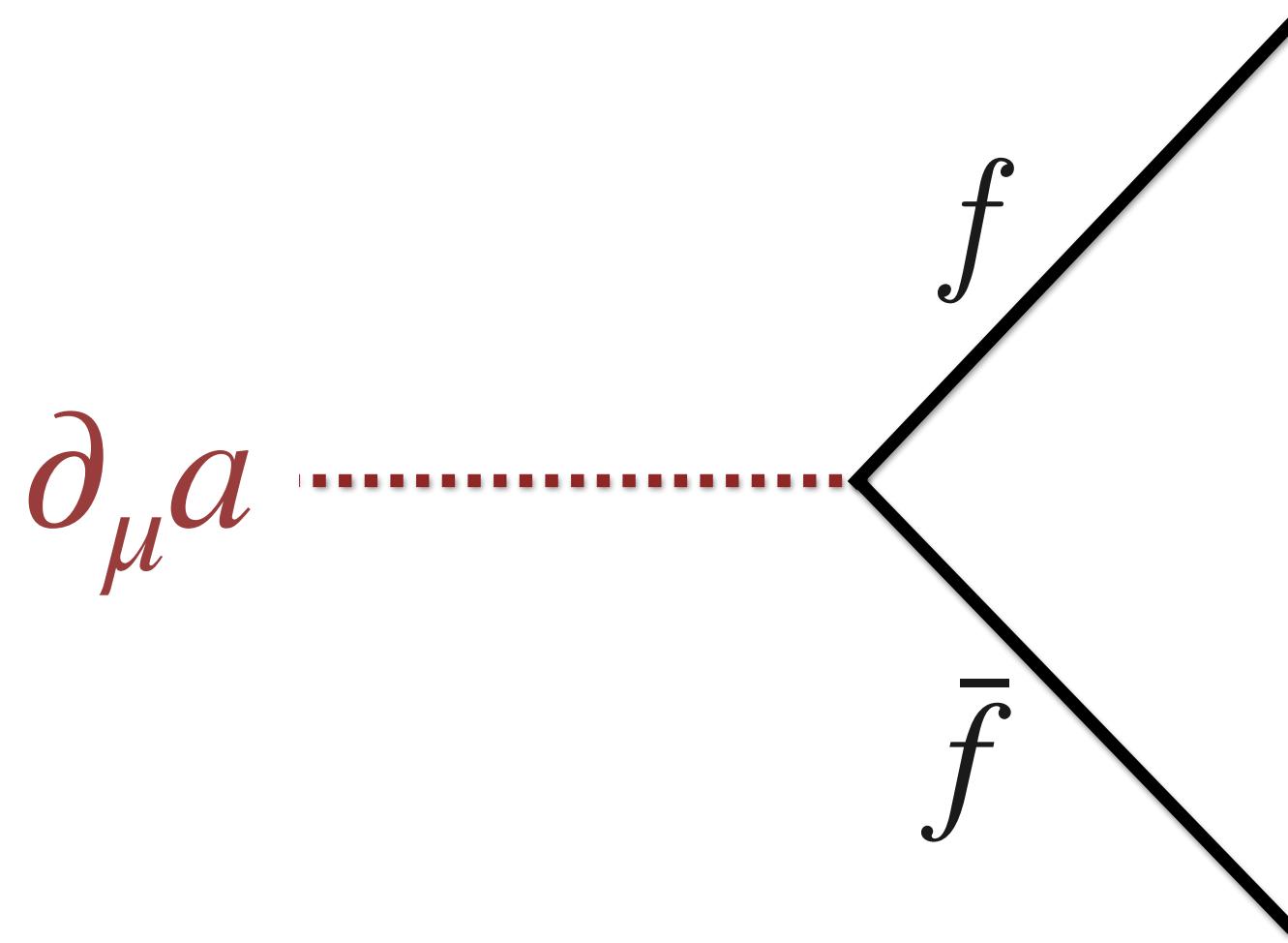


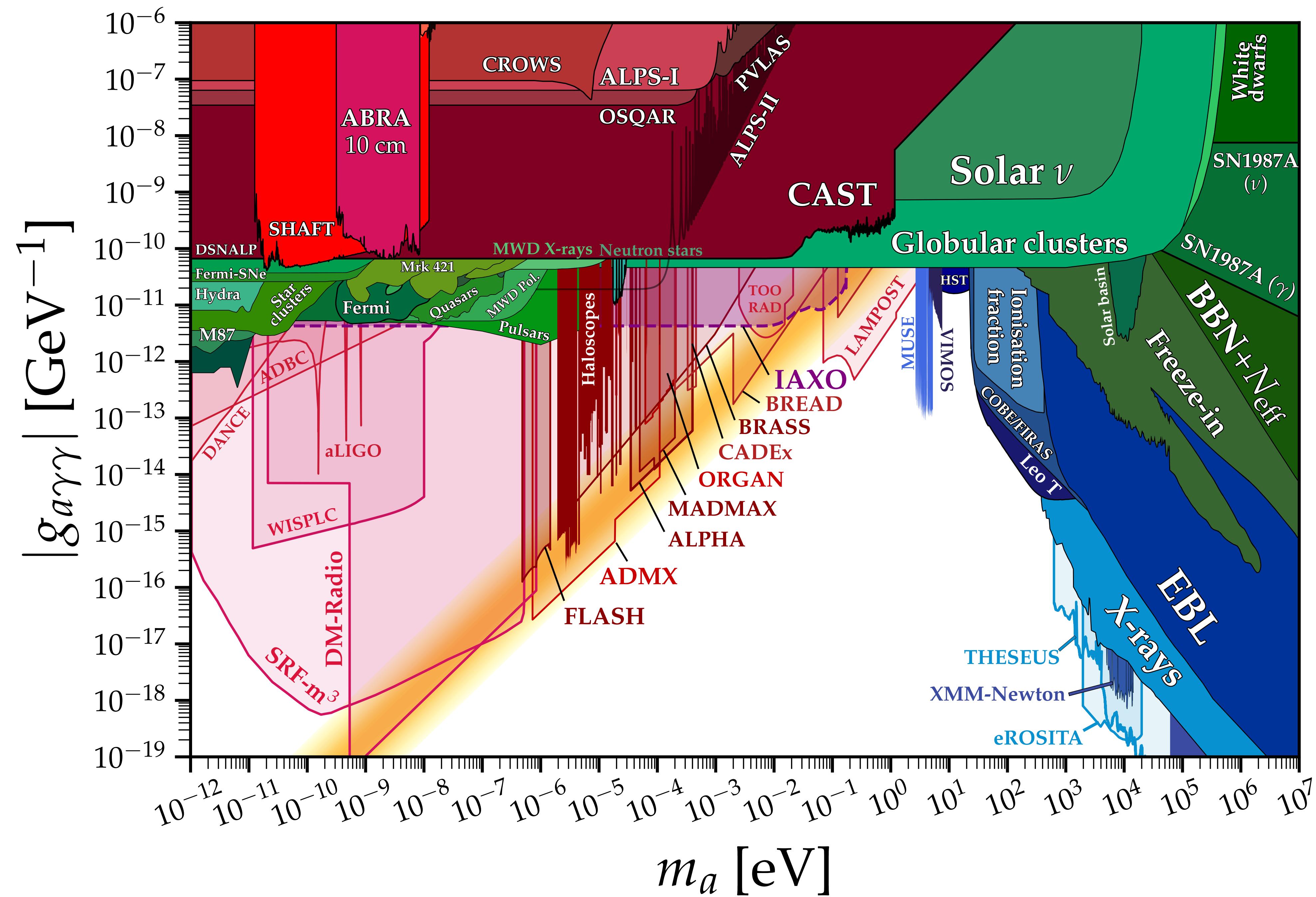


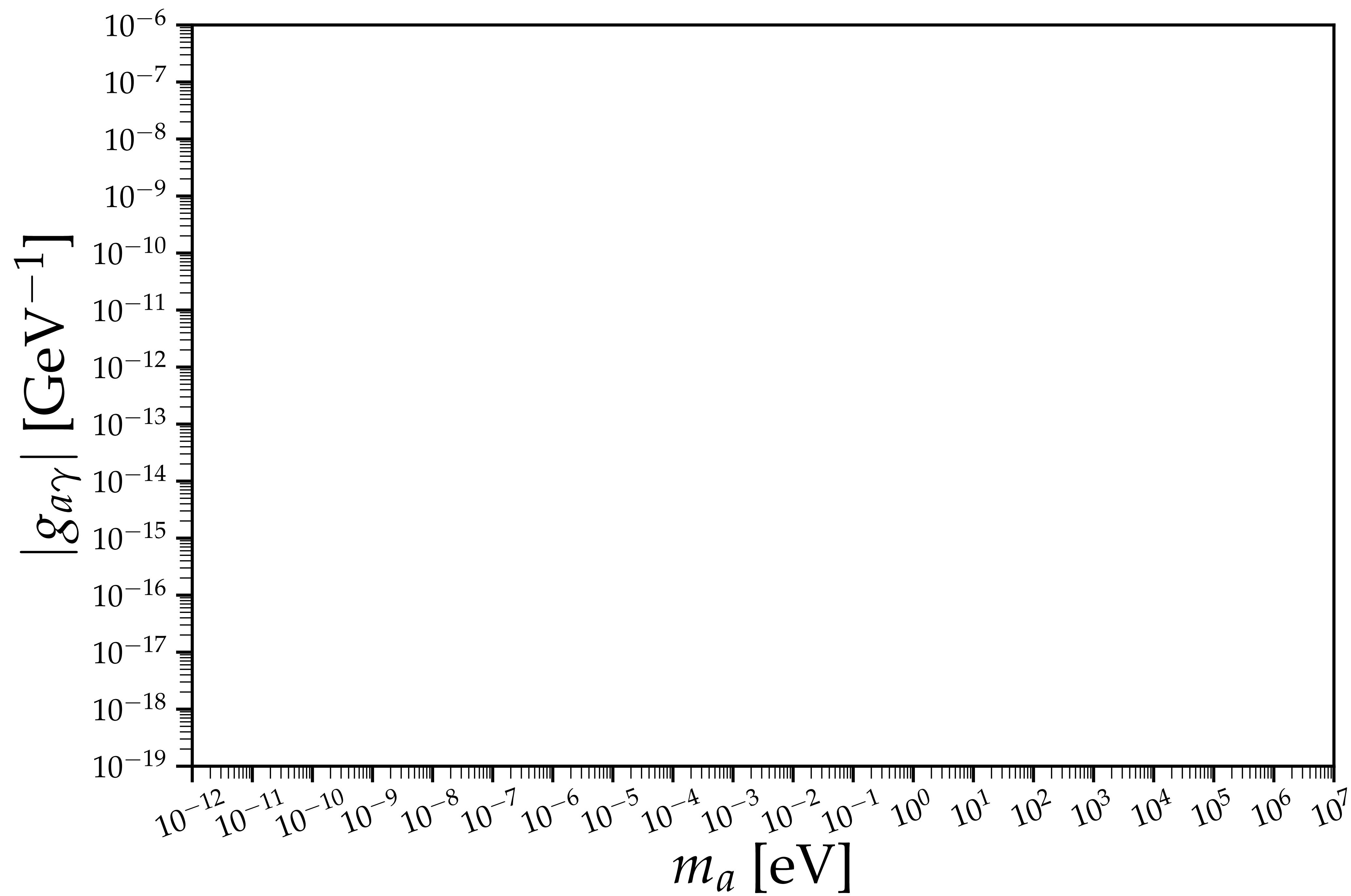
**Axion-photon**

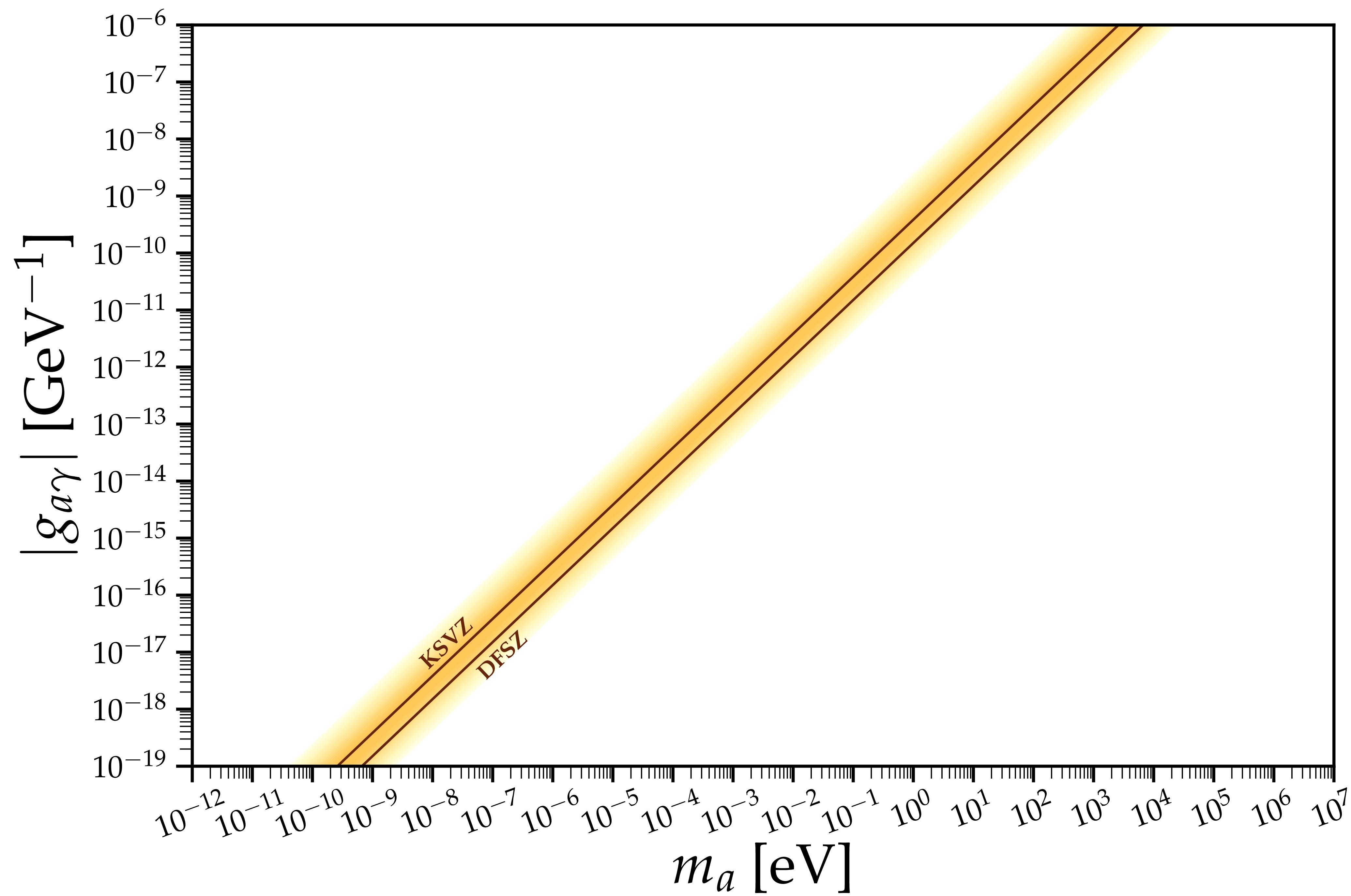


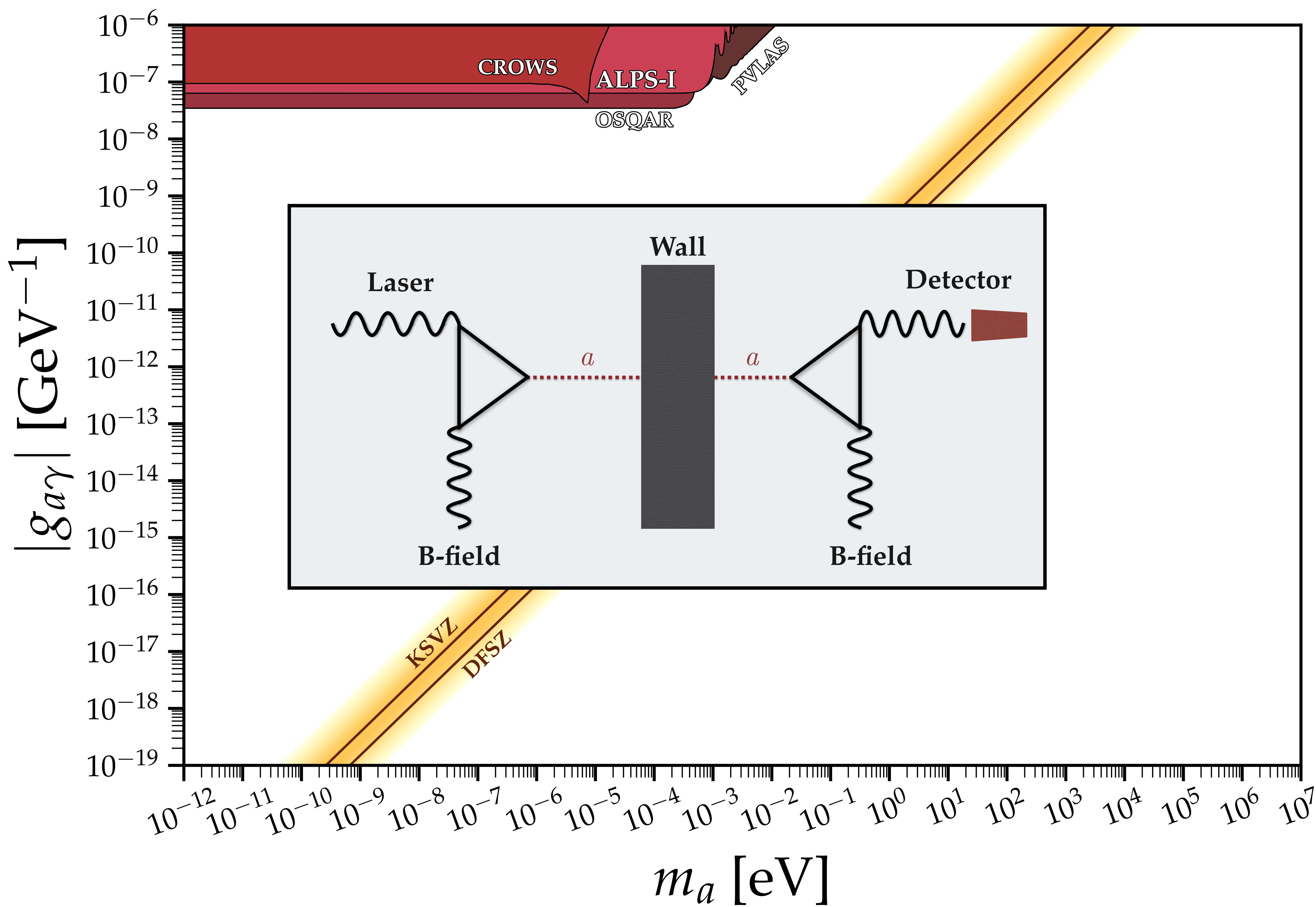
**Axion-fermion**

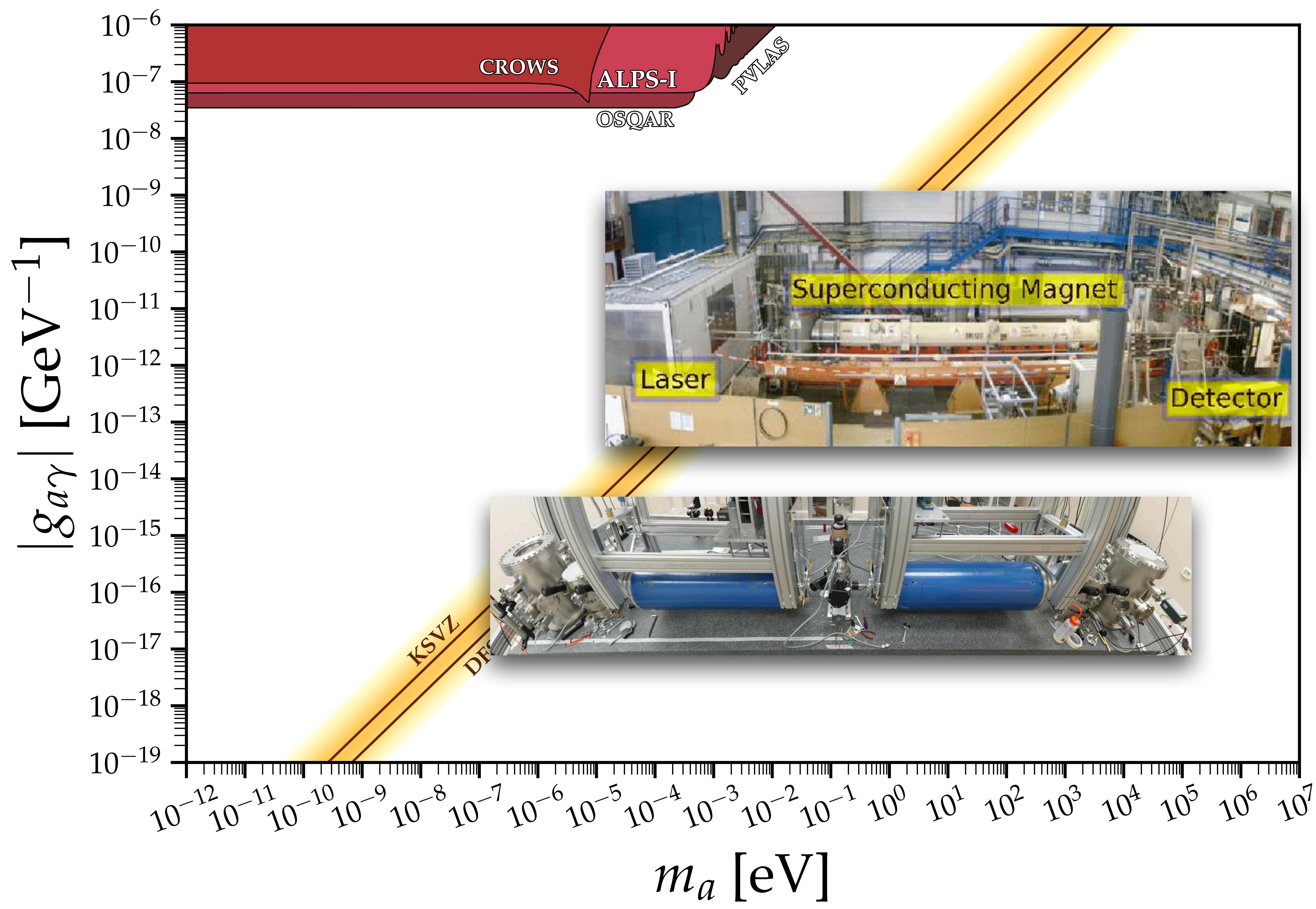


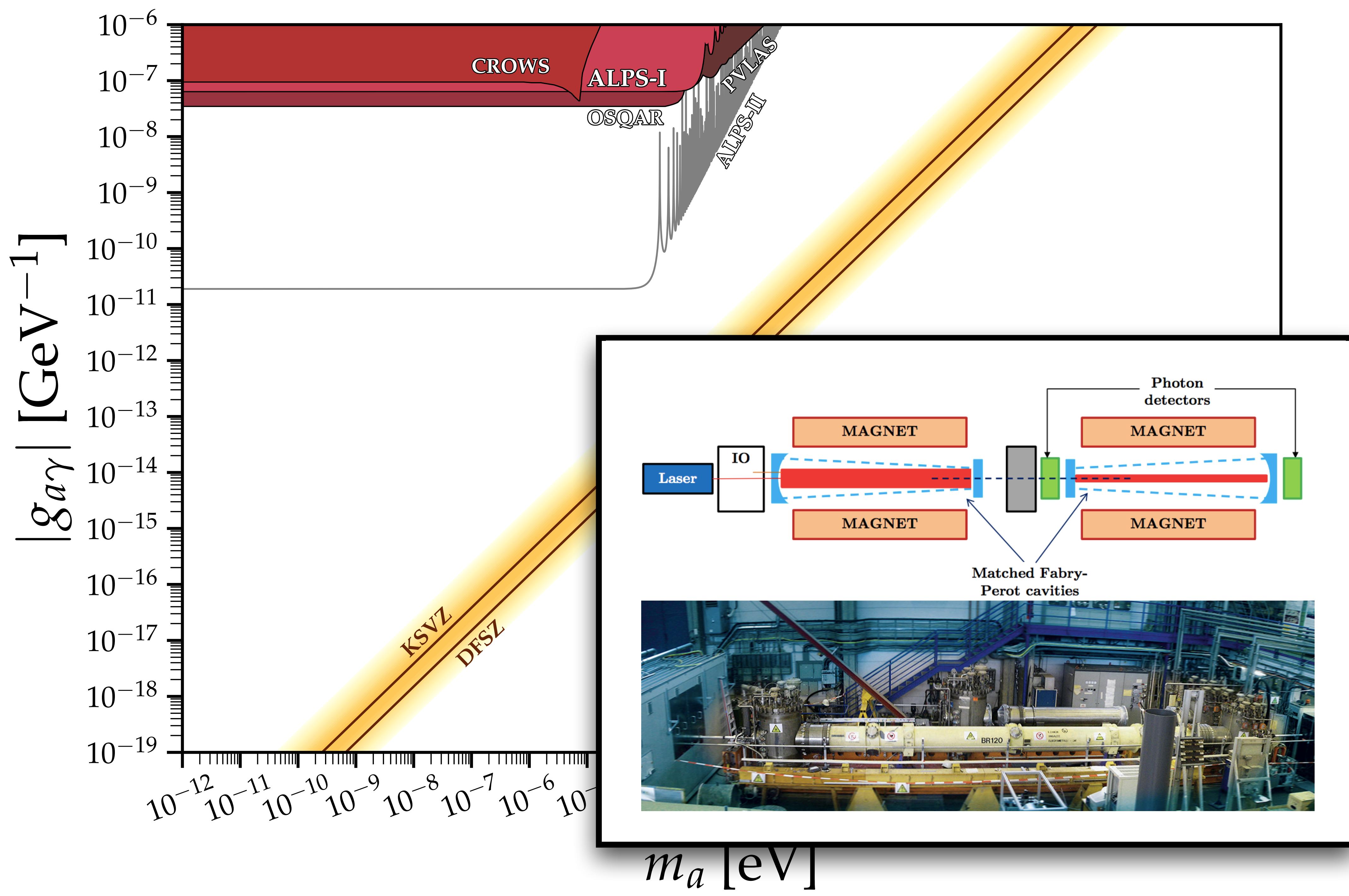


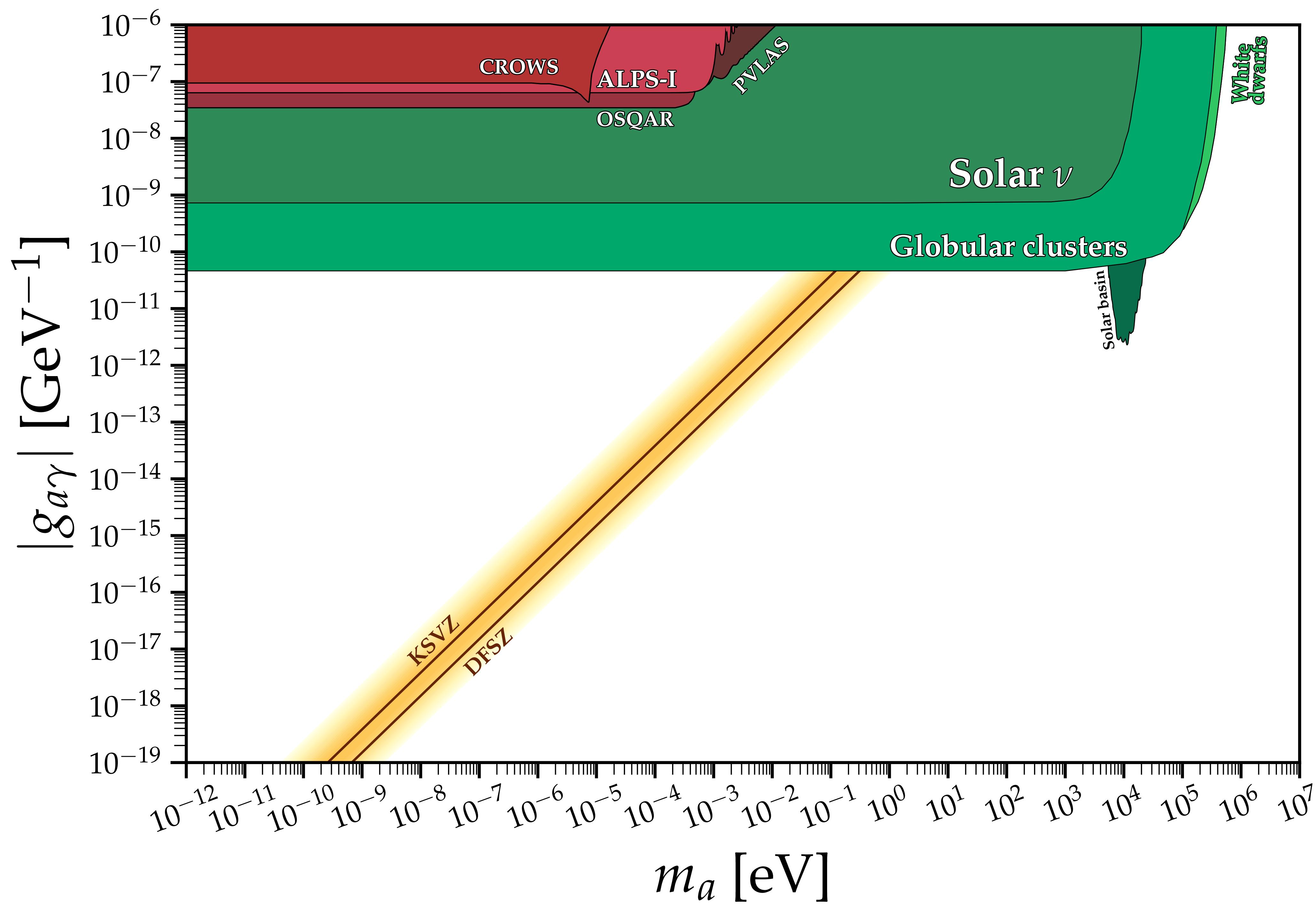


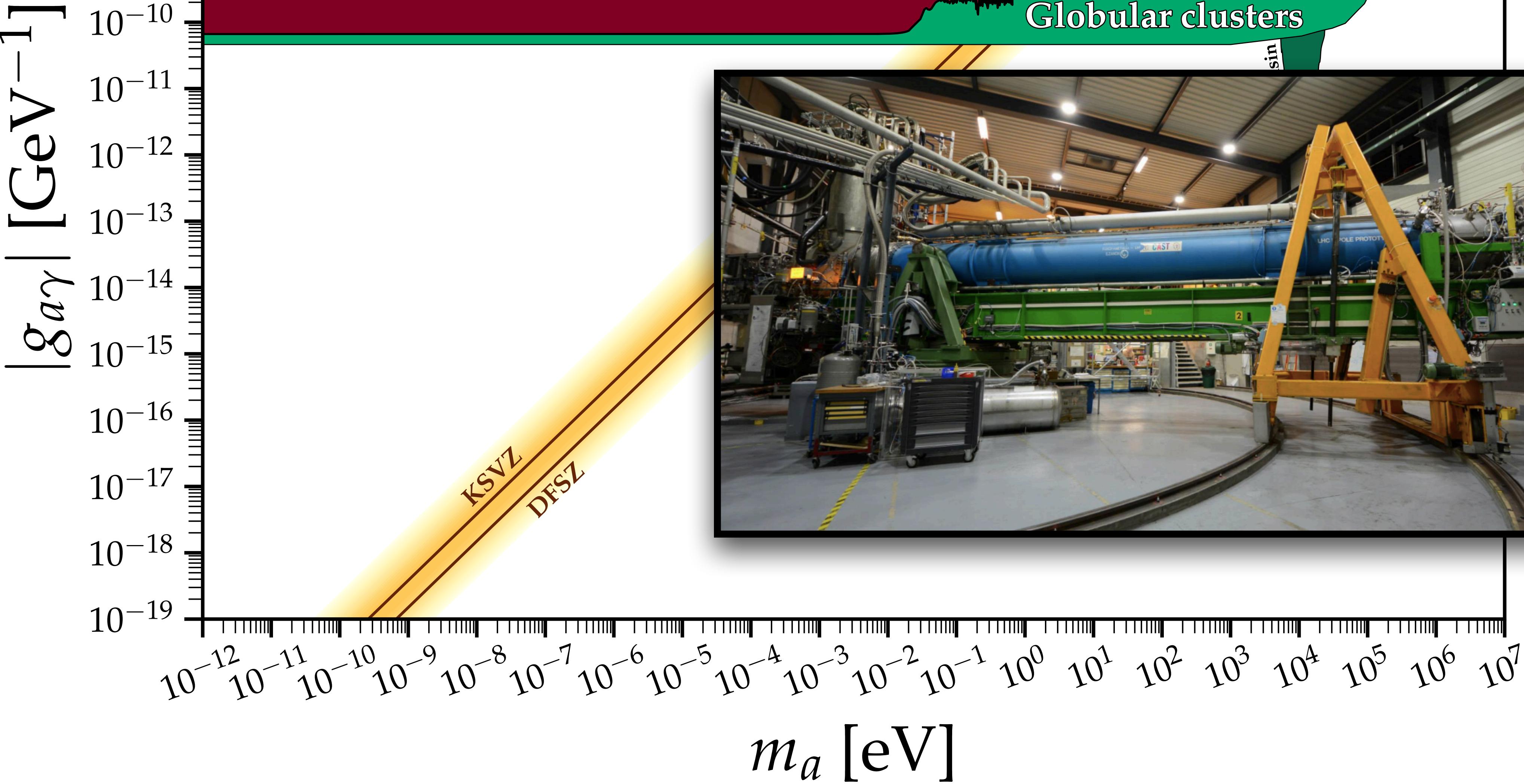
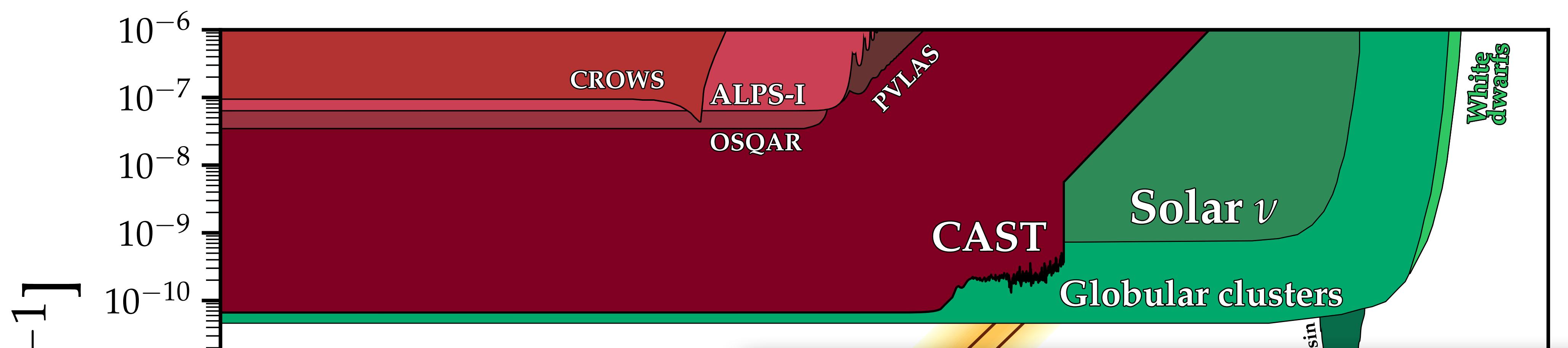


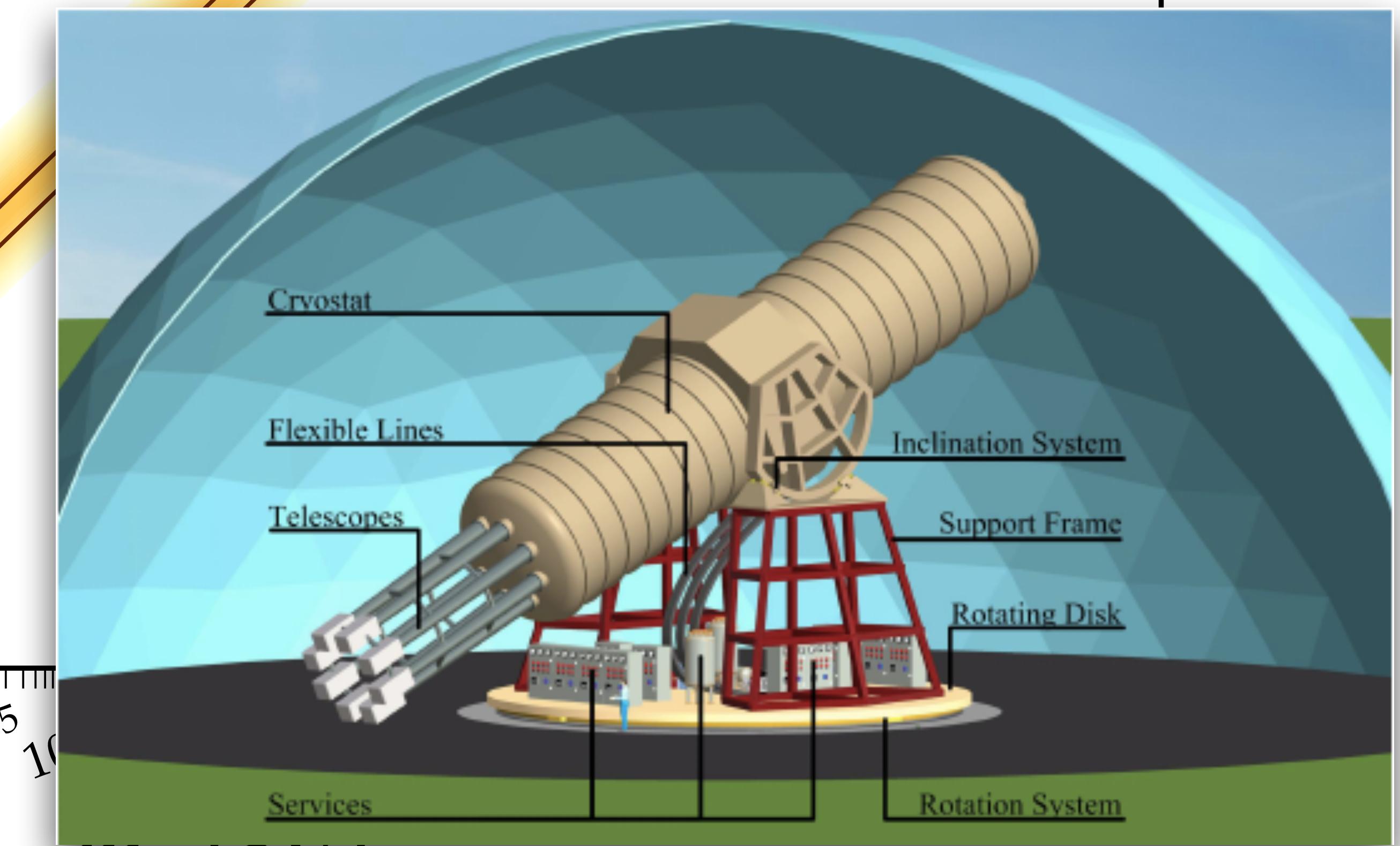
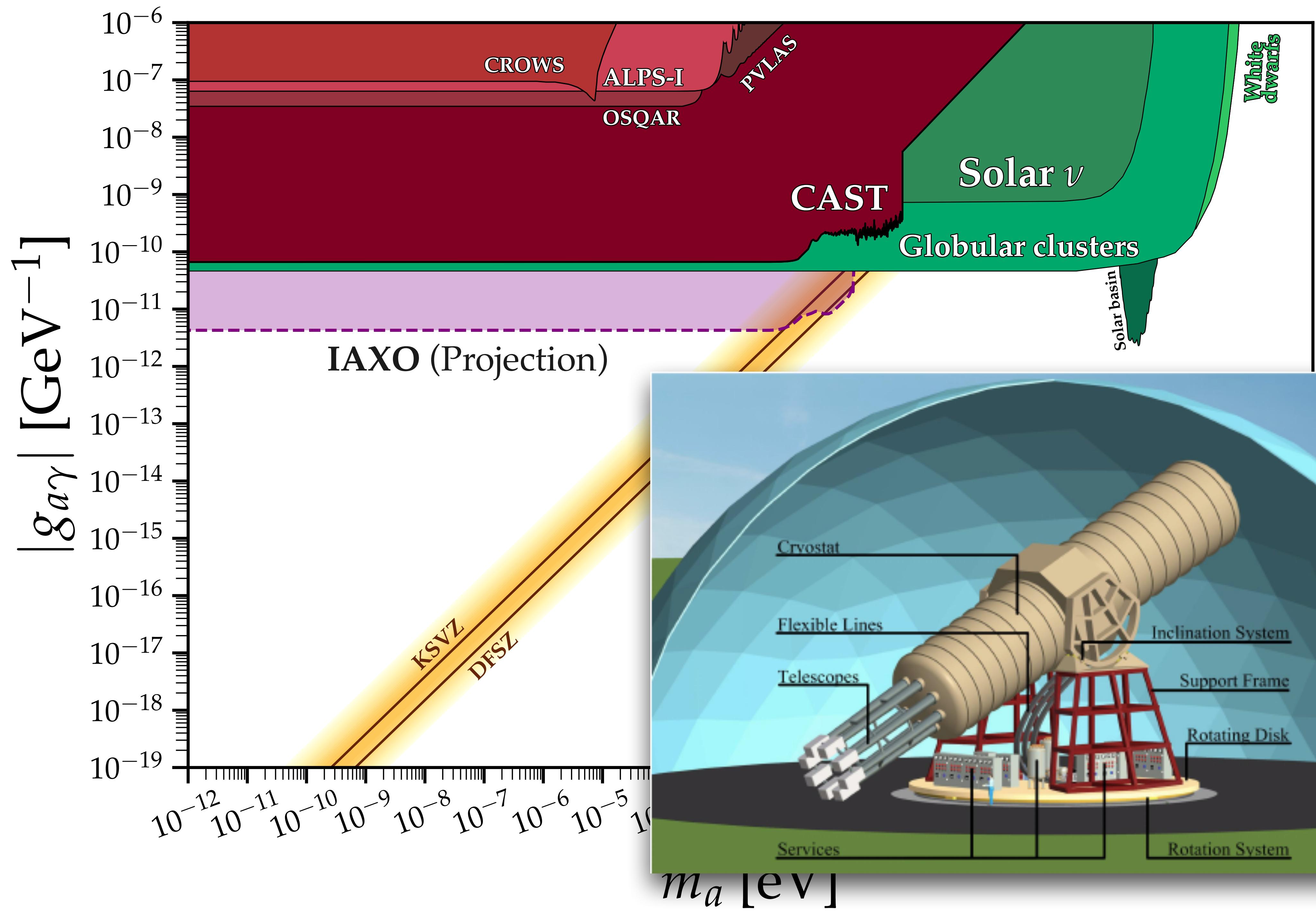




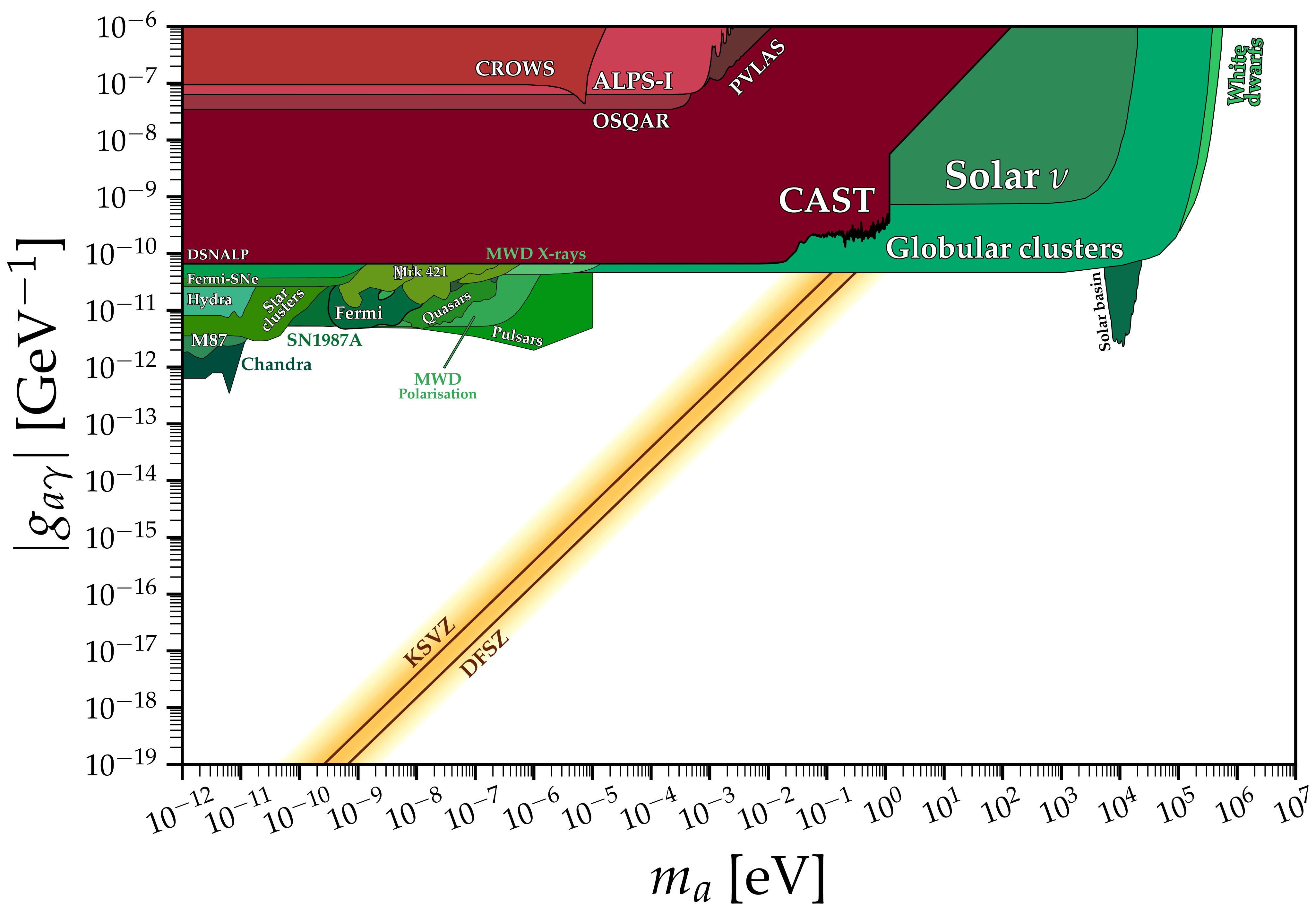


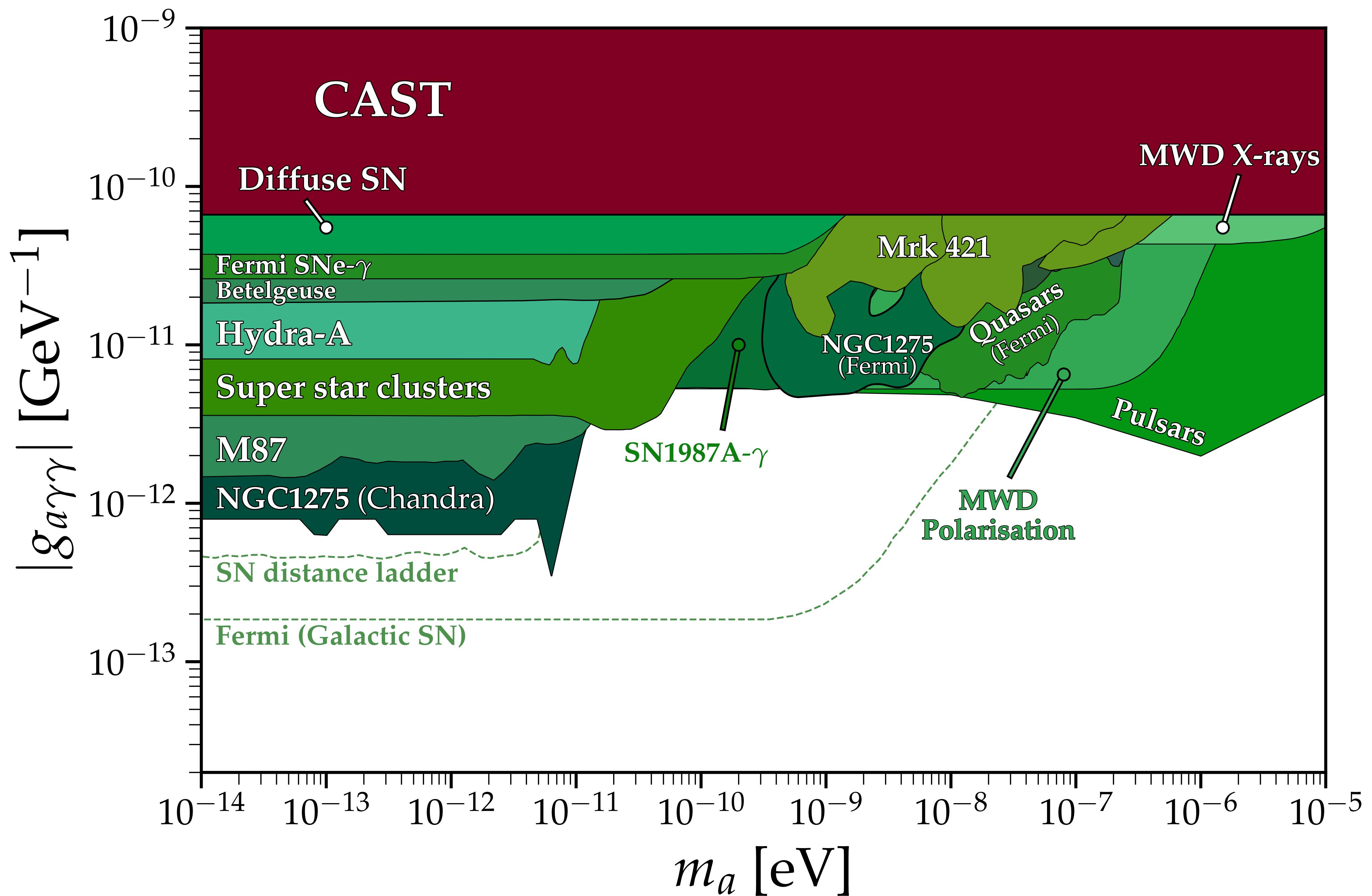


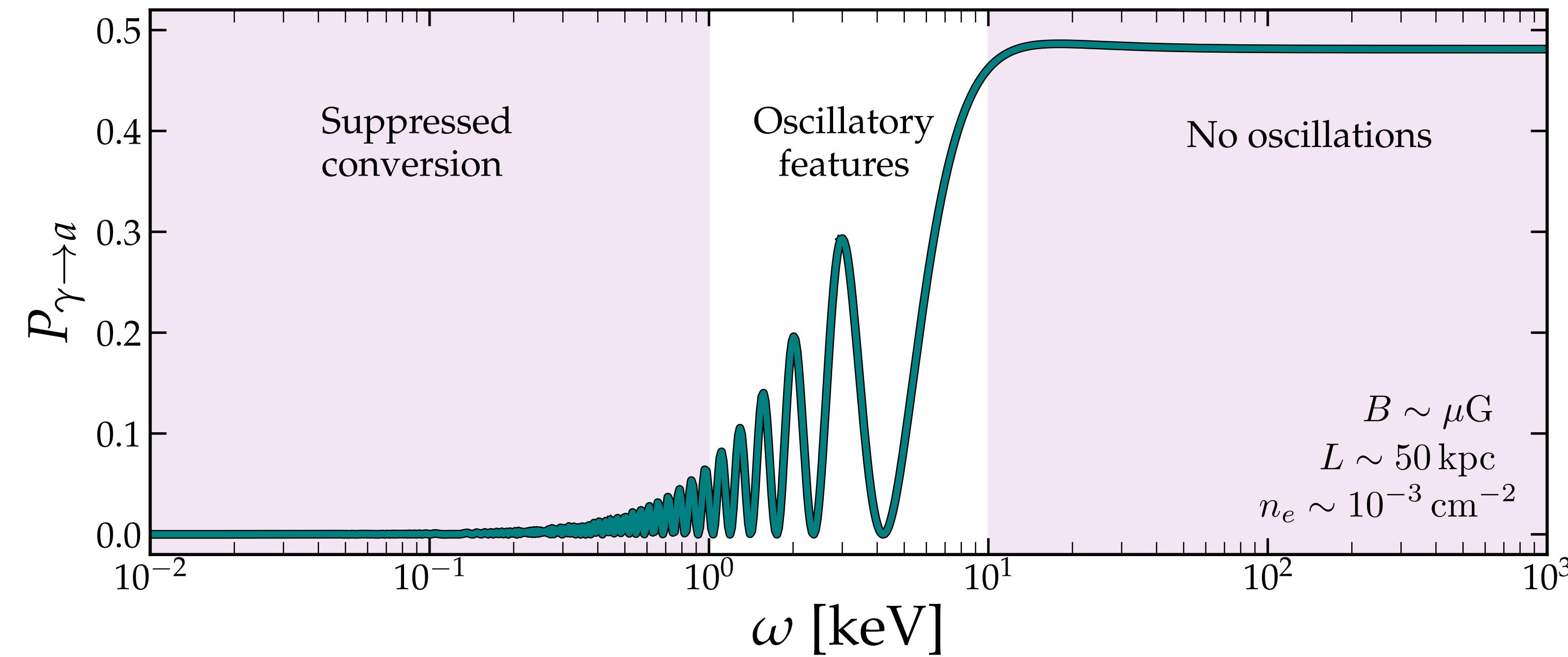
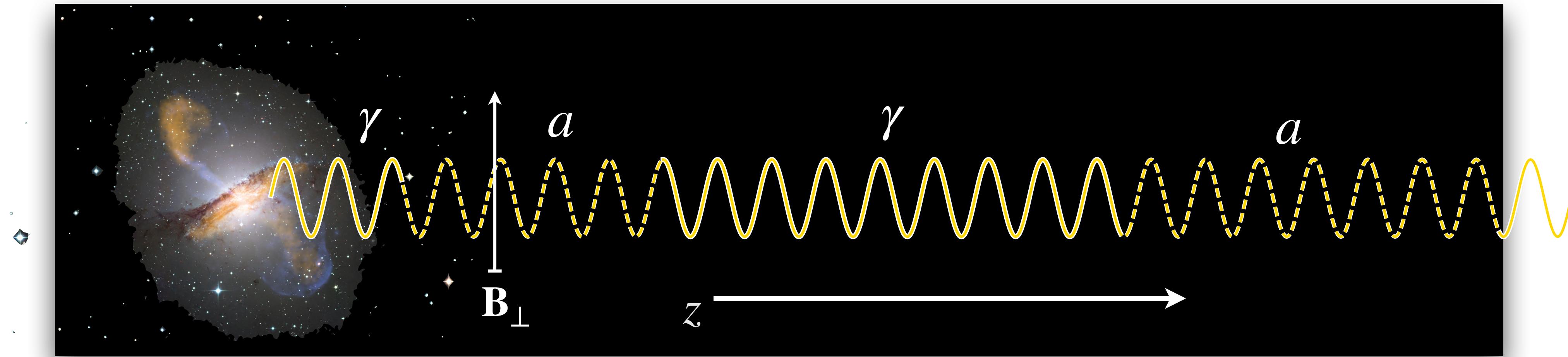


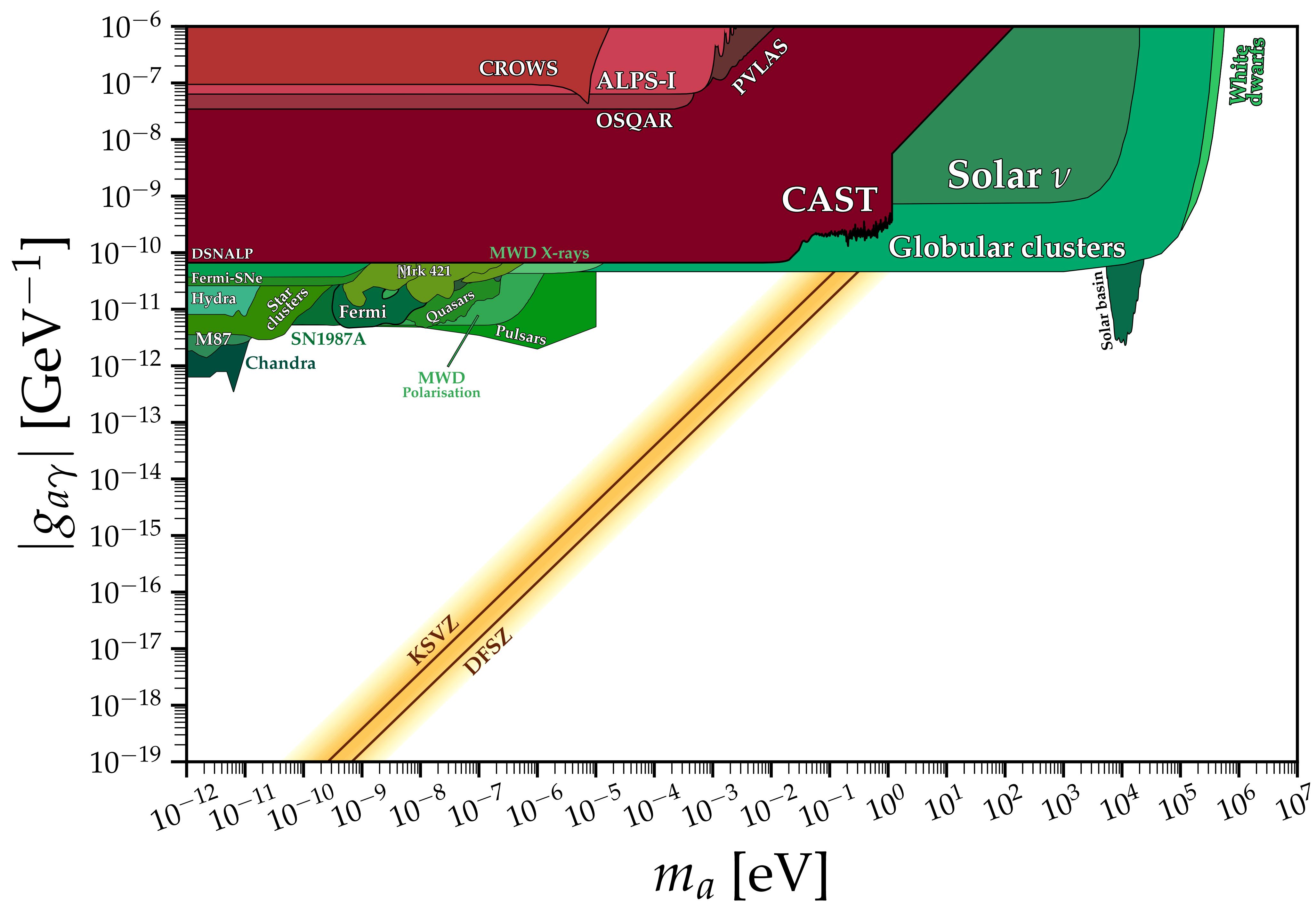


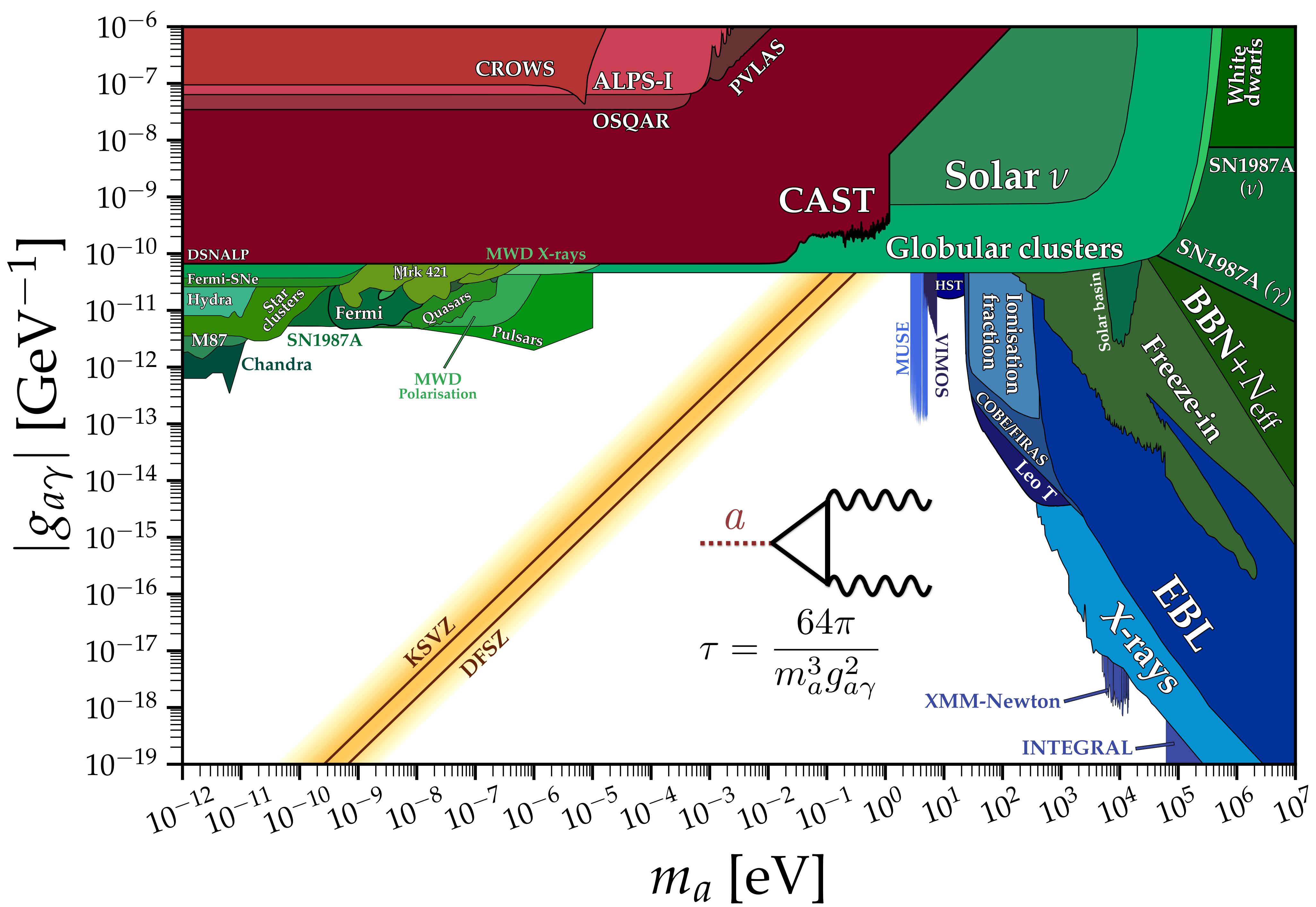
$m_a [\text{eV}]$

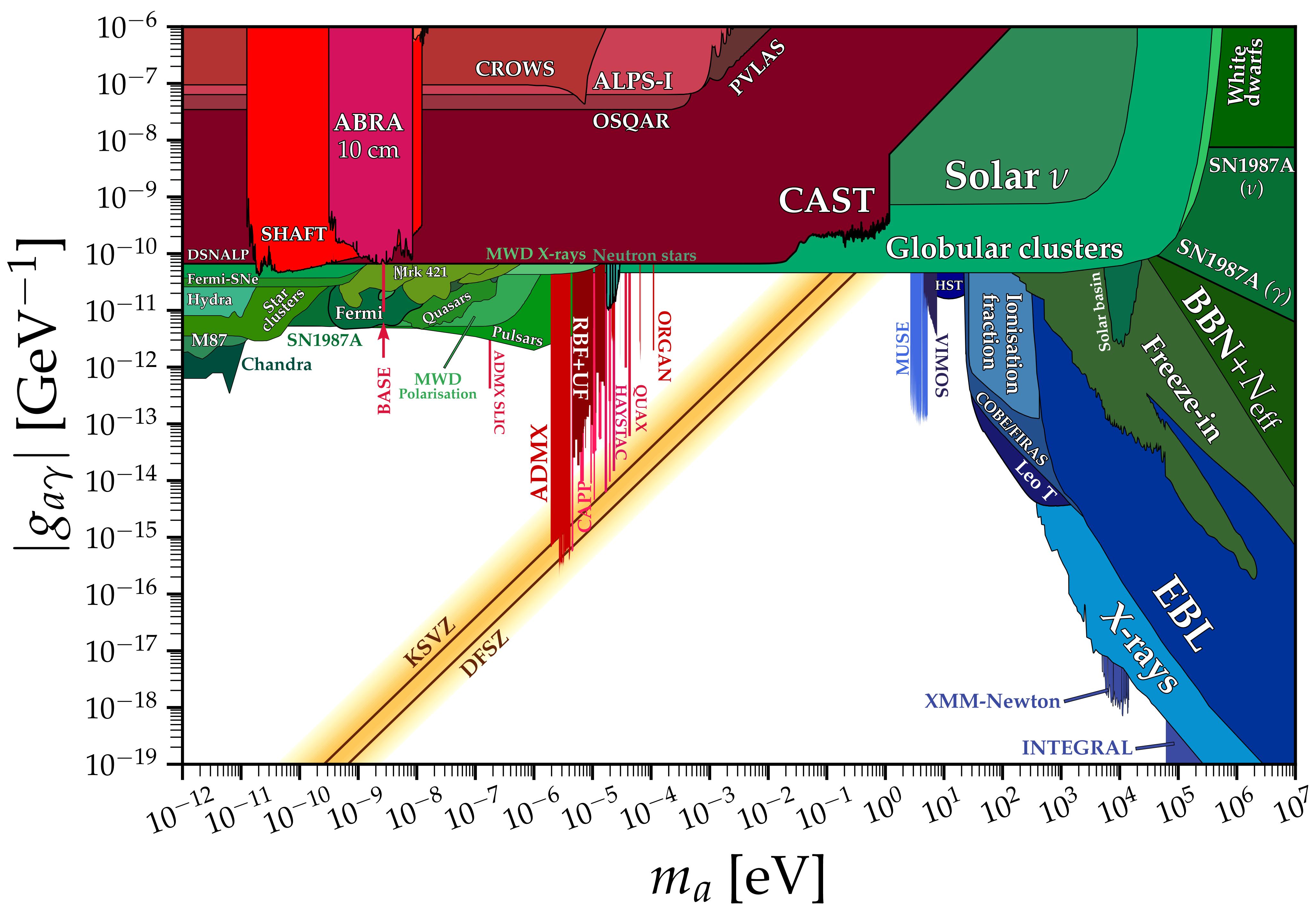


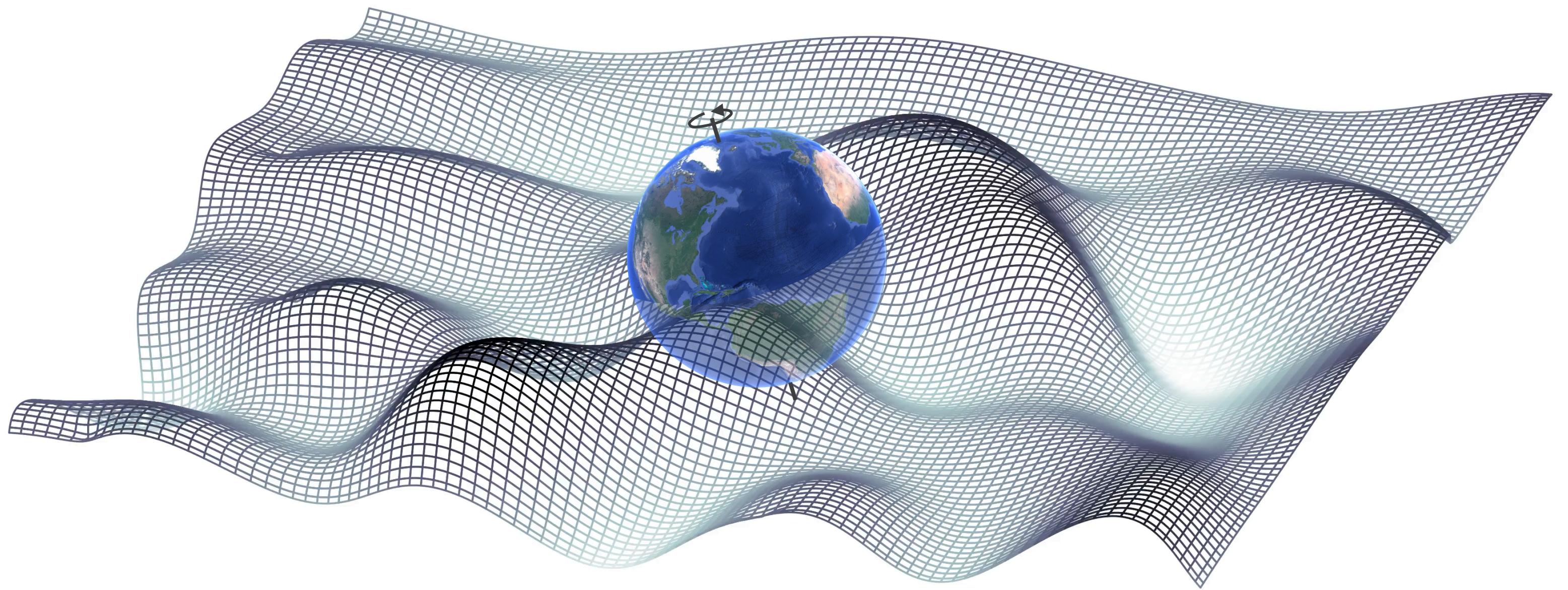






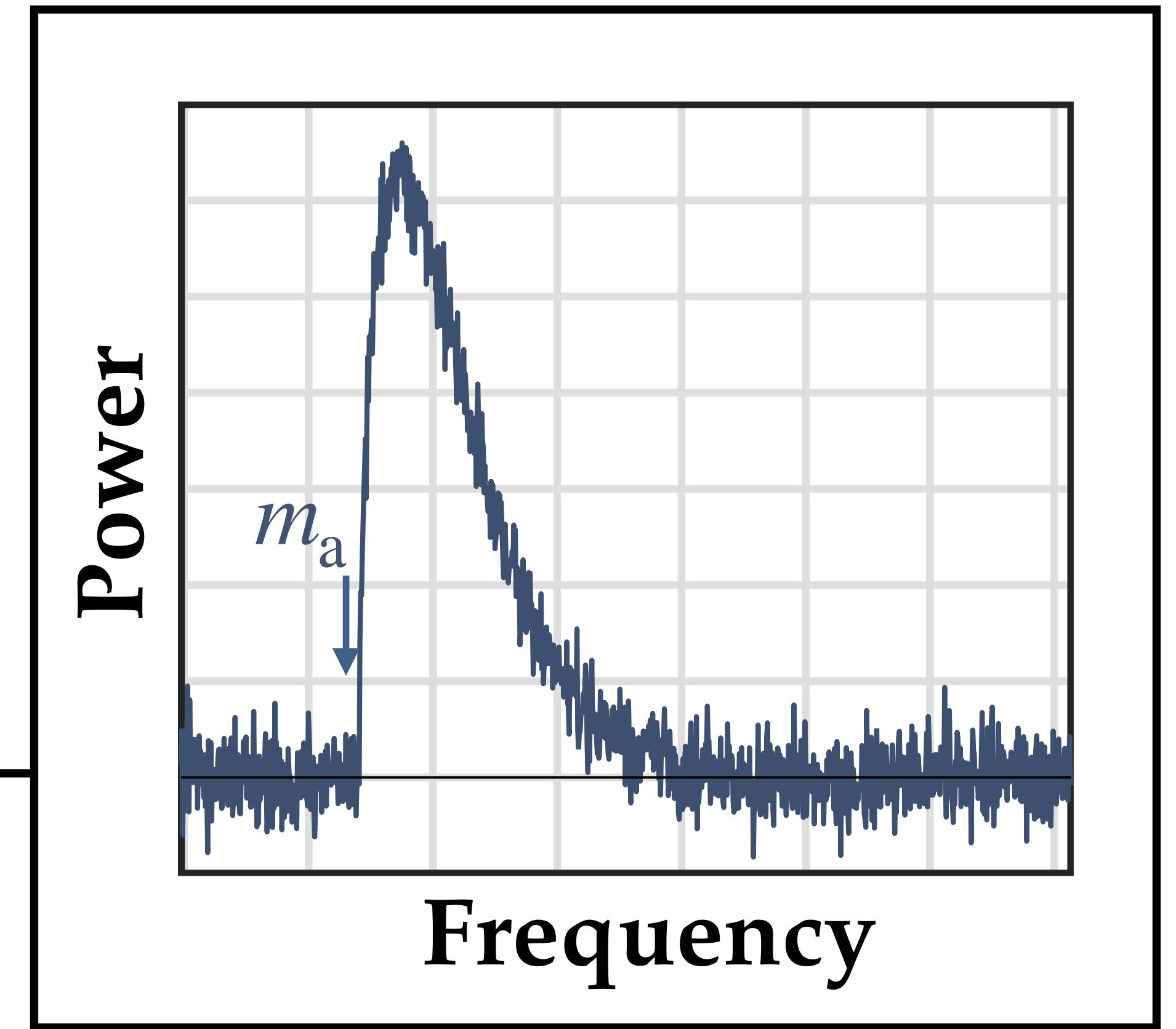
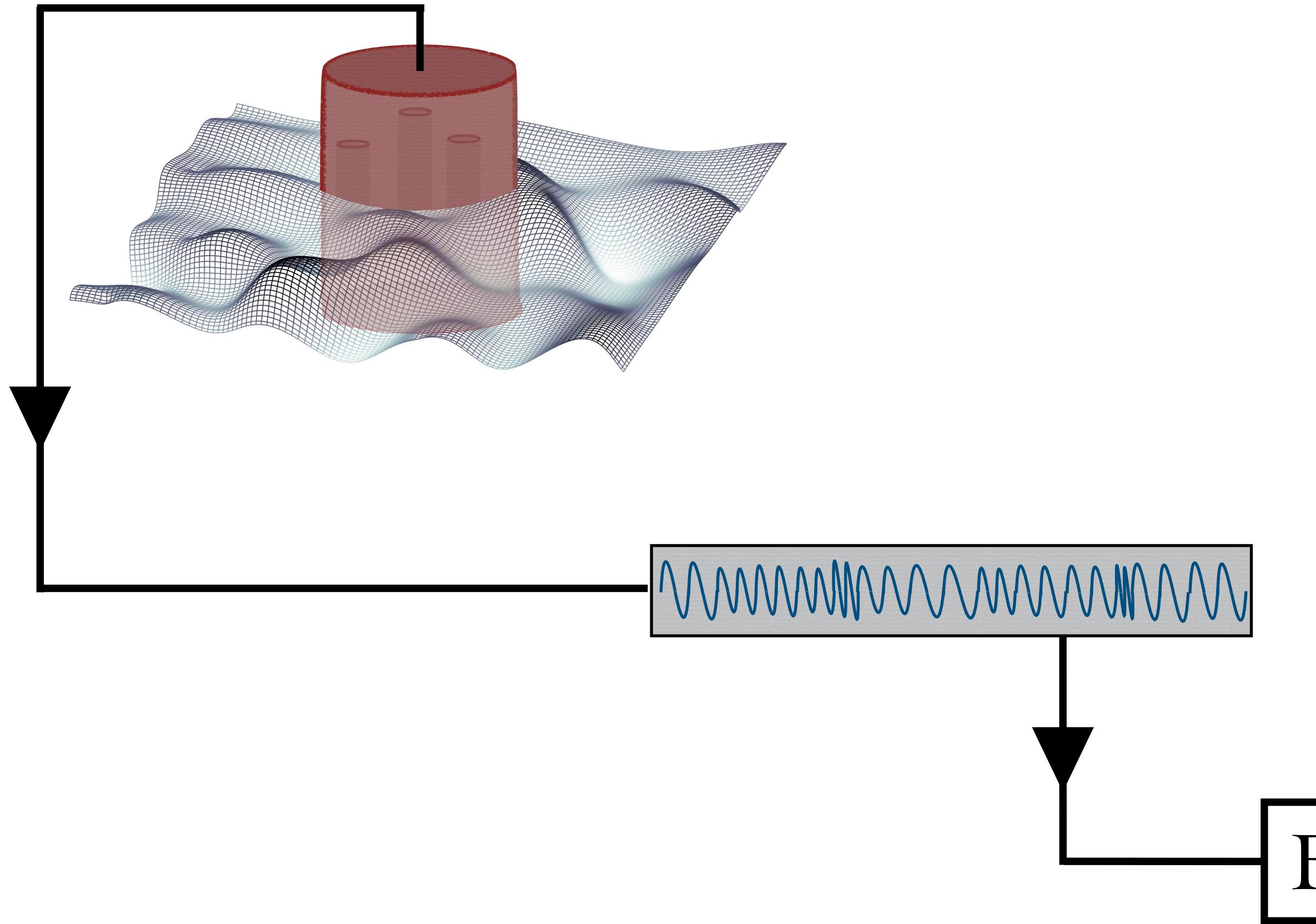


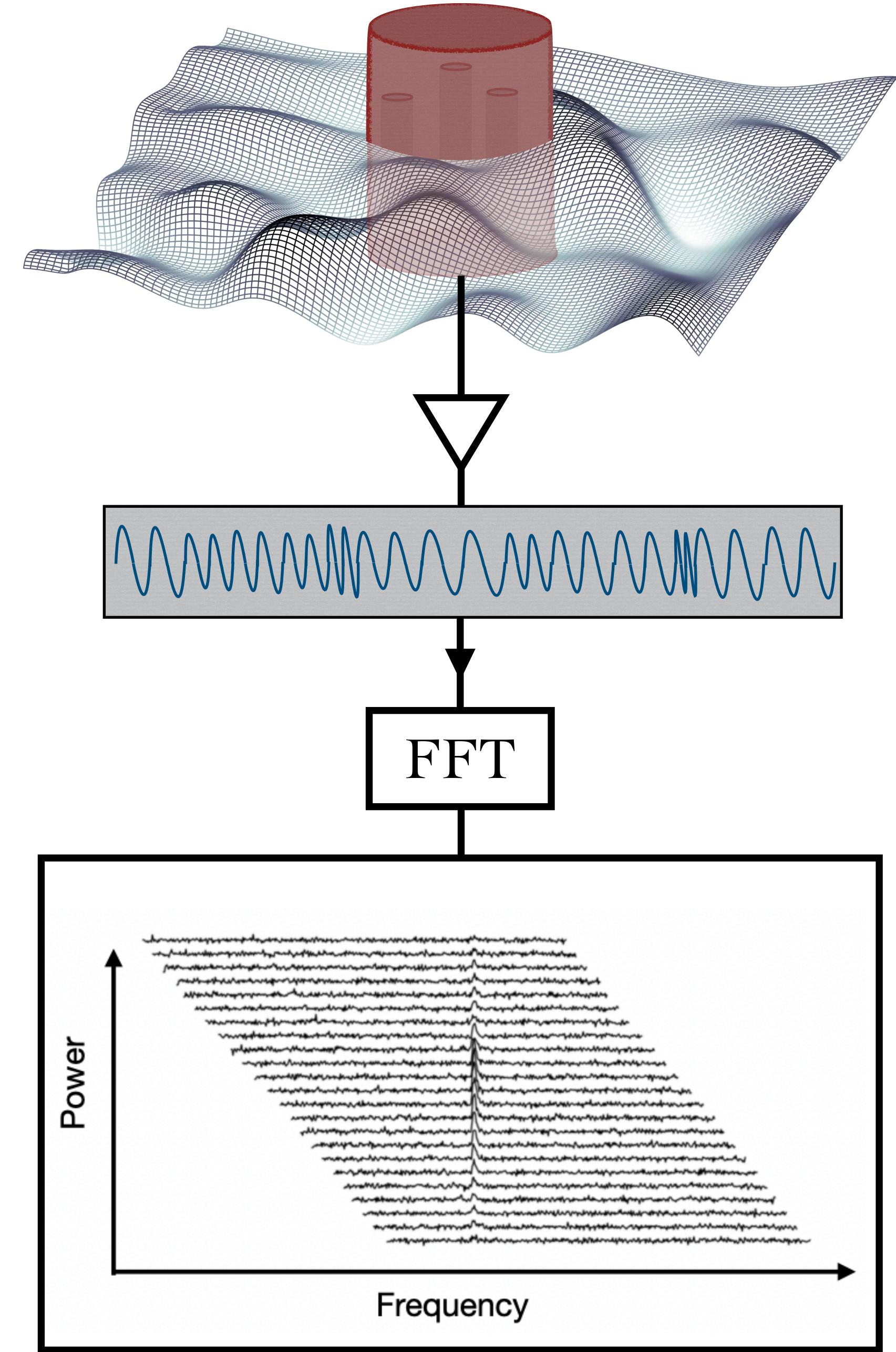
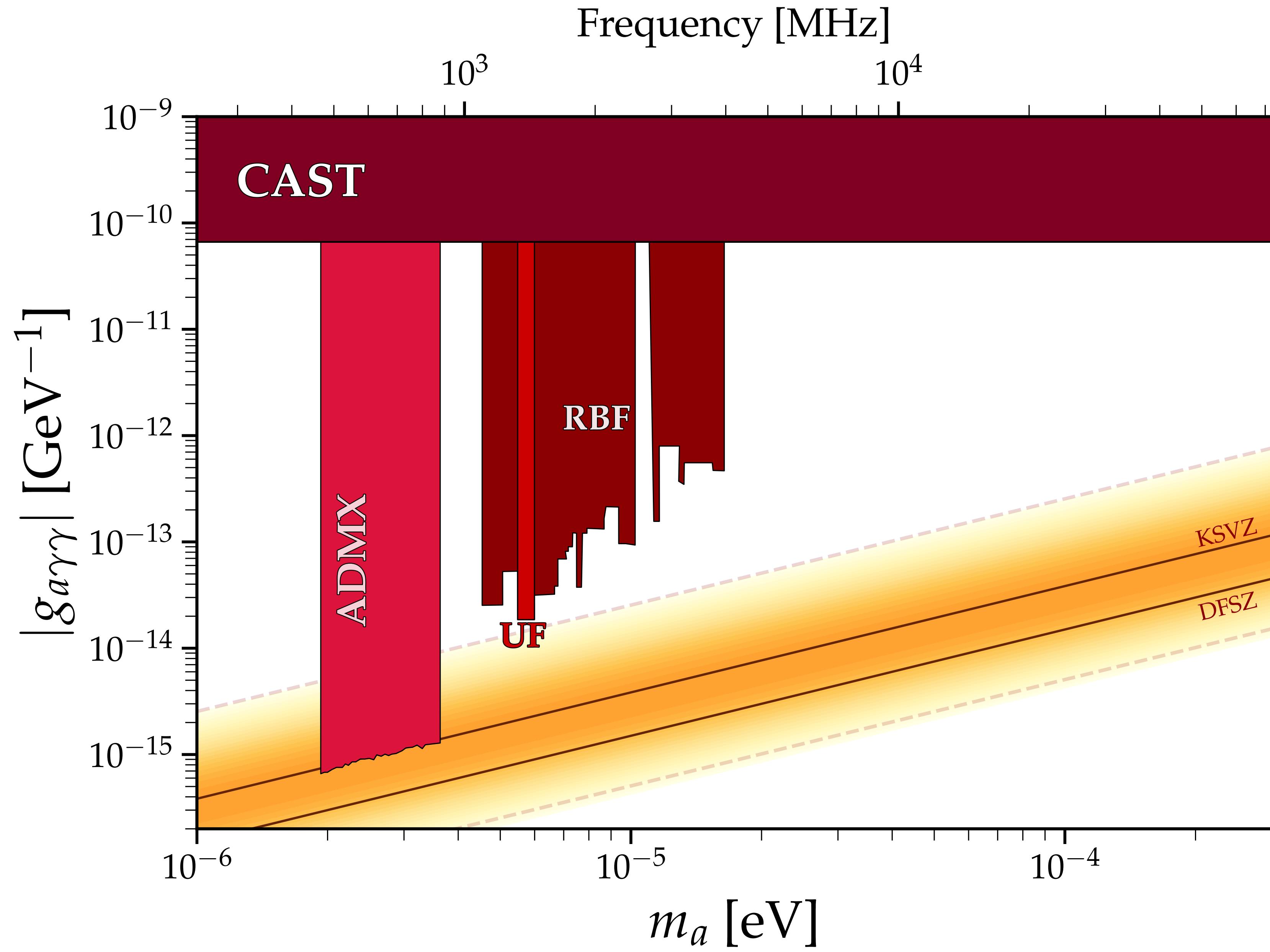


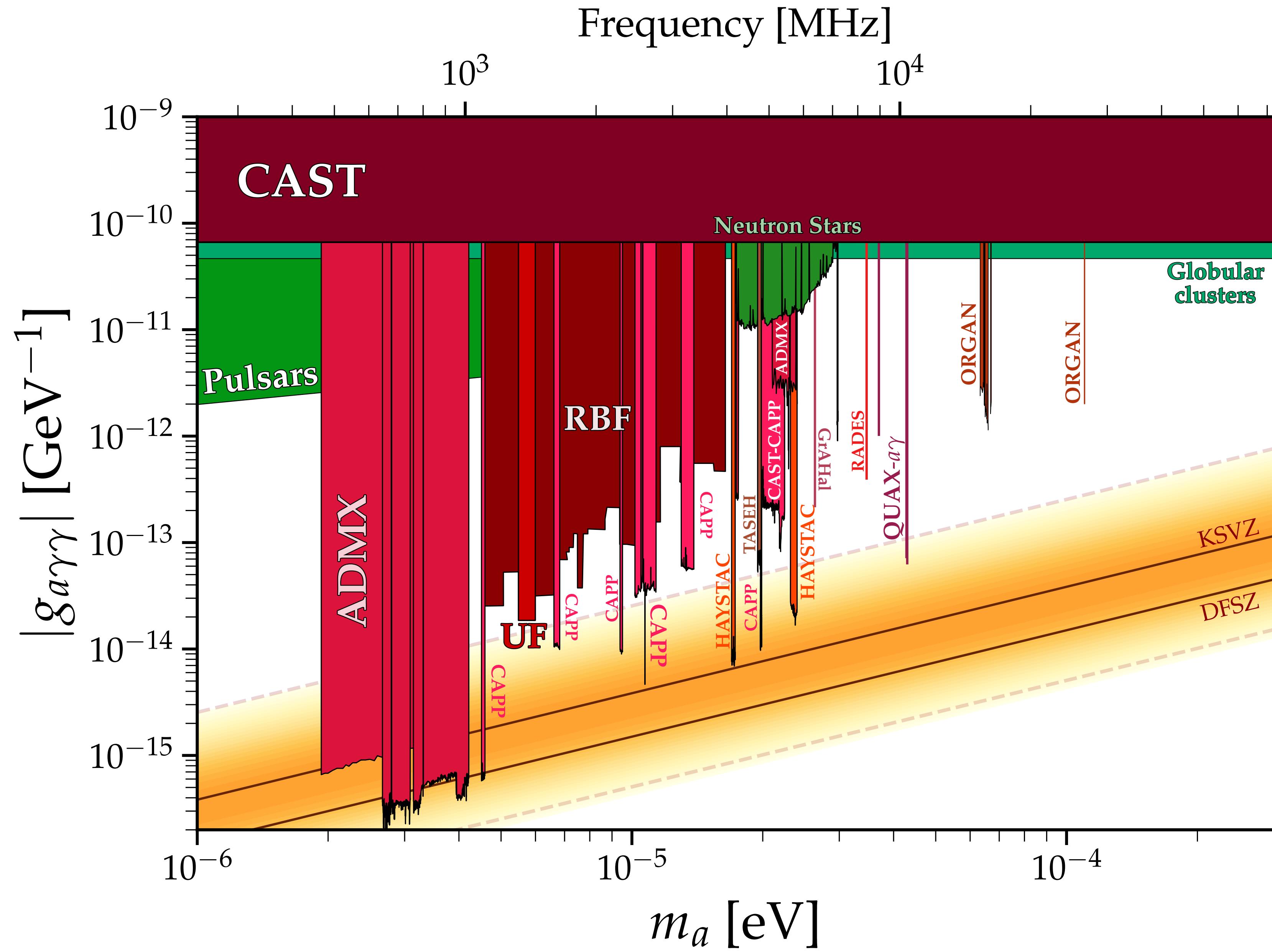


$$\textbf{Amplitude: } A = \frac{\sqrt{2\rho_a}}{m_a}$$

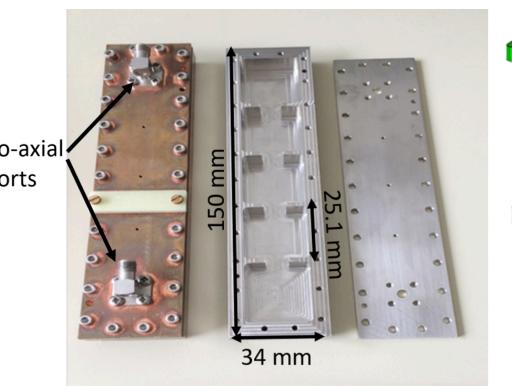
$$\begin{aligned}\textbf{Frequency: } \omega &= m_a + \frac{1}{2}m_a v^2 \\ &\approx m_a(1 + 10^{-6})\end{aligned}$$



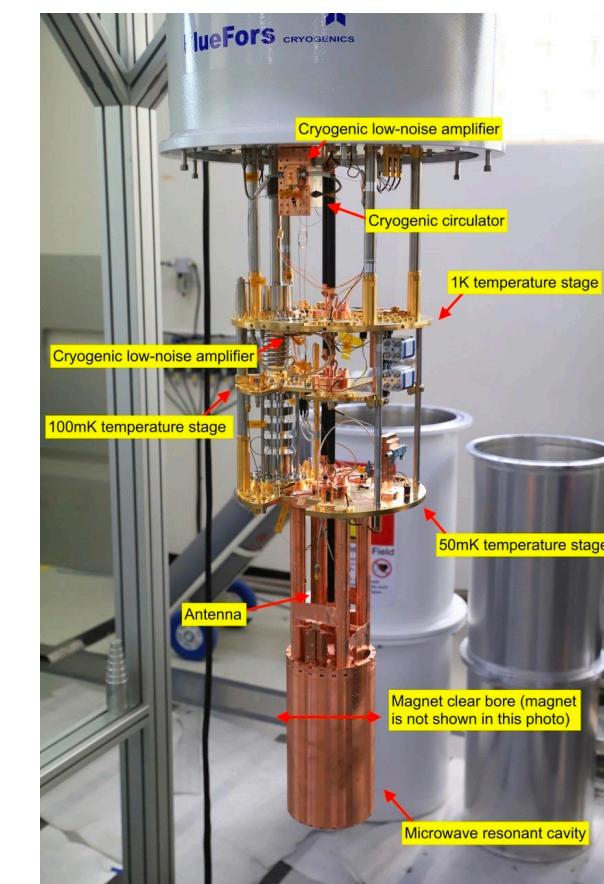




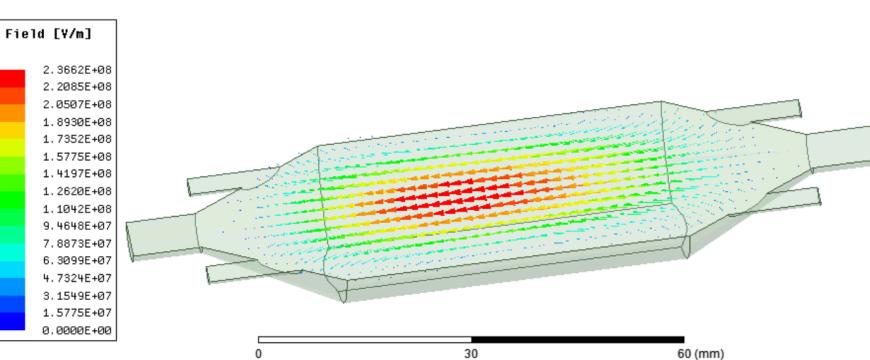
RADES (CERN)



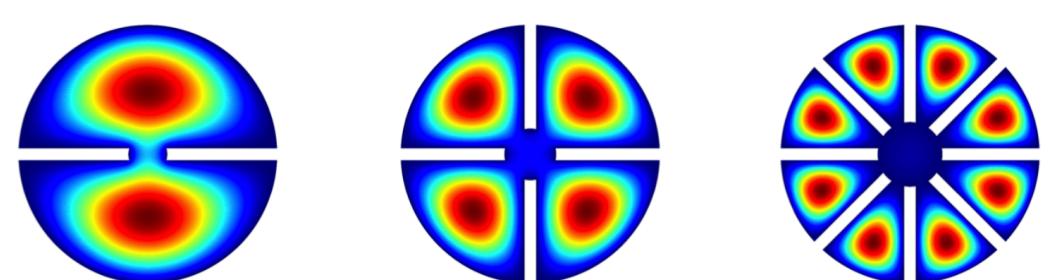
ORGAN (UWA)



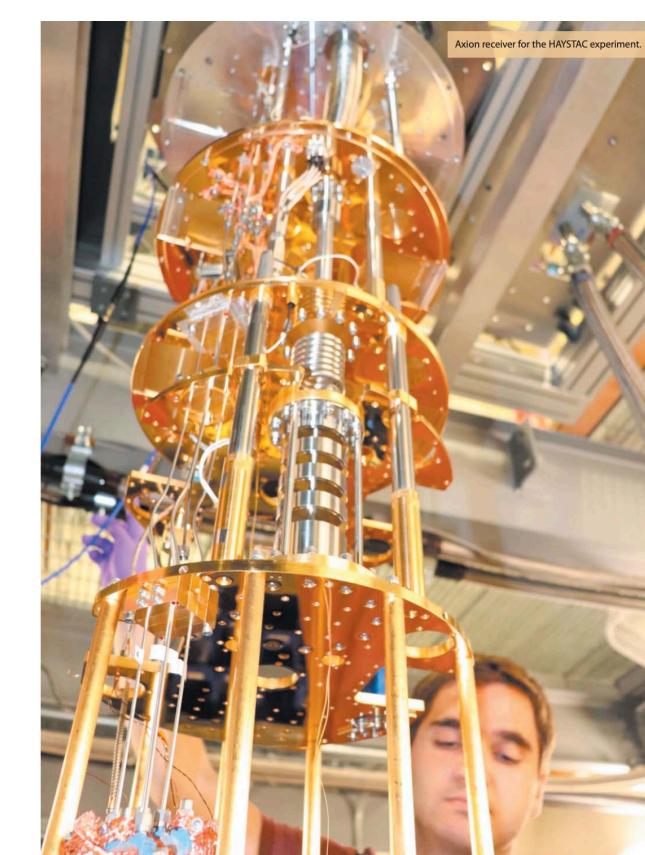
QUAX (Legnaro)



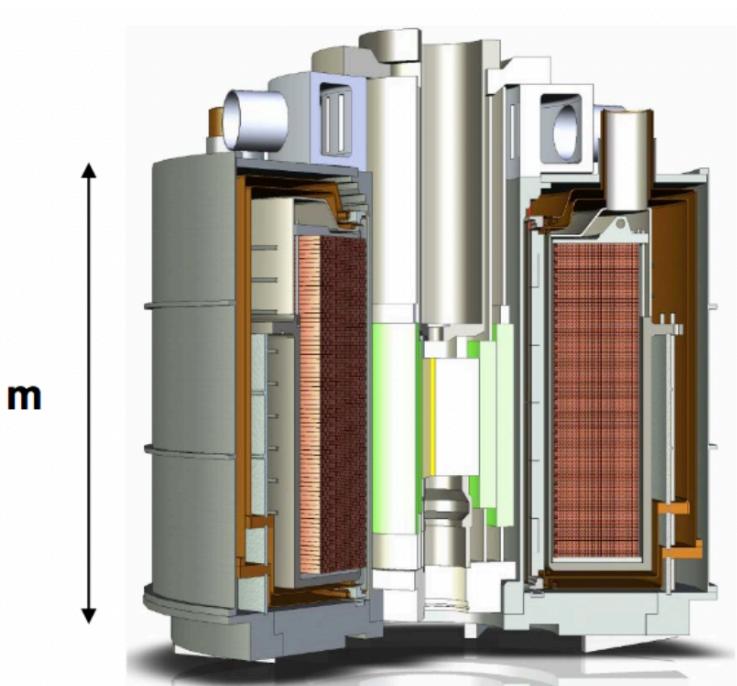
CAPP (IBS)

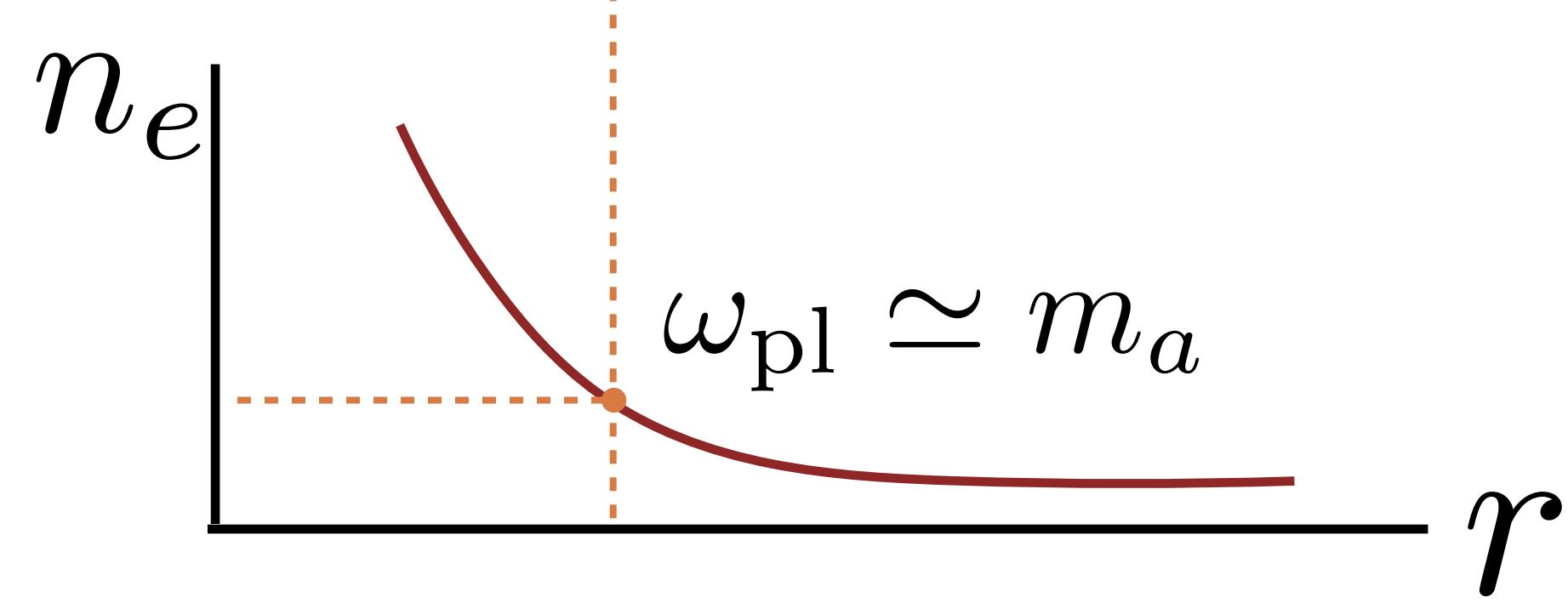
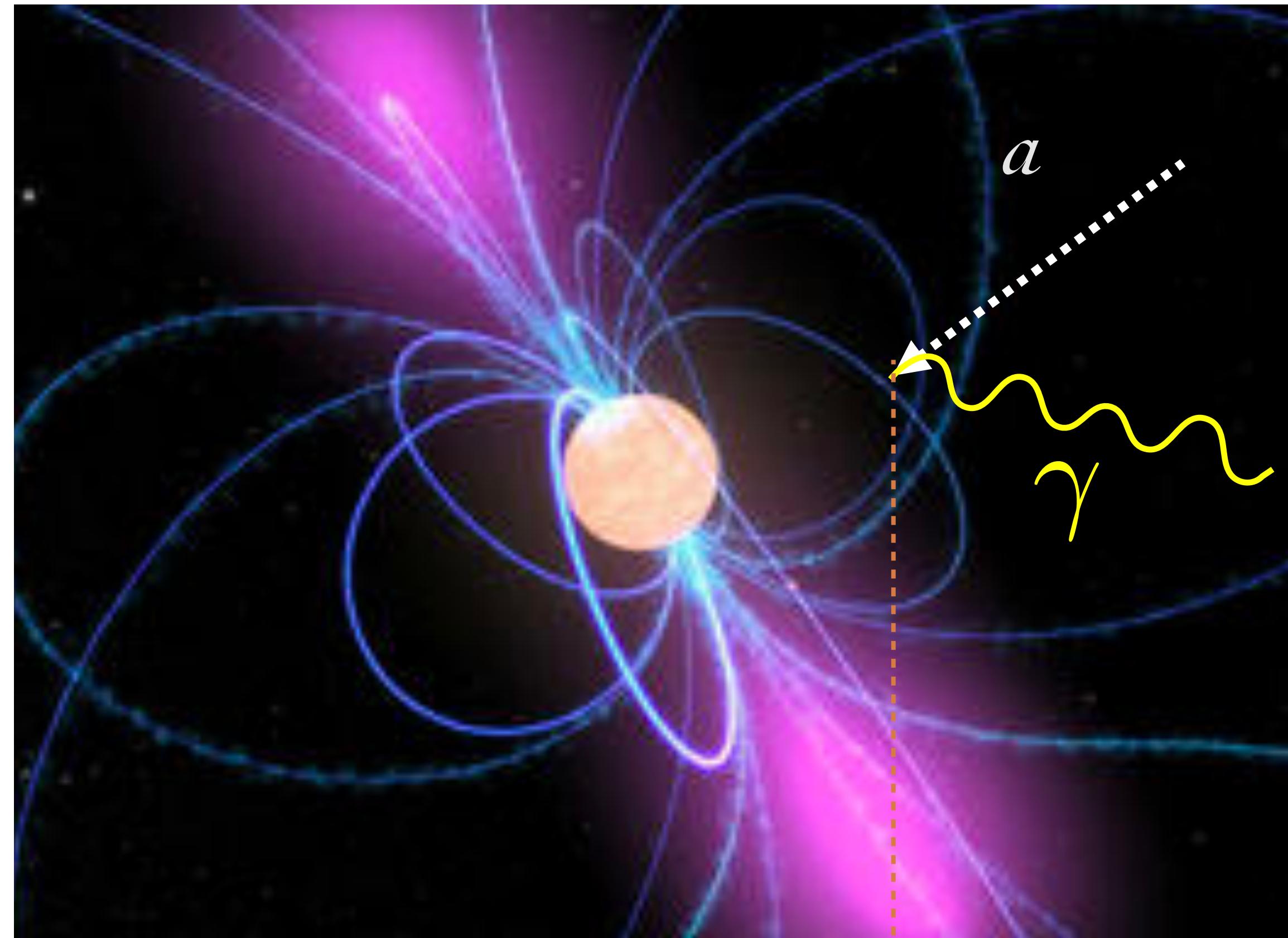


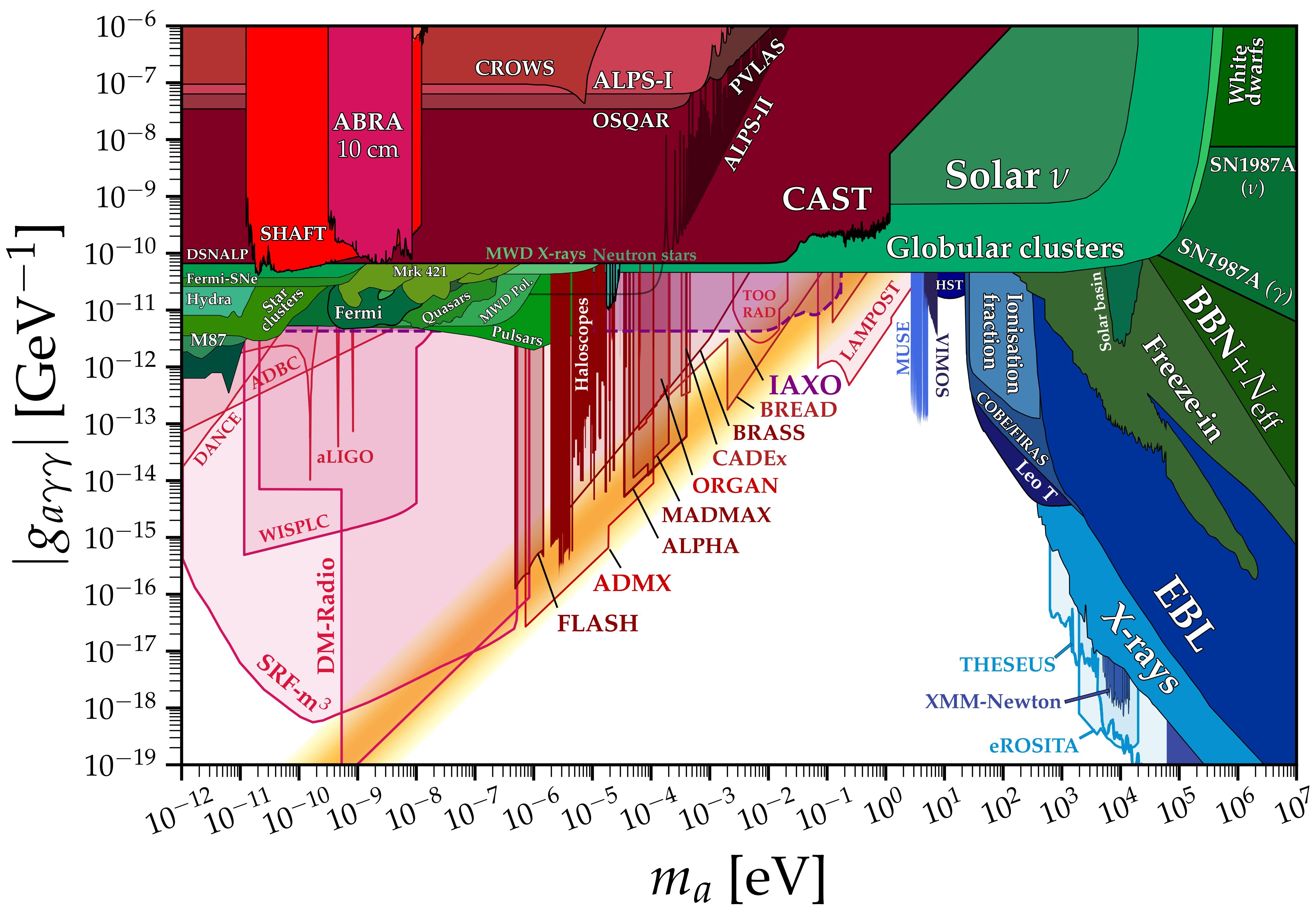
HAYSTAC (Yale)



GrAHal (Grenoble)

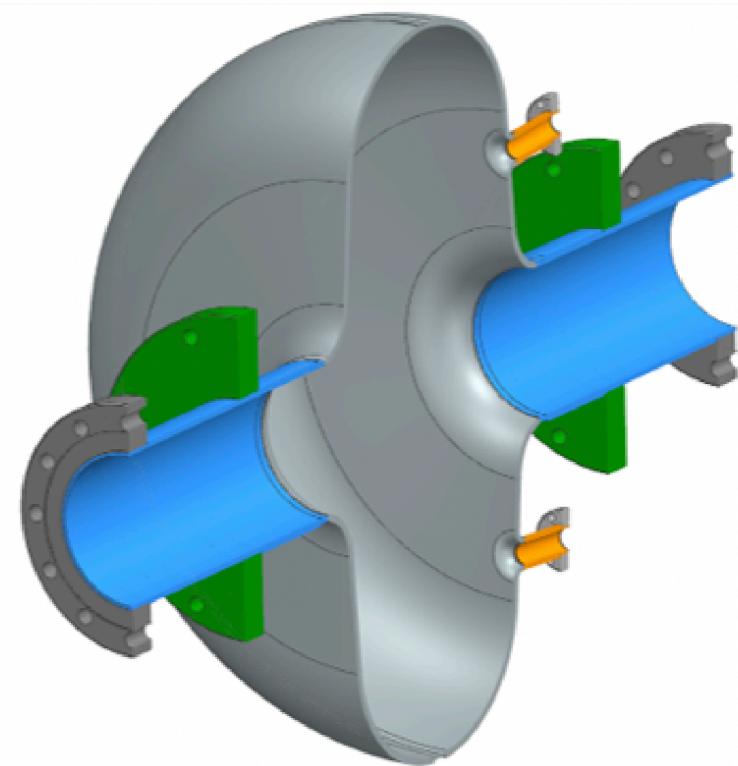






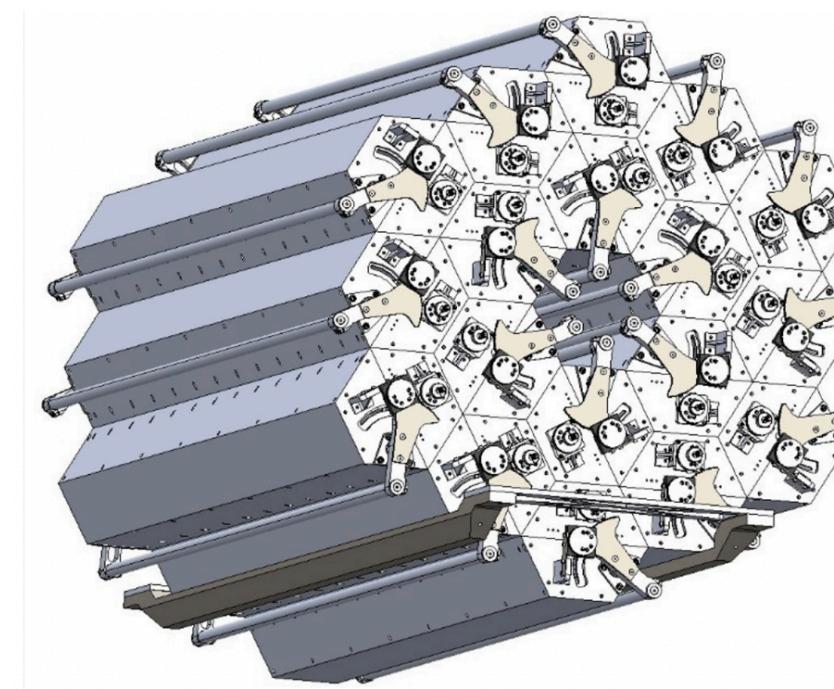
# Dark-SRF

2207.11346



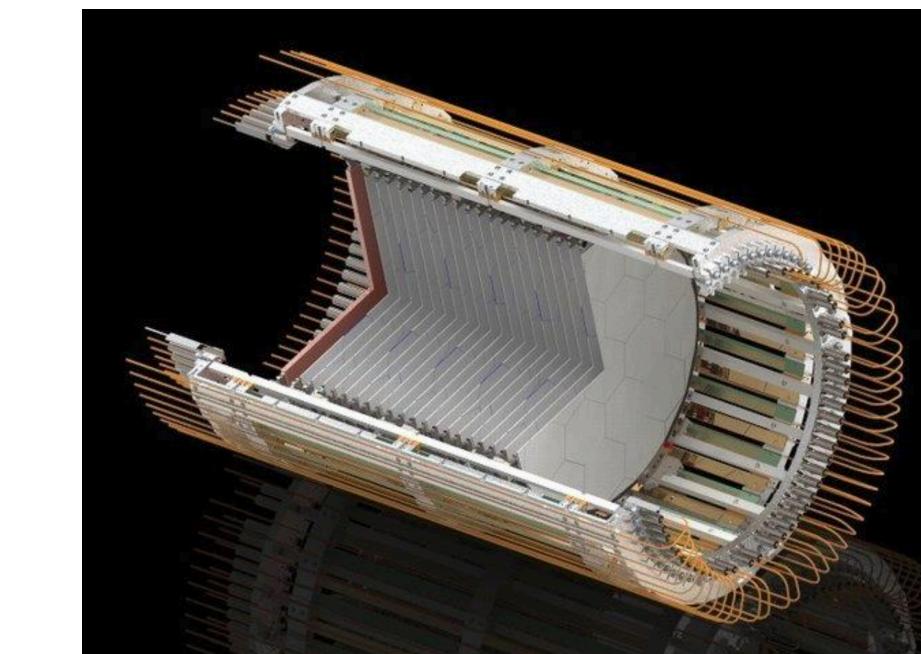
# ADMX-EFR

2203.14923



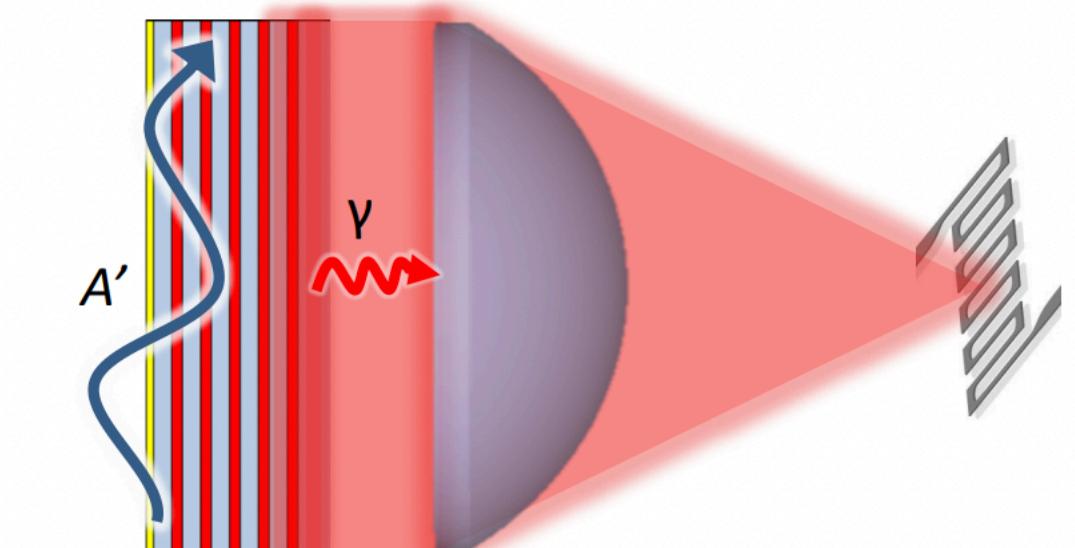
# MADMAX

2003.10894



# LAMPOST

2110.01582



MHz

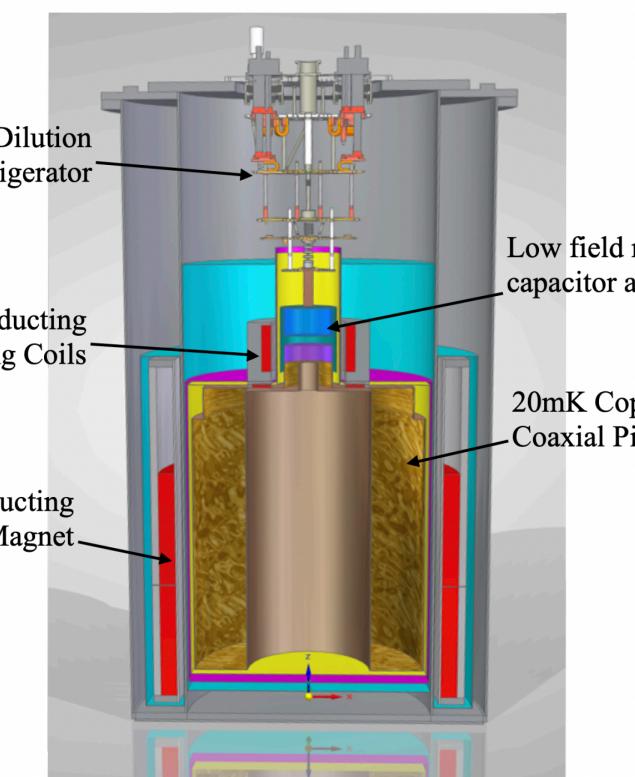
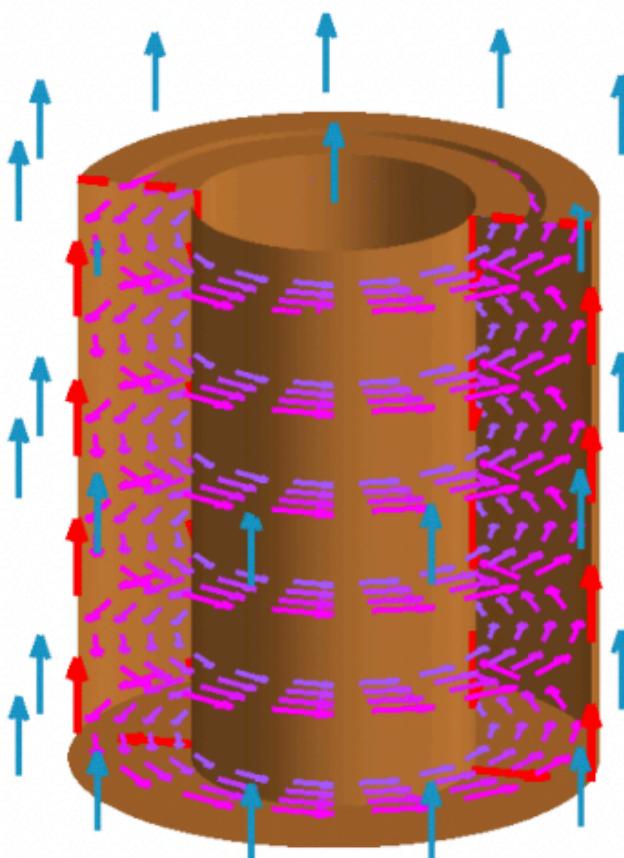
neV

GHz

$\mu\text{eV}$

THz

meV



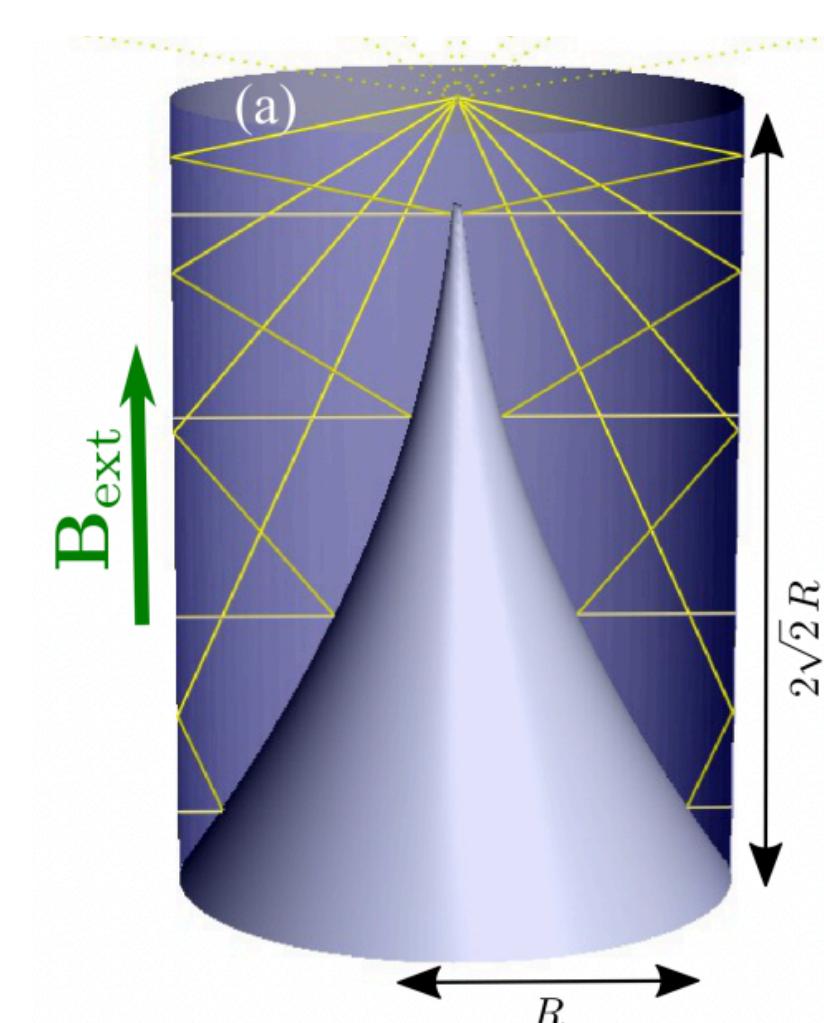
# DM-Radio

2204.13781



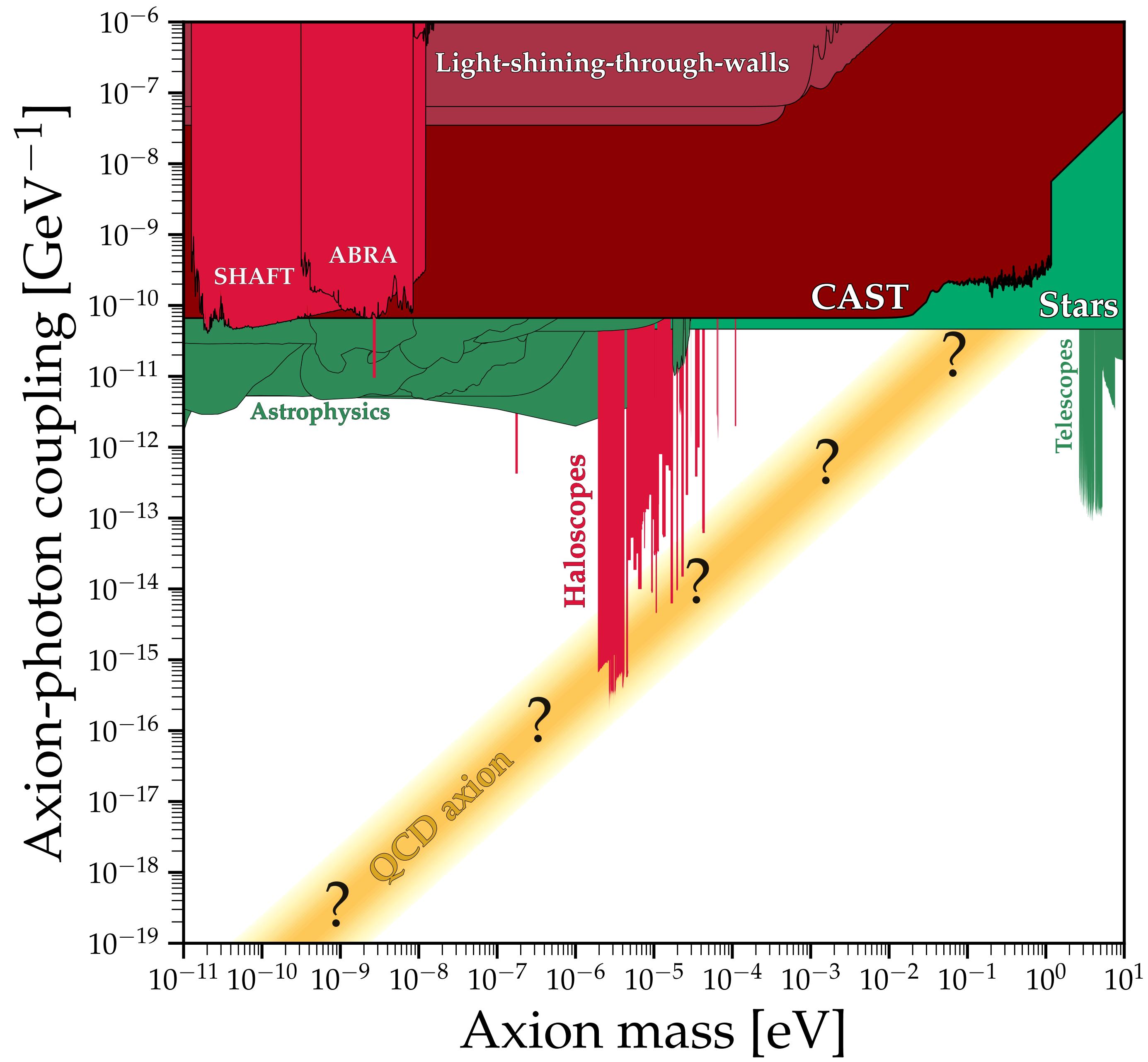
# ALPHA

2210.00017

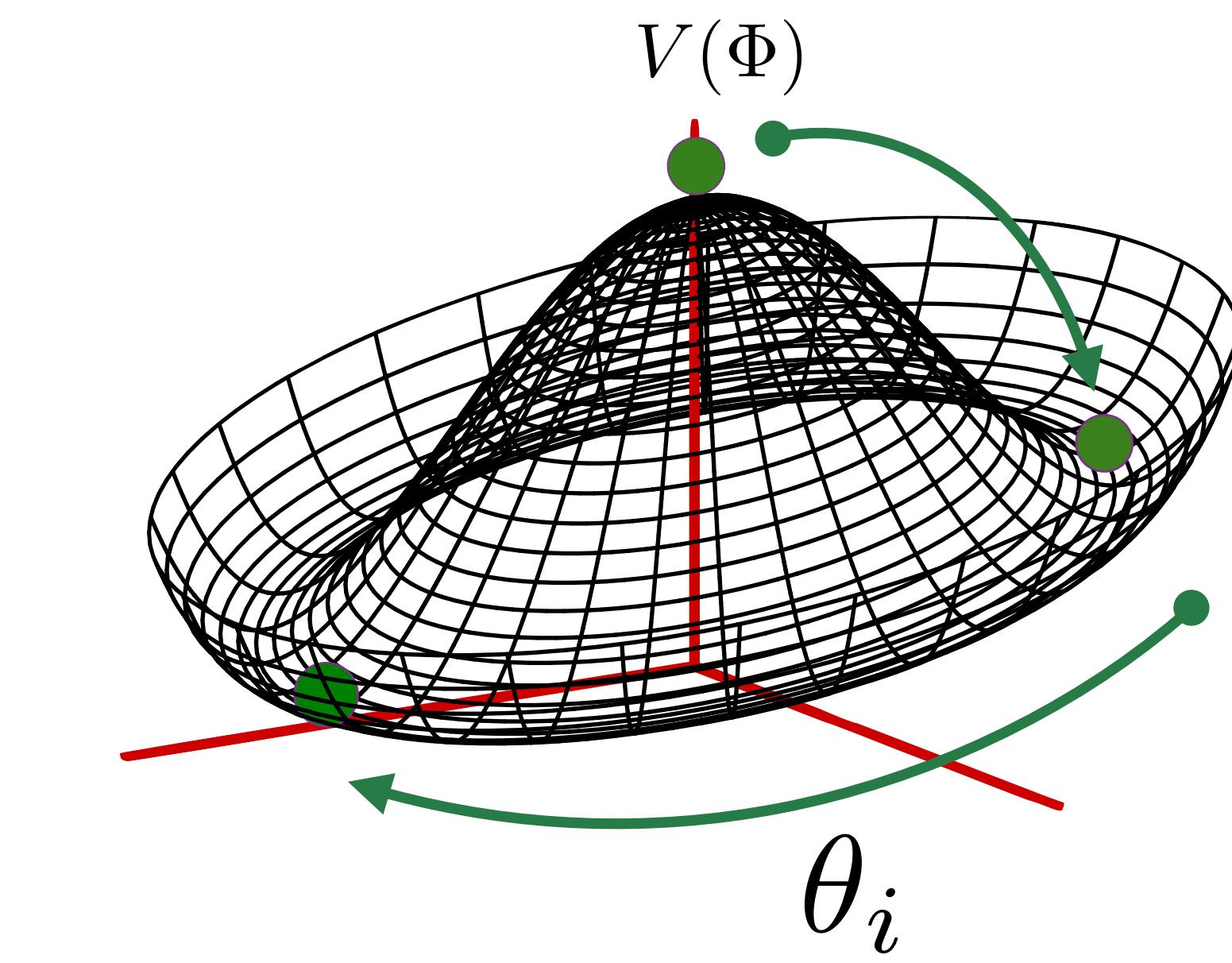
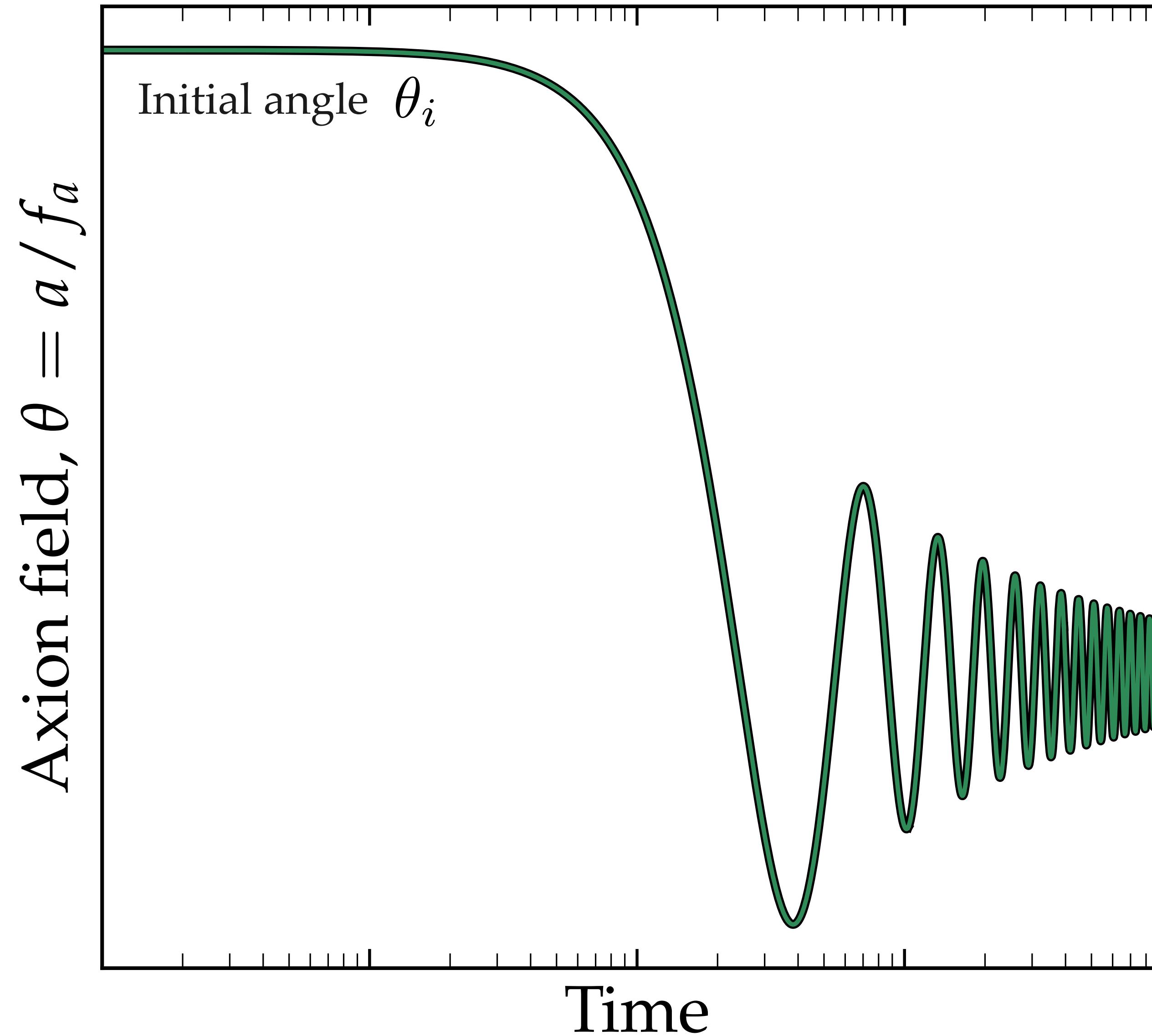


# BREAD

2111.12103

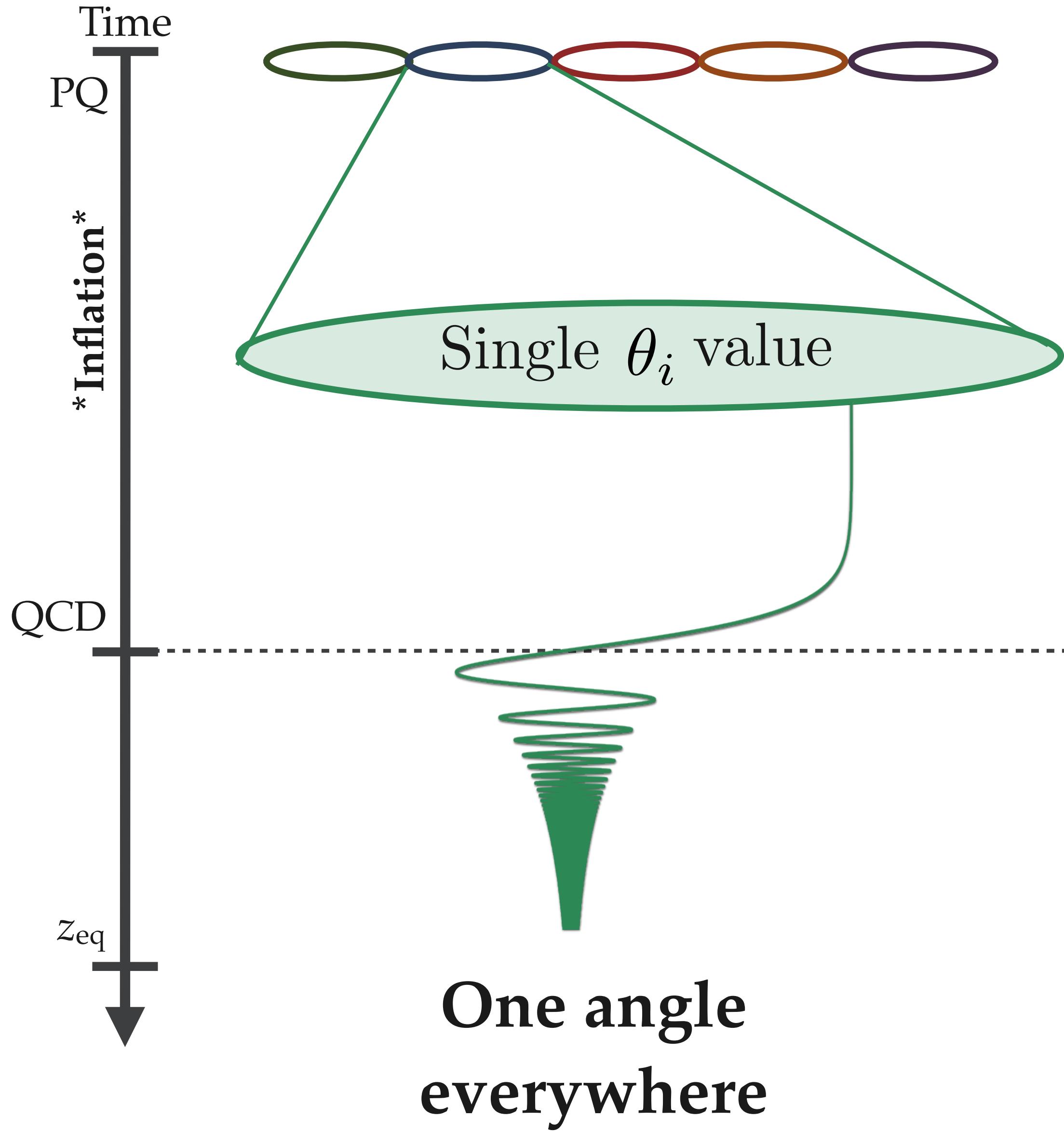


# Axion cosmology: the misalignment mechanism

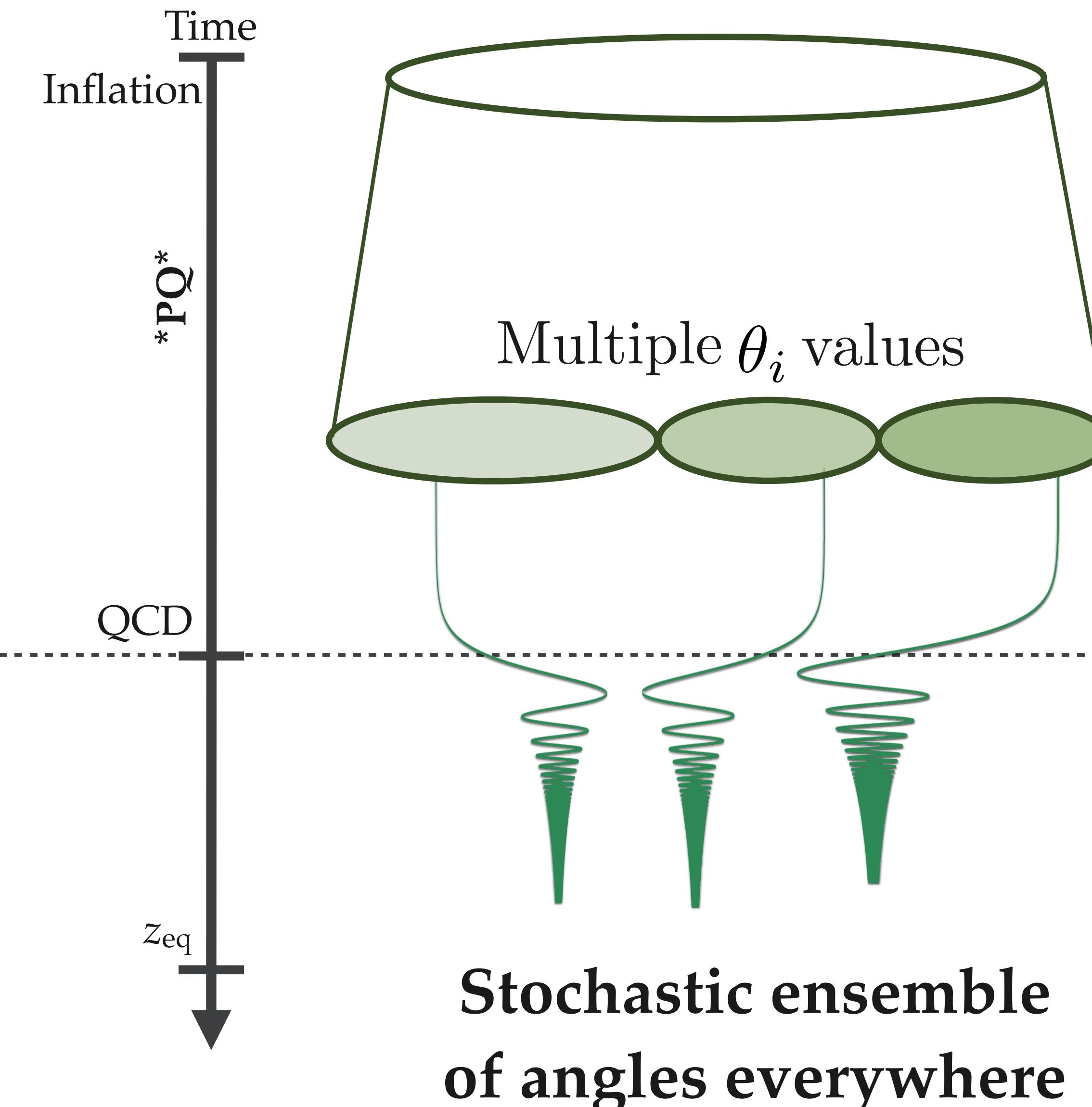


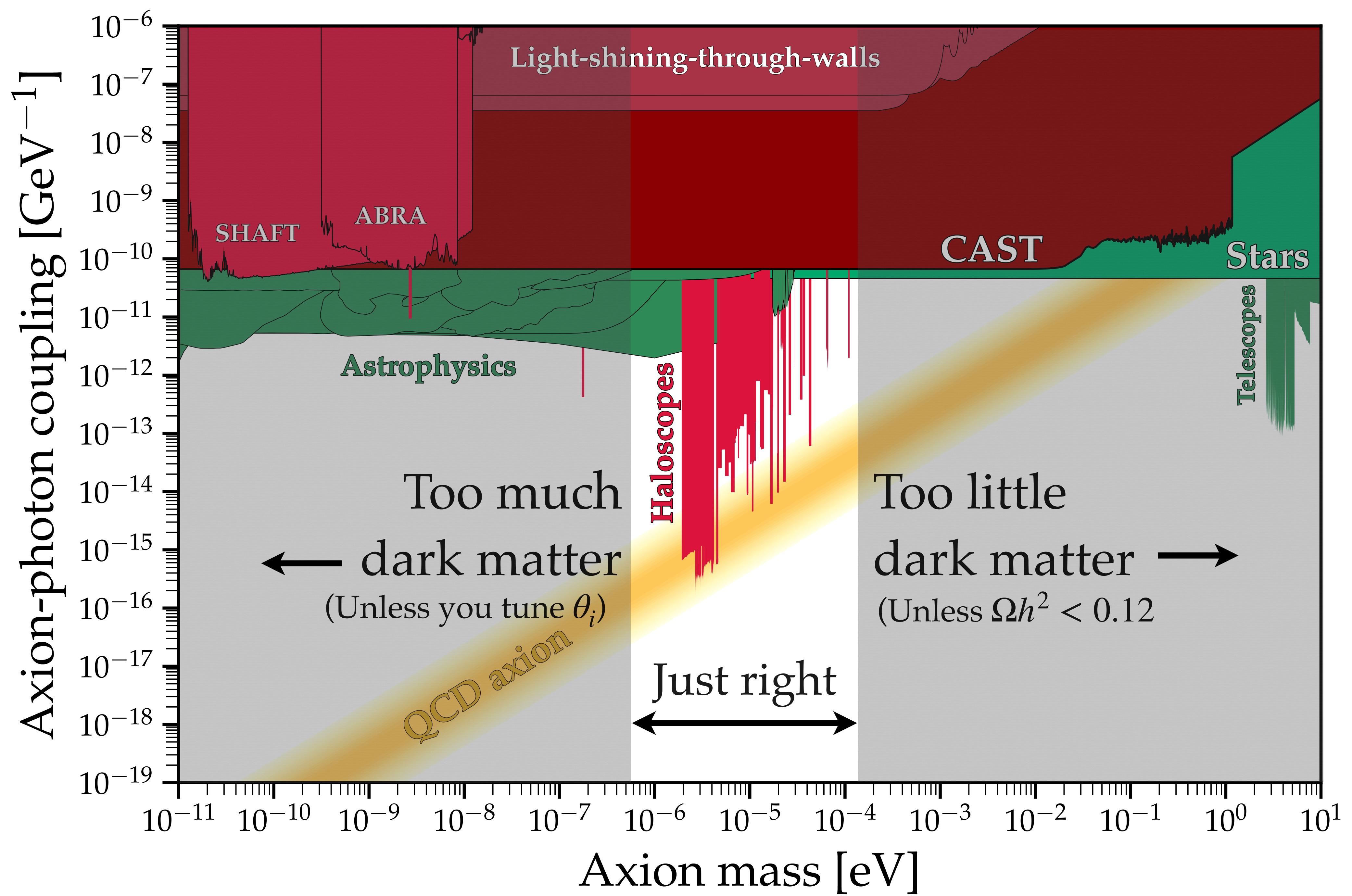
$$\text{Amount of DM} \propto \theta_i^2 m_a^{-7/6}$$

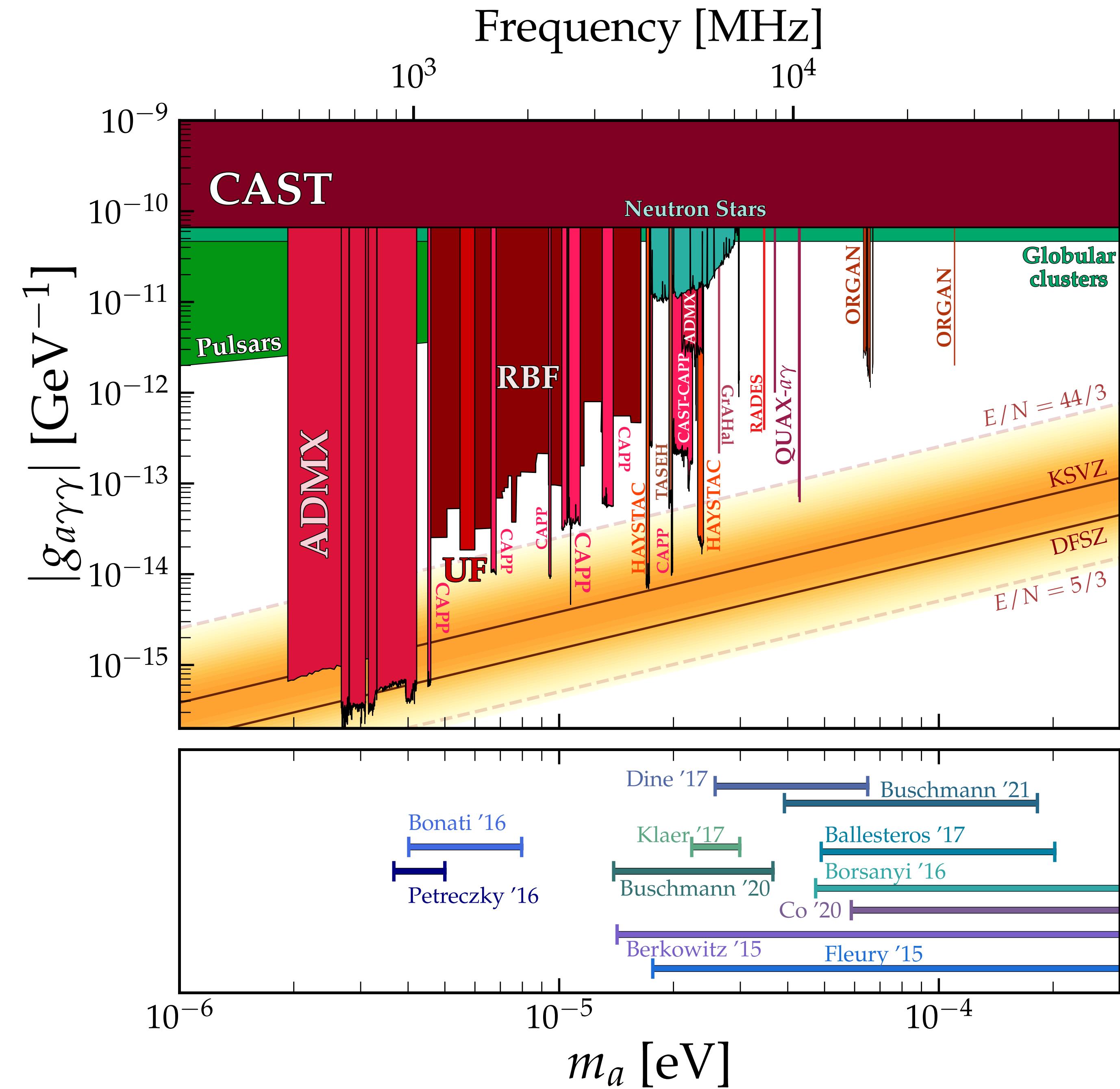
## Pre-inflationary axion



## Post-inflationary axion



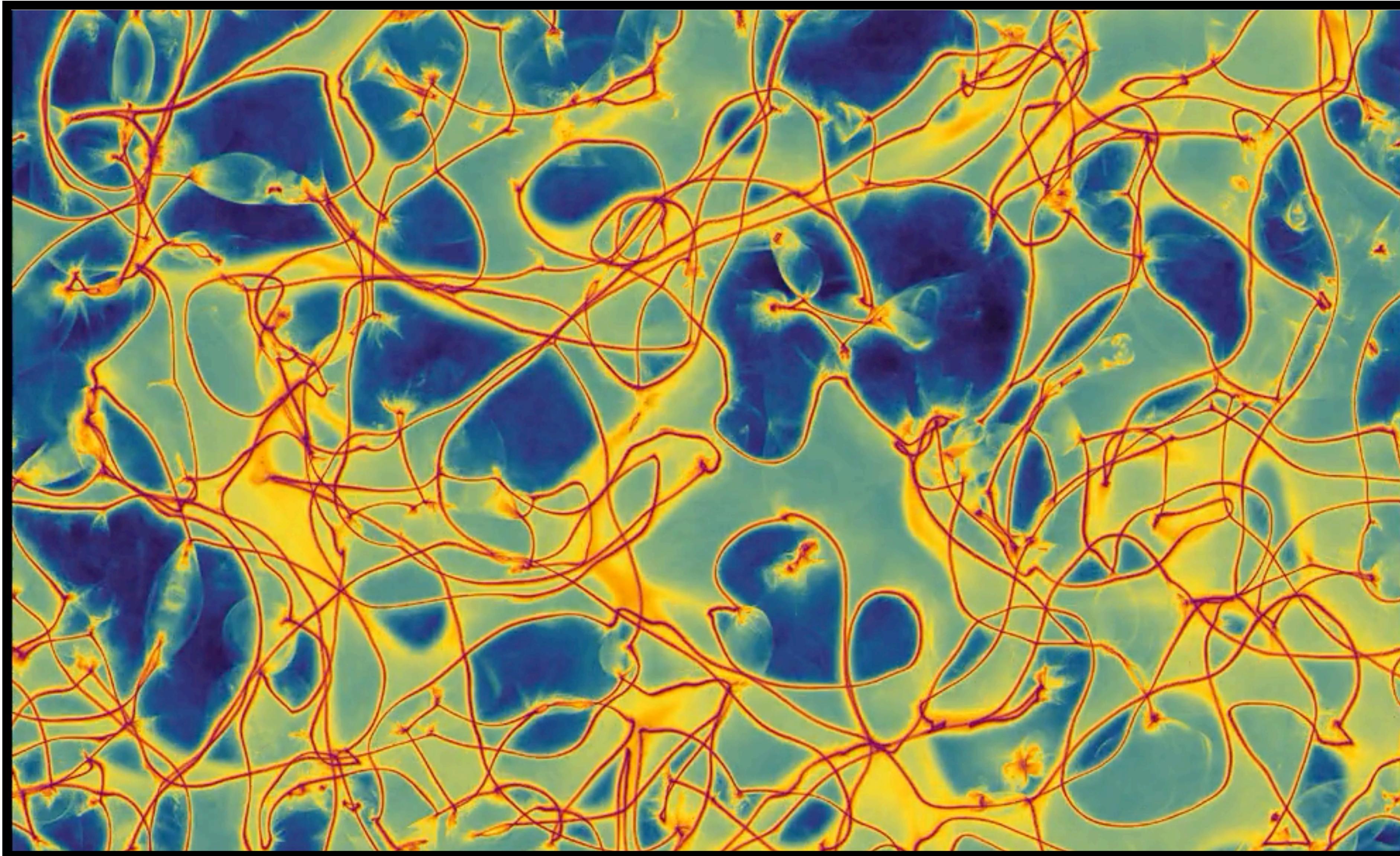


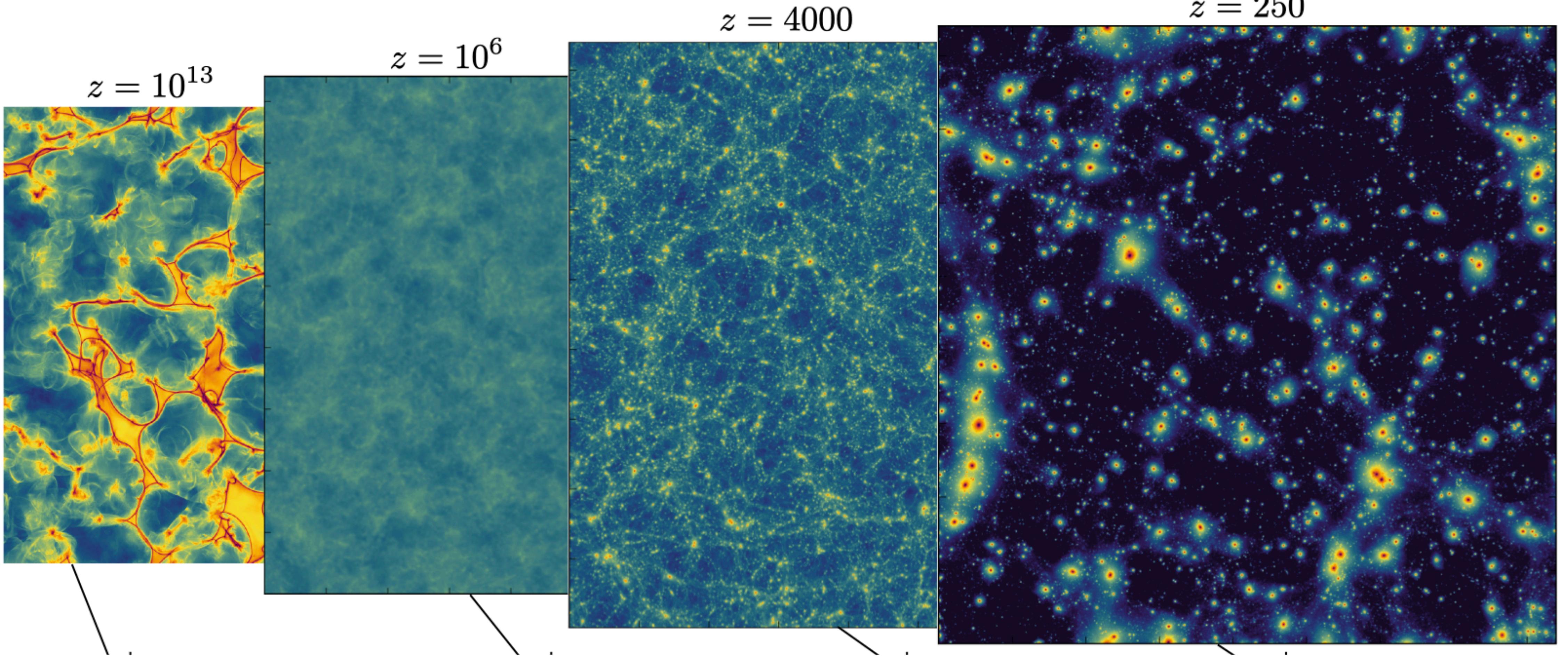


**Pre-inflation:** calculation is easy... but there is a free parameter  $\theta_i$  that we have no a priori knowledge of

**Post-inflation:** no free parameters... but things get complicated

# Topological defects in the axion field in the post-inflationary scenario





Topological  
defects produce  
relativistic axions

Axions free-  
stream until  
non-relativistic

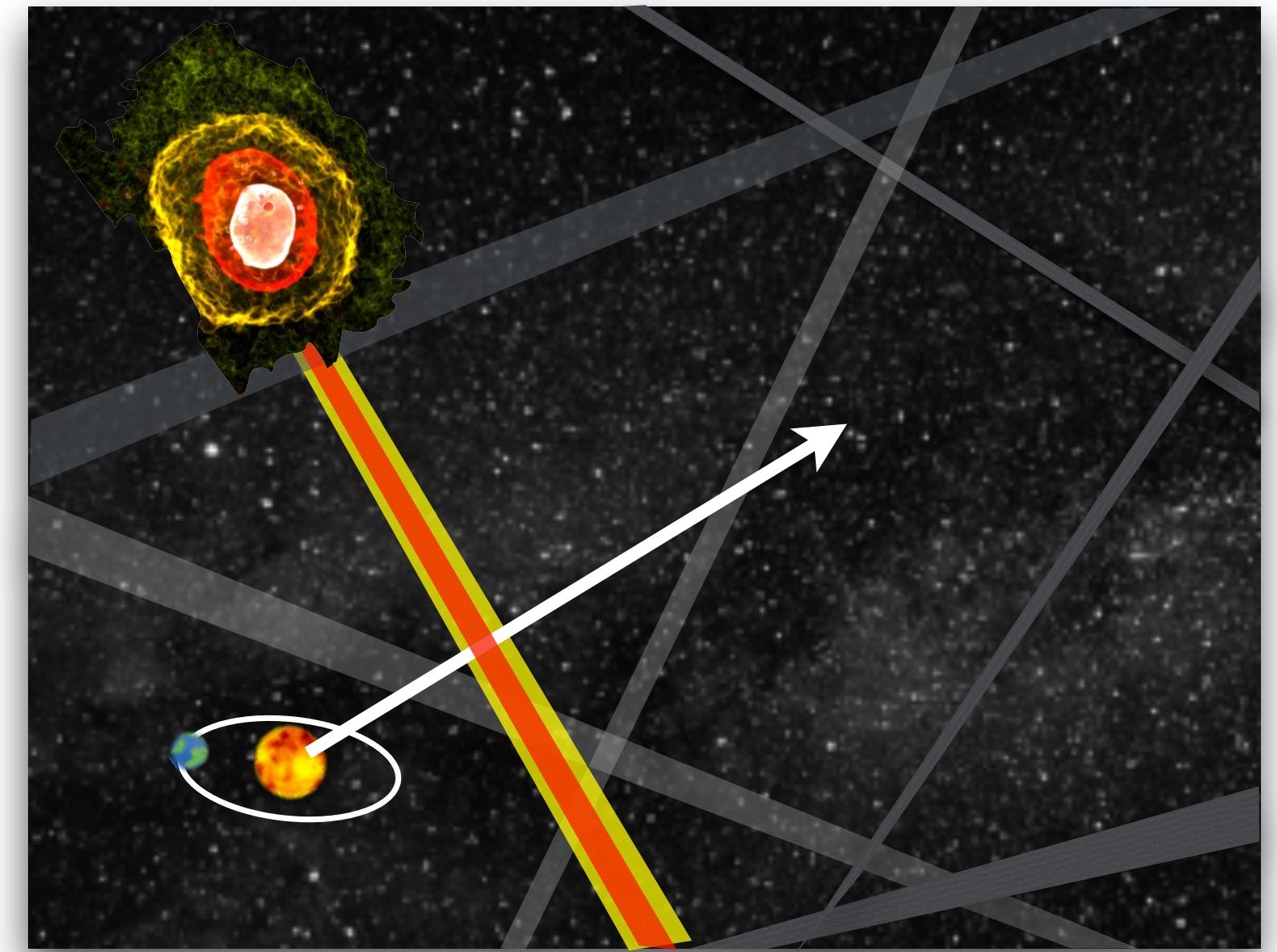
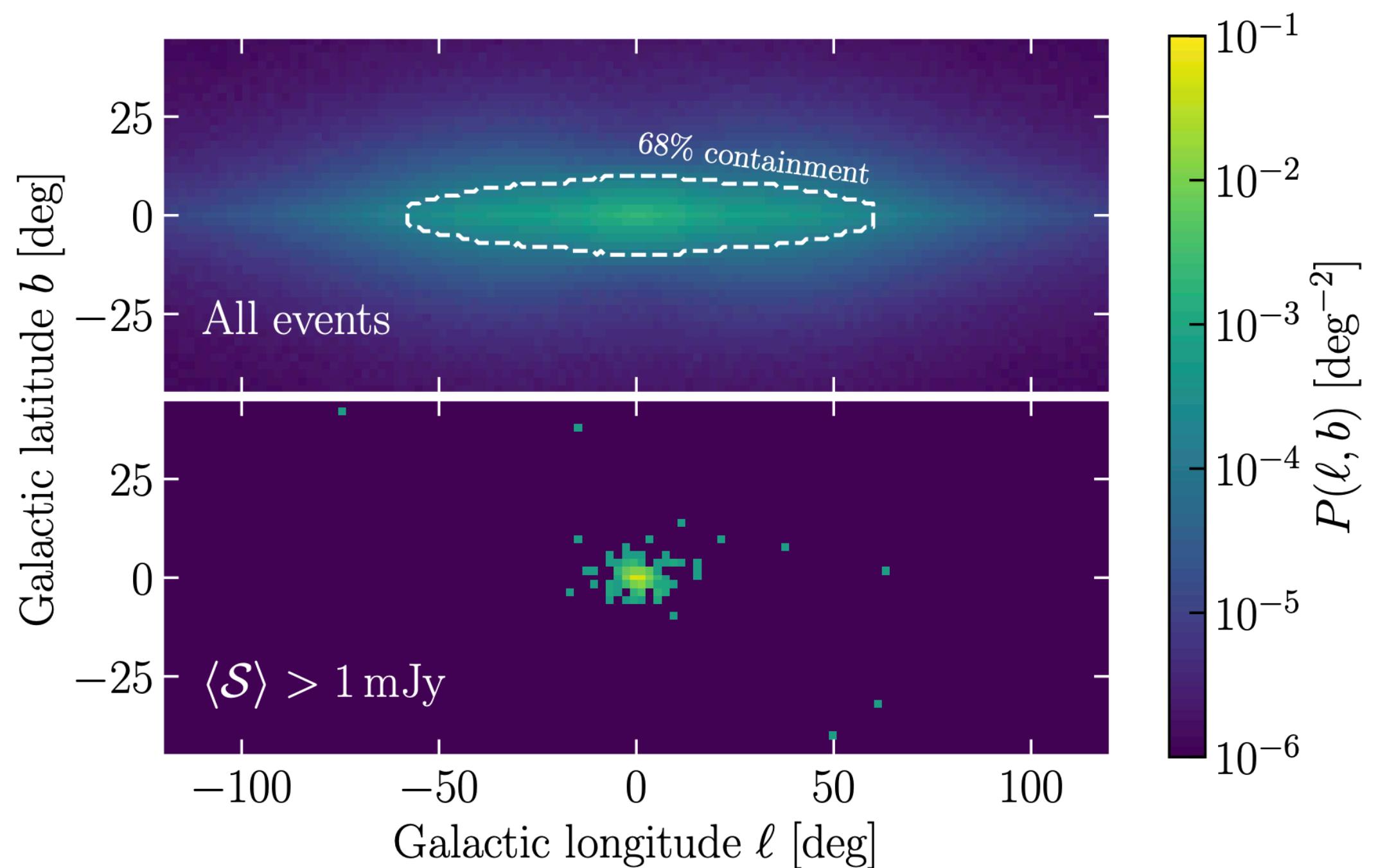
Ultra-small scale  
structures  
collapse

Axion  
miniclusters  
form

# Not the end of the story...

## Tidal disruption?

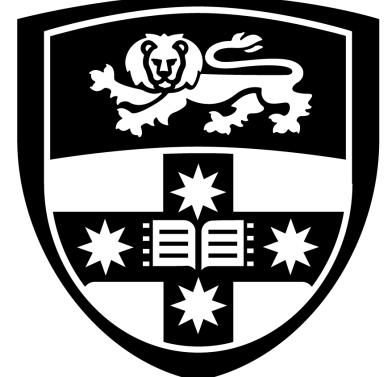
- About ~60% of the miniclusters should be disrupted at the solar position (big uncertainty on density profile)
- Disruption should re-fill phase space distribution, but by how much?



## Indirect signals?

- Collision of miniclusters with neutron stars, observe in radio [2011.05378]
- Some miniclusters may be concentrated enough to microlense [2204.13187]

# Axions

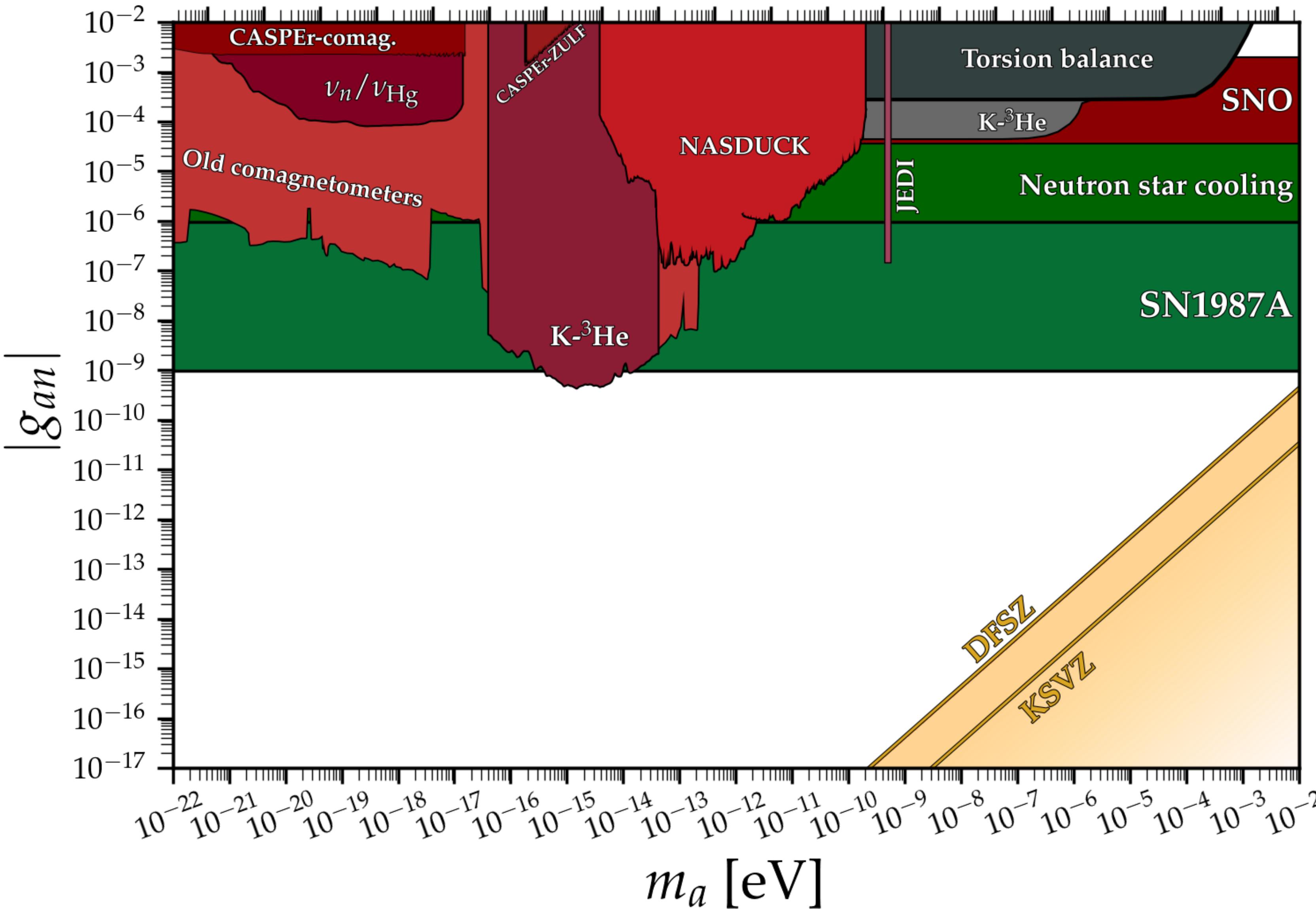


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# Back to axions... fermion couplings, e.g. neutron

→ Hard to beat astrophysical bounds



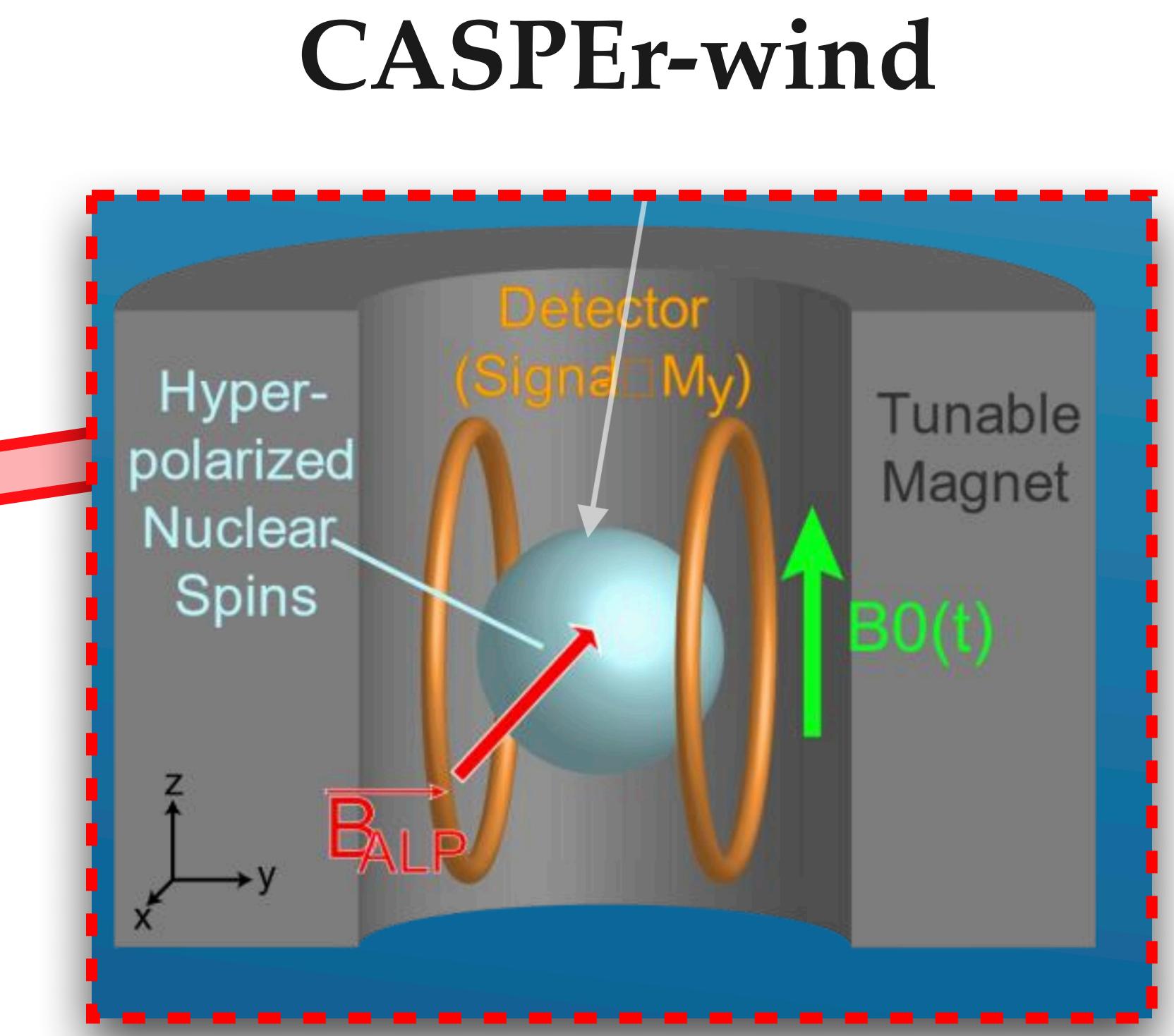
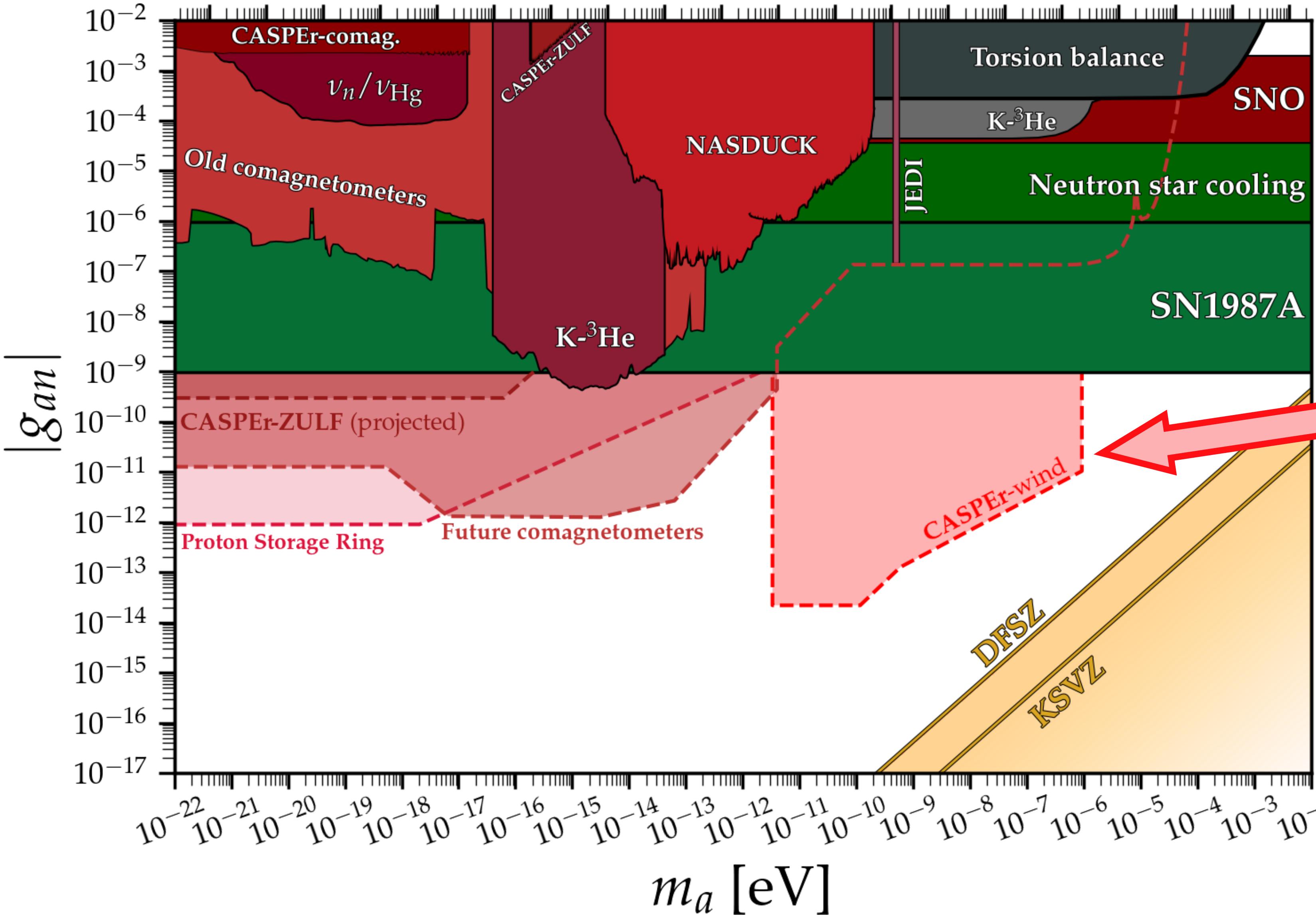
Why?

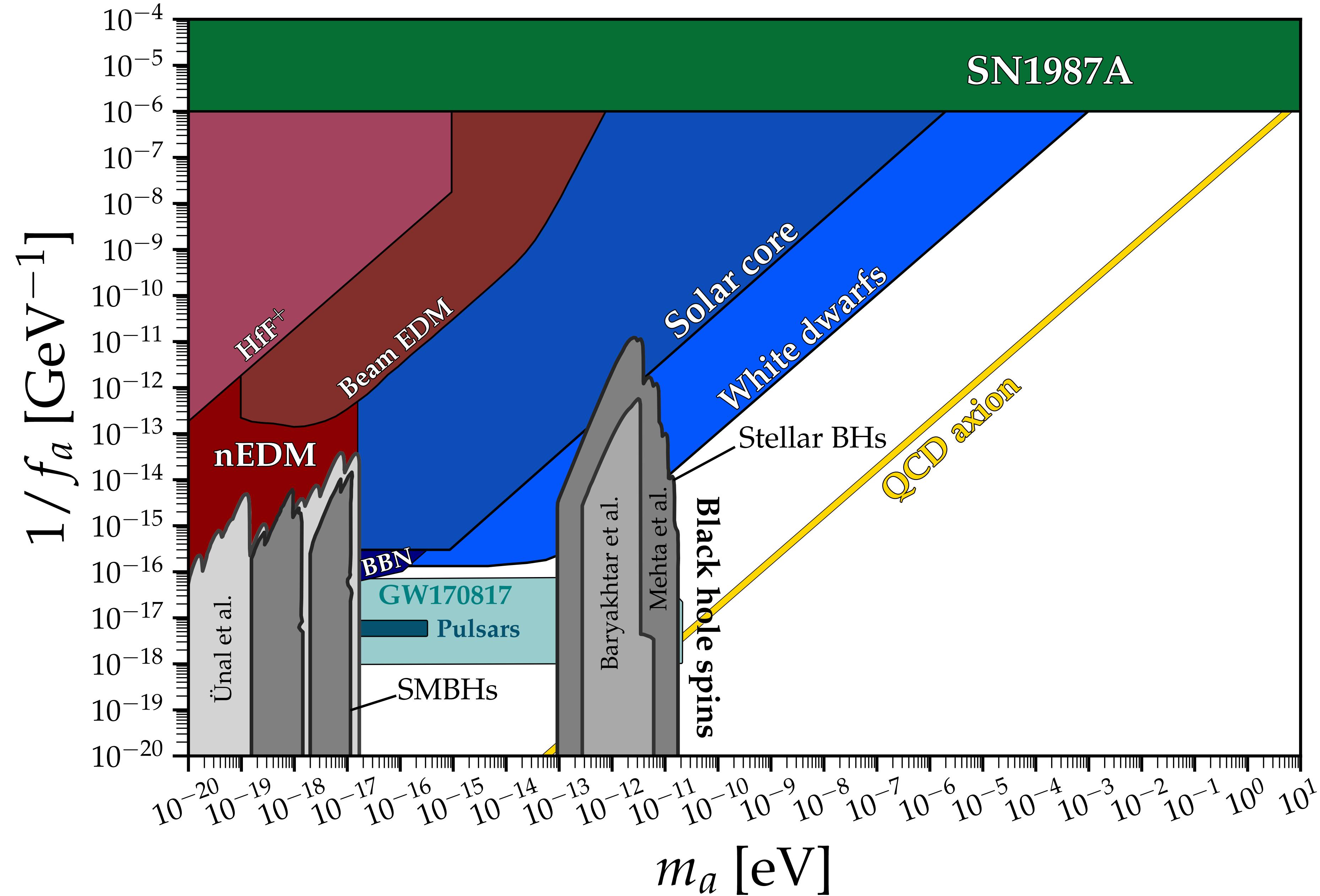
$$\mathcal{L} = -\frac{g_{an}}{2m_n} \partial_\mu a \underbrace{\bar{n} \gamma^5 \gamma^\mu n}_{\text{Spin-dependent}}$$

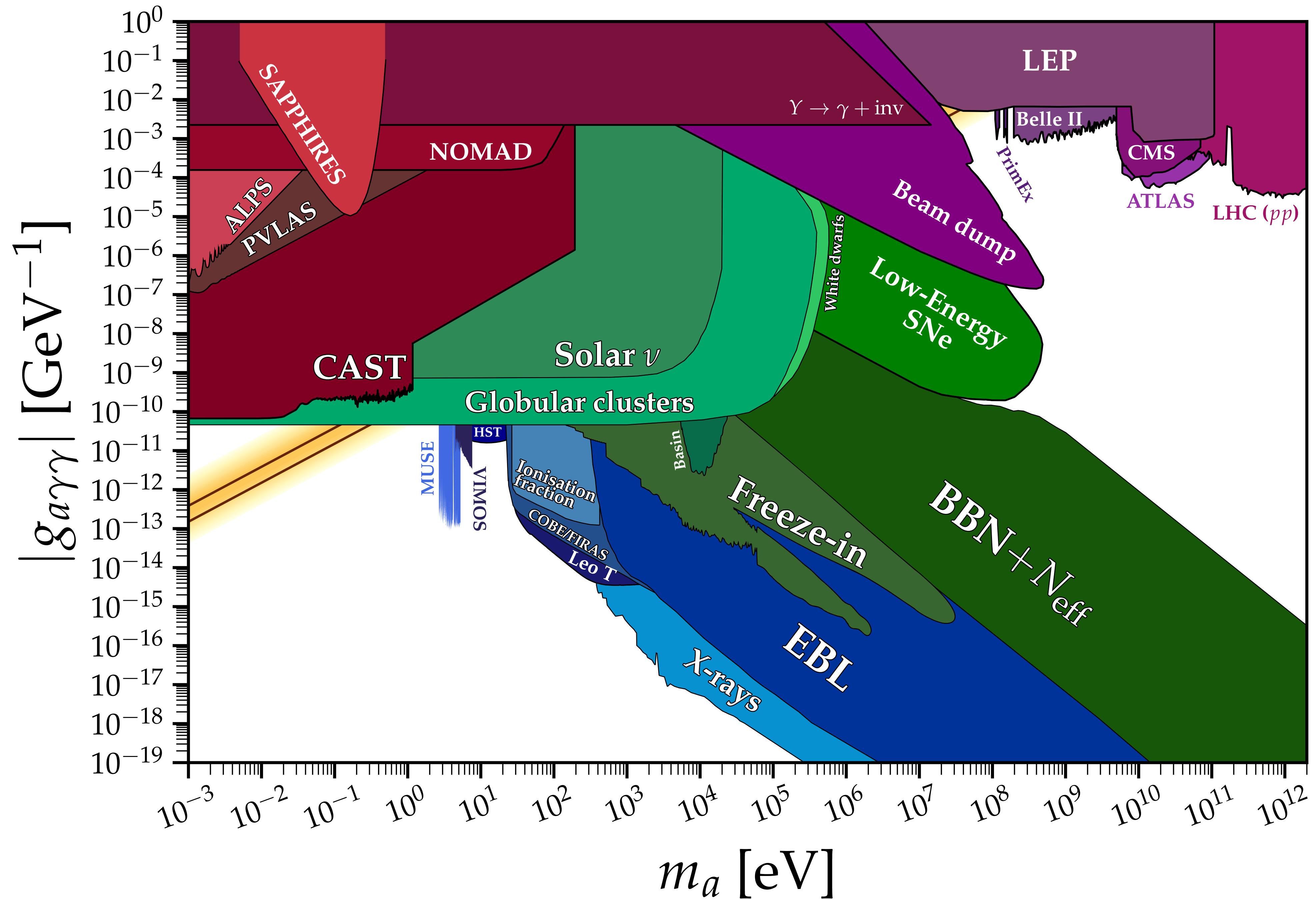
Hard to get large-enough samples of spin-polarised materials to compete with a star

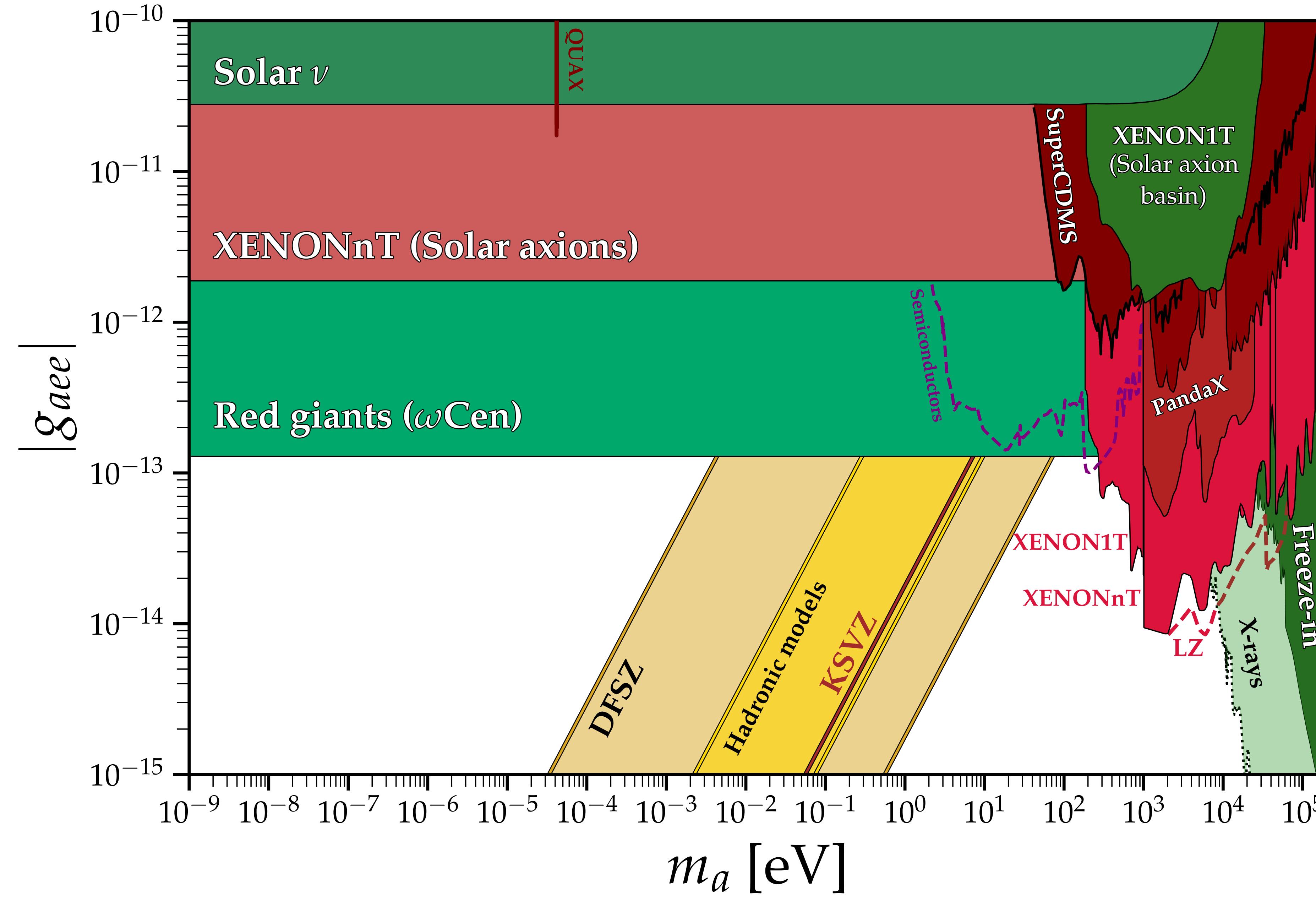
# Back to axions... fermion couplings, e.g. neutron

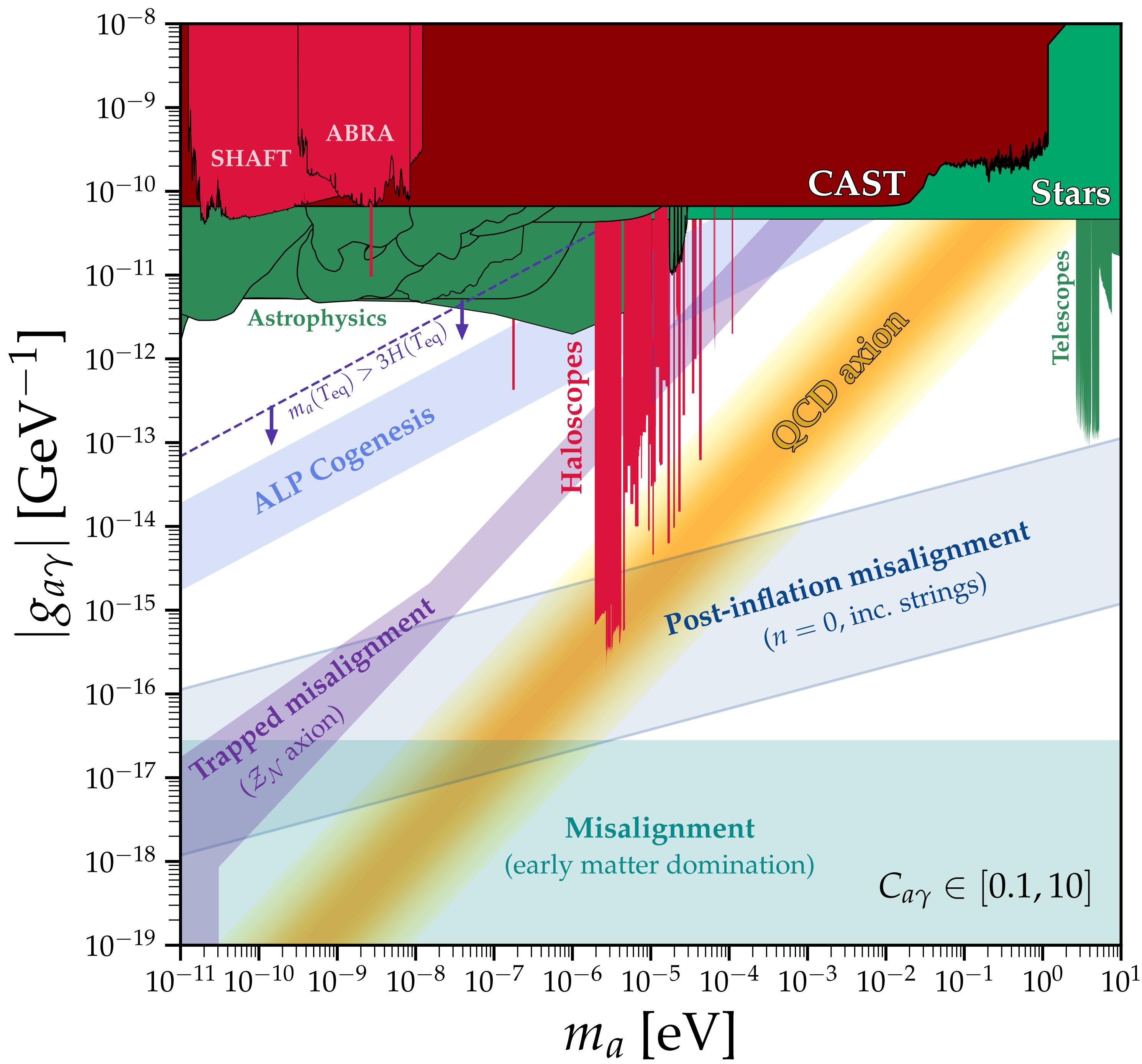
→ Hard to beat astrophysical bounds

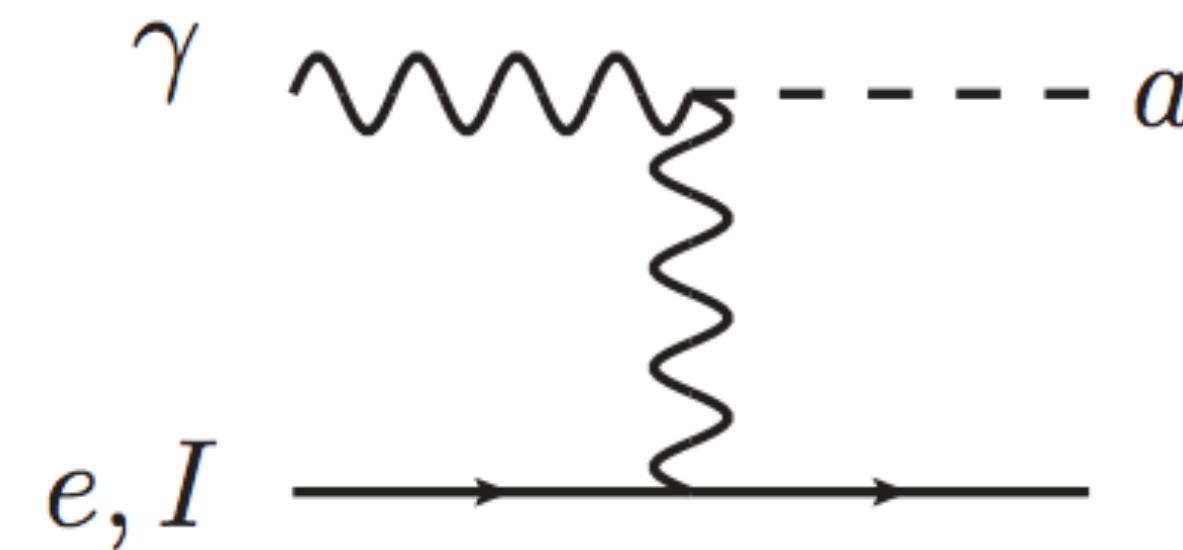




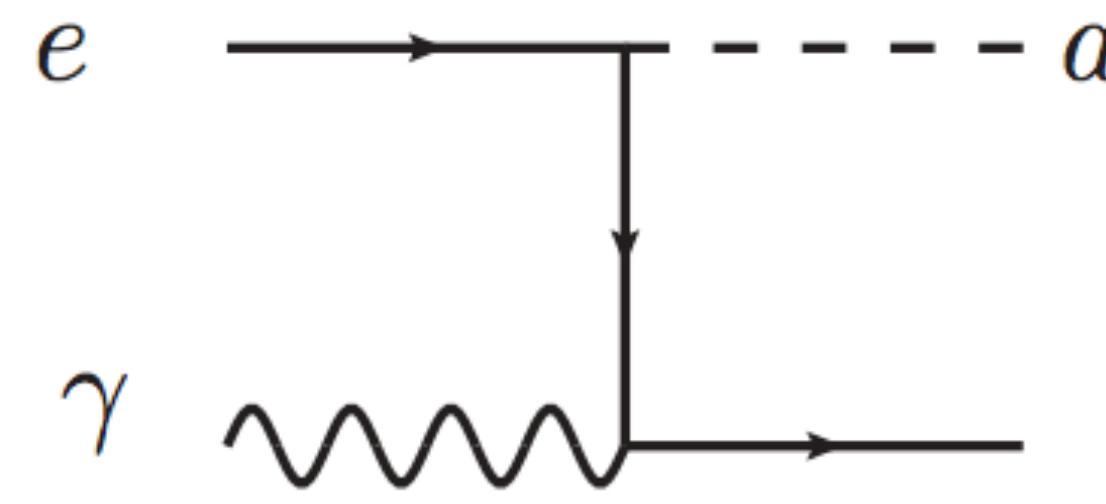




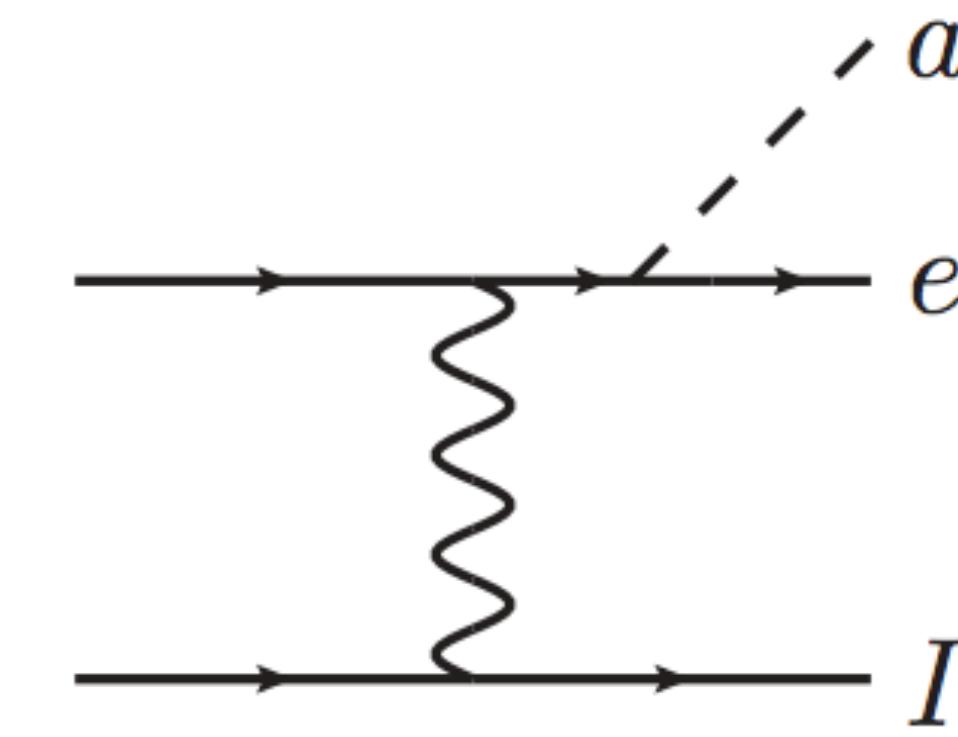




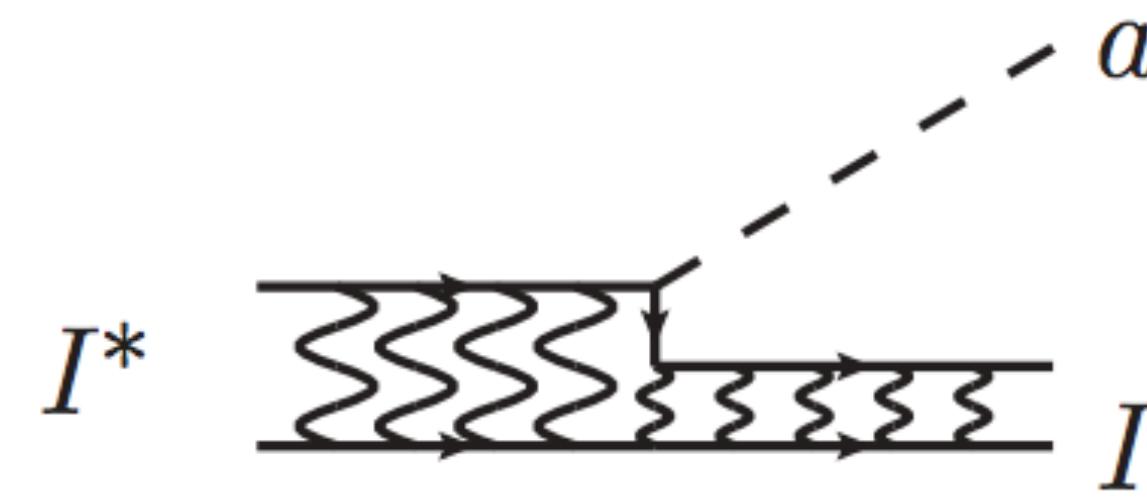
Primakoff



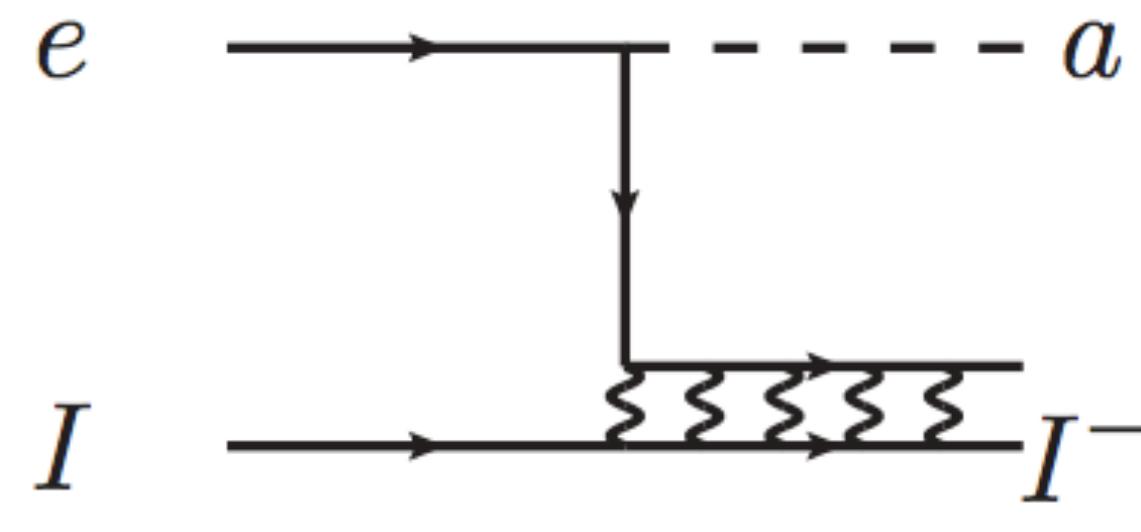
Compton



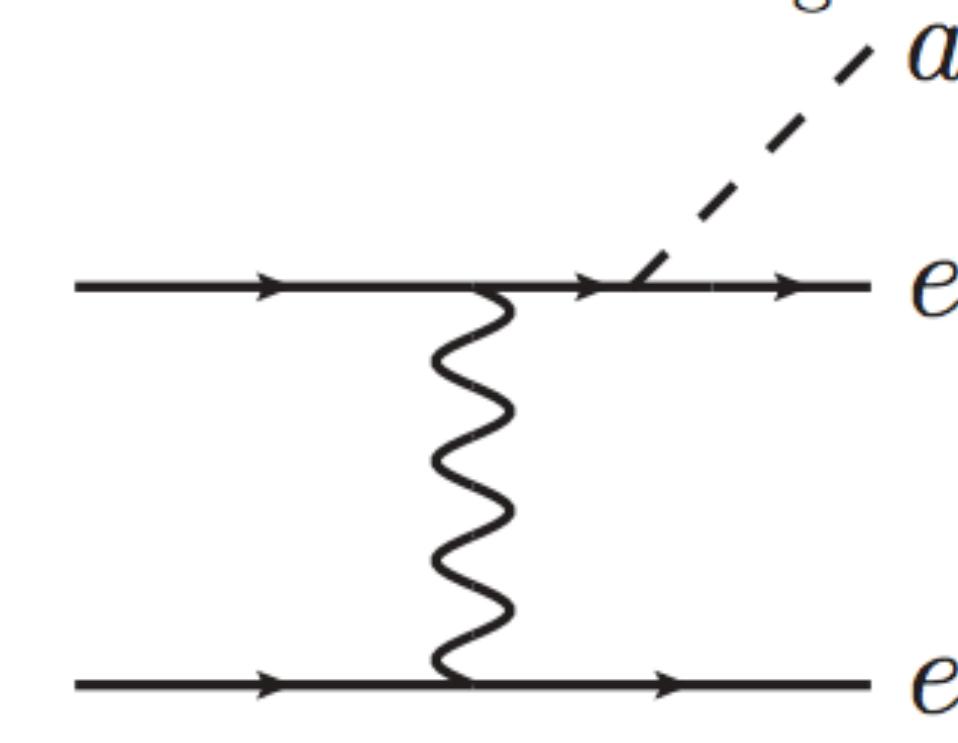
$e - I$  bremsstrahlung



axio – deexcitation



axiorecombination



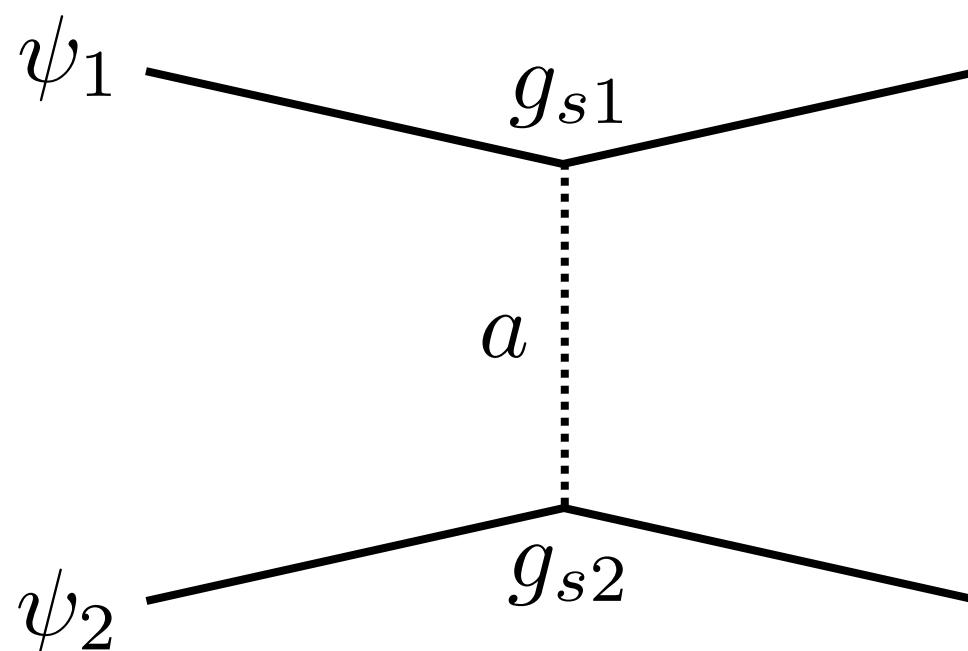
$e - e$  bremsstrahlung

[1310.0823]

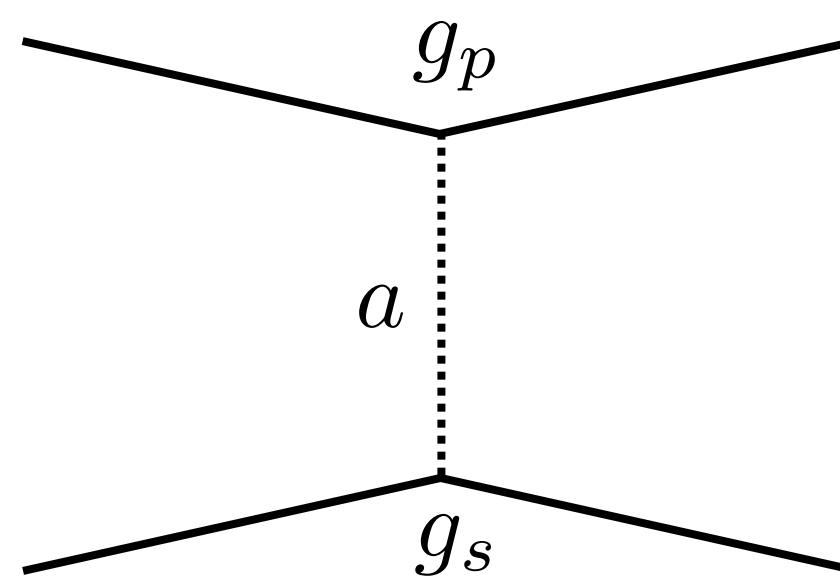
# Pure laboratory tests of axion-fermion fifth forces

Key point: any SM or BSM CP-violation (could be e.g. size of Jarlskog invariant of CKM matrix) could shift axion vev and generate **CP-violating** axion-fermion couplings in addition to the **CP-conserving** ones

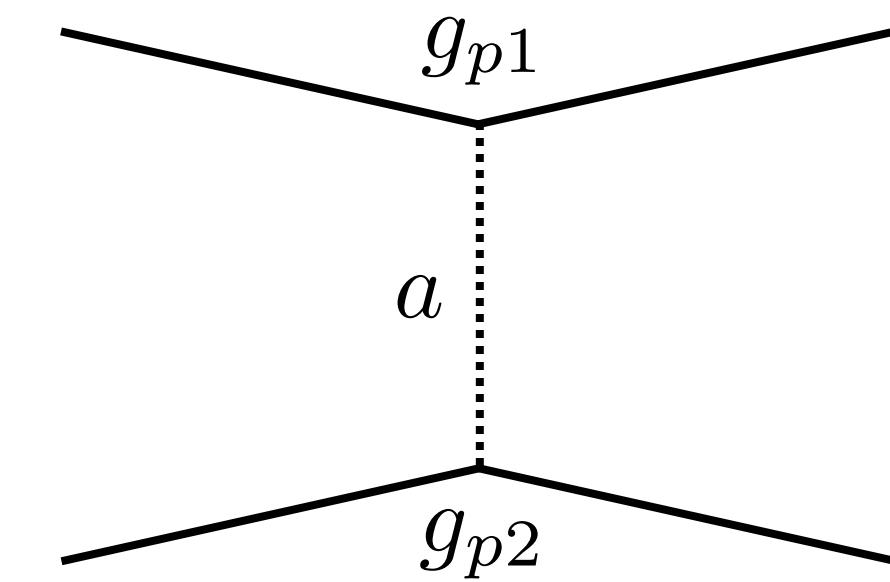
$$\mathcal{L} \supset -a \sum_{\psi} \underbrace{g_p^\psi (i\bar{\psi}\gamma^5\psi)}_{\text{green}} - a \sum_{\psi} \underbrace{g_s^\psi (\bar{\psi}\psi)}_{\text{red}}$$



**Monopole-monopole  
(Spin independent)**  
→ e.g. tests of inverse square law / WEP



**Monopole-dipole  
(Spin dependent)**  
→ Spin-mass forces e.g.  
ARIADNE / QUAX



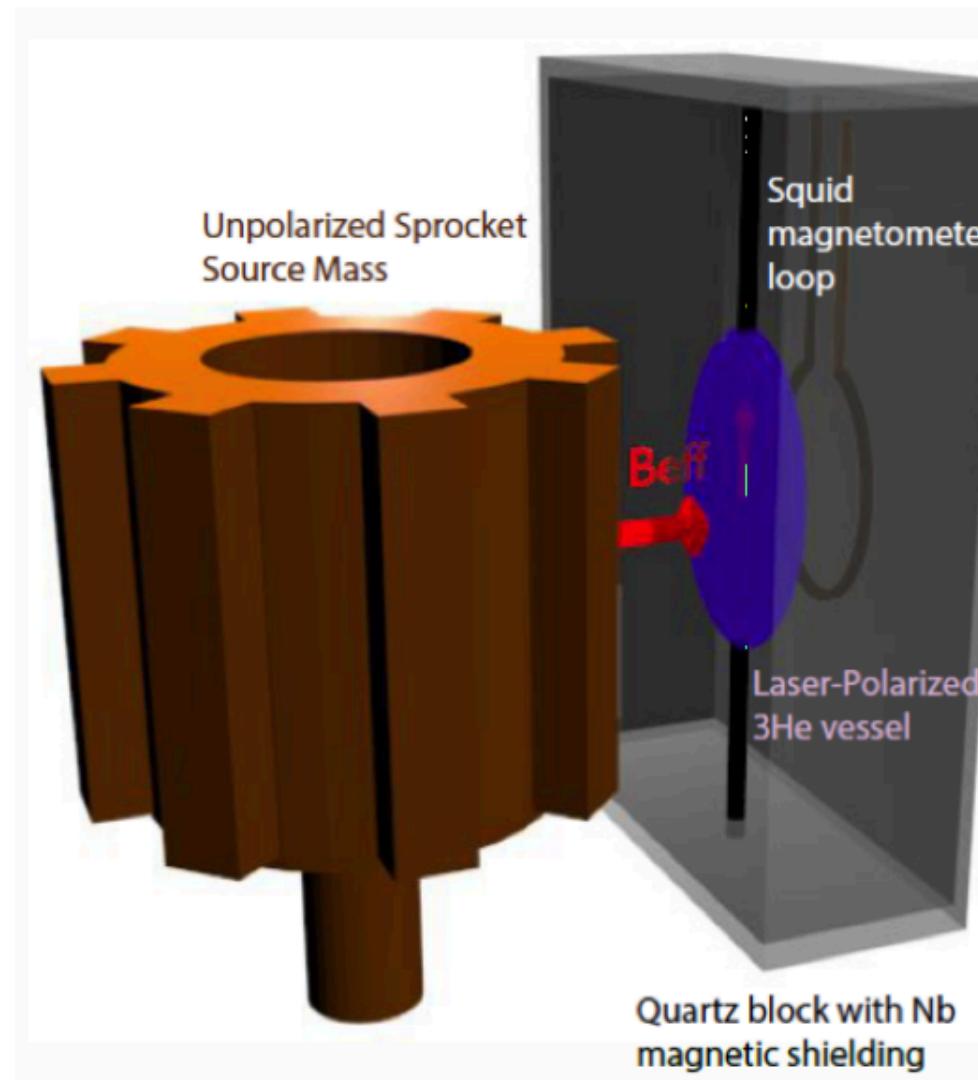
**Dipole-dipole  
(Spin dependent)**  
→ Forces between spin-polarised samples

# Monopole-dipole searches

Conceptually similar, spin an unpolarised source mass near to a spin-polarised target

## ARIADNE

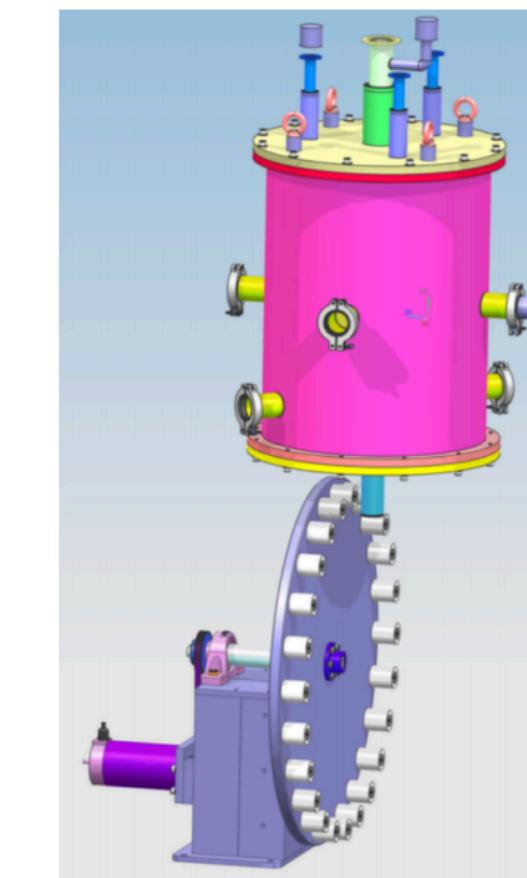
Constrains:  $g_p$   $g_s$  (nucleon-nucleon)



## QUAX

Constrains:  $g_p$   $g_s$  (electron-nucleon)

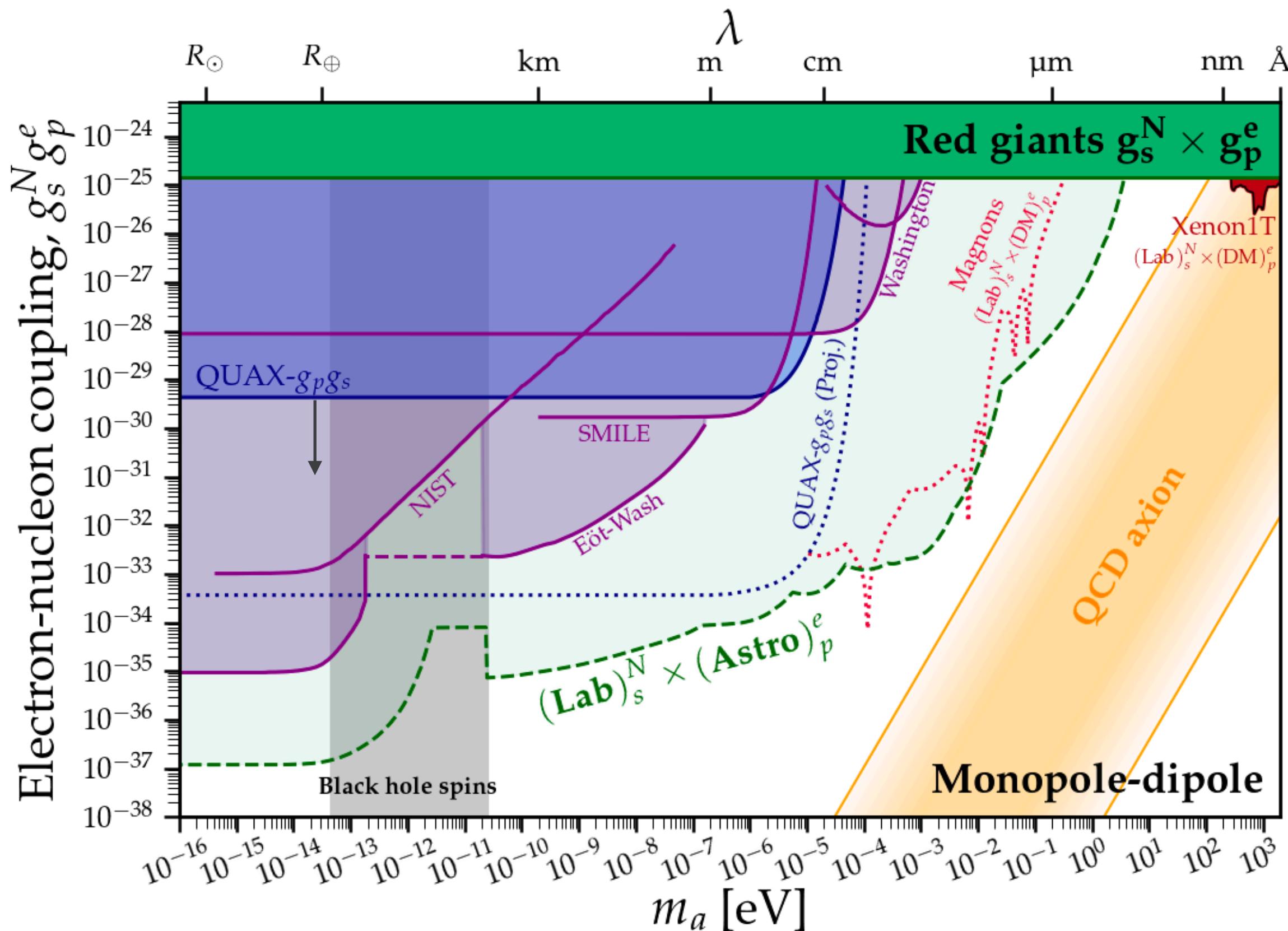
(Latest QUAX result 2011.07100)



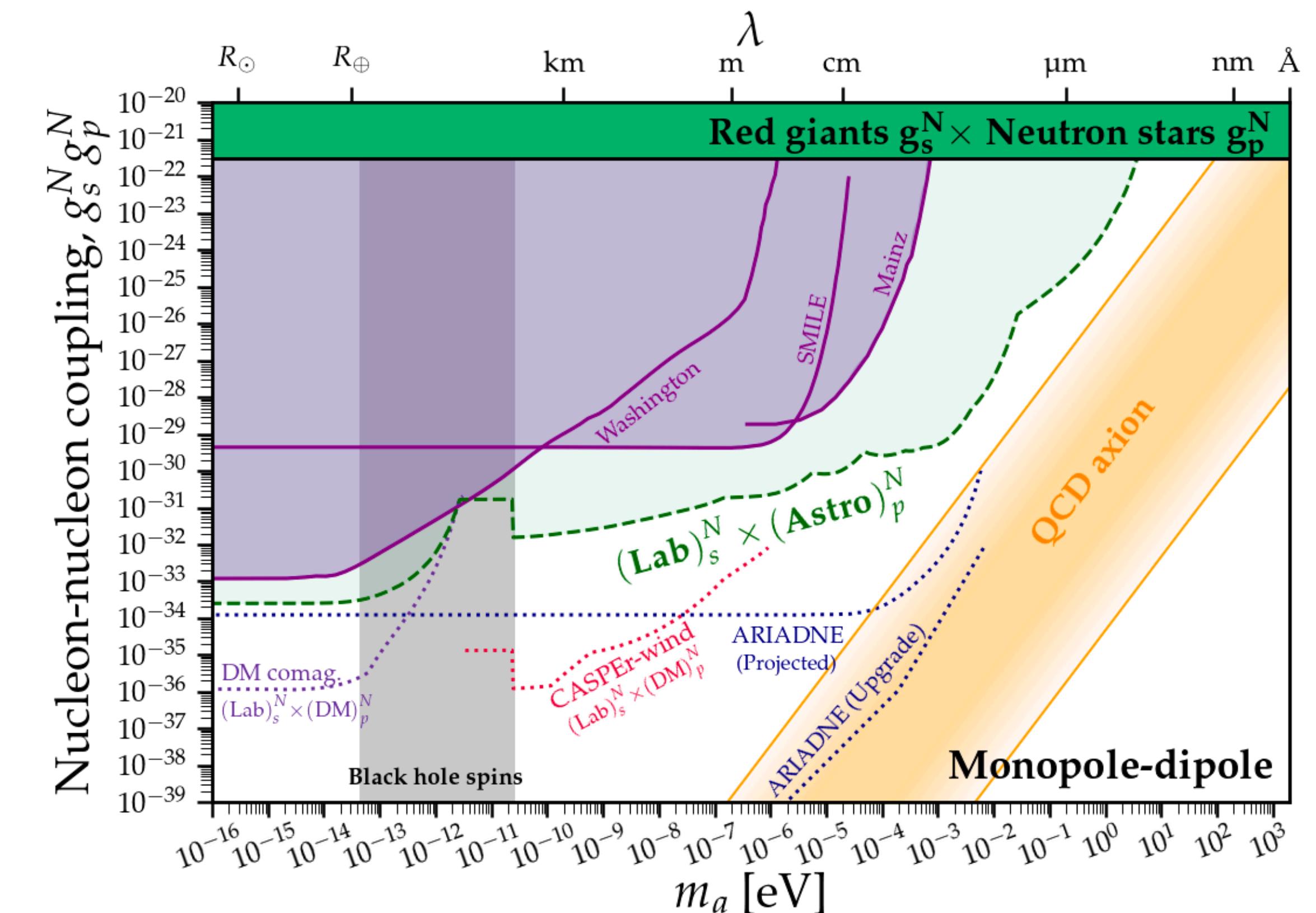
**Challenge:** stellar bounds tightly constrain  $g_p$ , and spin independent fifth force tests easily constrain  $g_s$ : so Astro x Lab bound on these coupling combos are very strong

# Pure laboratory tests for monopole-dipole axion-mediated forces

## Electron-nucleon coupling



## Nucleon-nucleon coupling



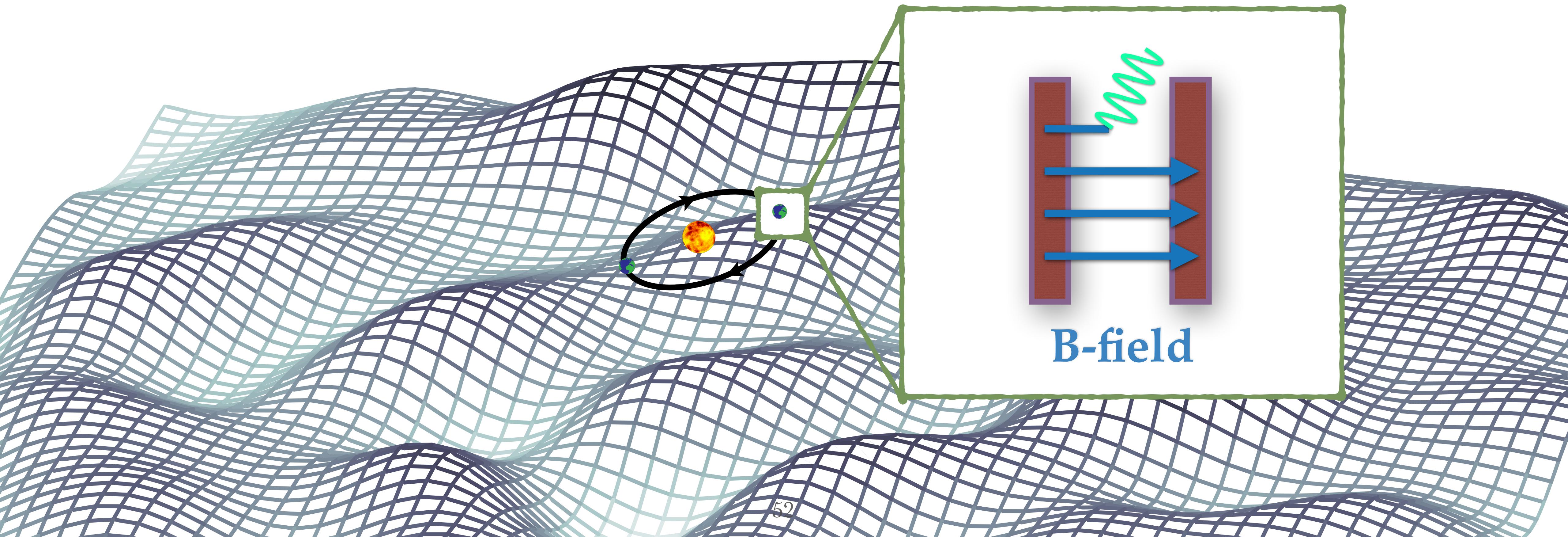
Hard to beat the **astrophysical bounds**, but **ARIADNE** projects that it will

# Dark matter axions

behave like a classical field :  $a(\mathbf{x}, t) \approx \frac{\sqrt{2\rho_a}}{m_a} \cos(\omega t - \mathbf{p} \cdot \mathbf{x} + \alpha)$

$$\omega \approx m_a$$

Oscillating at the axion mass



$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$\tilde{F}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}$$

$$\mathbf{E} = -\nabla A_0 - \dot{\mathbf{A}}$$

$$\mathbf{B} = \nabla \times \mathbf{A}$$

# Axion electrodynamics

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - J^\mu A_\mu - \frac{g_{a\gamma}}{4}F_{\mu\nu}\tilde{F}^{\mu\nu}a$$

- EL equation for photon shows we can interpret axion as the source of an effective current:
- $$\partial_\mu F^{\mu\nu} = J^\nu - \underbrace{g_{a\gamma}\tilde{F}_{\mu\nu}\partial_\mu a}_\downarrow$$
- $$J_a^\mu = g_{a\gamma}(-\mathbf{B} \cdot \nabla a, -\mathbf{E} \times \nabla a + \partial_t a \mathbf{B})$$

- Rewrite Maxwell's equations with  $J \rightarrow J + J_a$ :

$$\nabla \cdot \mathbf{E} = \rho$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \frac{\partial \mathbf{E}}{\partial t} + \mathbf{J}$$

# Ampere's law

$$\nabla \times \mathbf{B} = \frac{\partial \mathbf{E}}{\partial t} + \cancel{\mathbf{J}} - g_{a\gamma} \left( \mathbf{E} \cancel{\times} \nabla a - \frac{\partial a}{\partial t} \mathbf{B} \right)$$

Usually not important unless experiment larger than

$$\lambda_{dB} \sim (\nabla a)^{-1} \sim (m_a \mathbf{v})^{-1} \sim 10^3 \lambda_c$$

(Most experiments are actually around  $\lambda_c \sim 1/m_a$  or smaller)

# Ampere's law

$$\nabla \times \mathbf{B} = \frac{\partial \mathbf{E}}{\partial t} + g_{a\gamma} \frac{\partial a}{\partial t} \mathbf{B}$$

Axion-induced magnetic field      Axion-induced electric field      Oscillating axion field

**What kind of experiment do we need to measure this?**

→ Depends on size of Compton wavelength ( $1/m_a$ ) relative to the size of some instrument, say  $\mathcal{O}(\text{metres})$

# Haloscope strategies

< $\mu\text{eV}$ : Compton wavelength long relative to experiment. DC magnetic field induces oscillating magnetic field

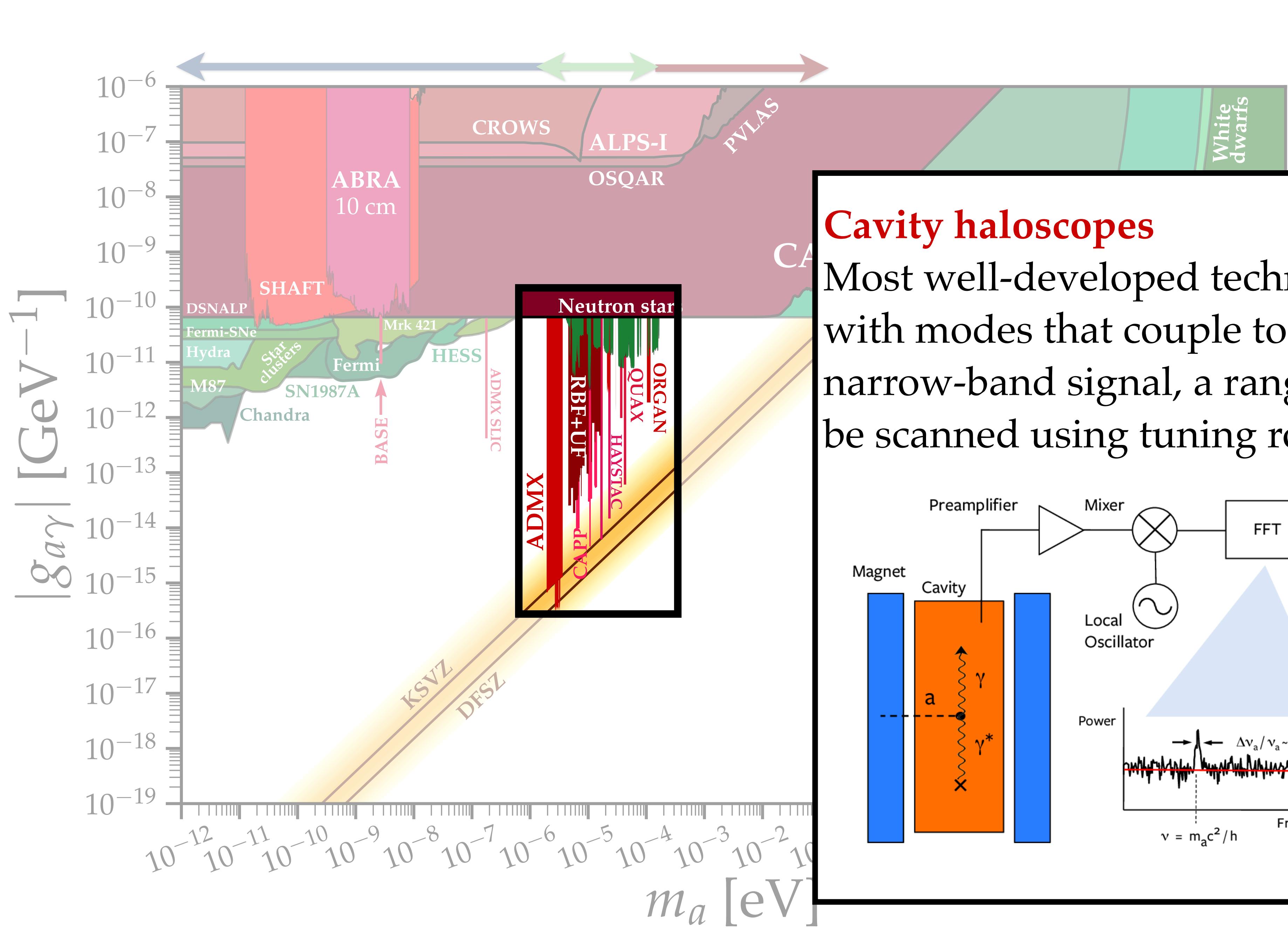
$$\nabla \times \mathbf{B}_a = \frac{\partial \mathbf{E}_a}{\partial t} - g_{a\gamma} \mathbf{B}_0 \frac{\partial a}{\partial t}$$

$\sim 1\text{-}100 \mu\text{eV}$ : Compton wavelength similar scale to experiment. Axion sources oscillating E&M-fields  $\rightarrow$  couple to an EM mode inside a cavity

$$\nabla \times \mathbf{B}_a = \frac{\partial \mathbf{E}_a}{\partial t} - g_{a\gamma} \mathbf{B}_0 \frac{\partial a}{\partial t}$$

$\gtrsim 100 \mu\text{eV}$ : Compton wavelength short relative to experiment. Axion generates radiation  $\rightarrow$  arrange experiment to have constructive interference

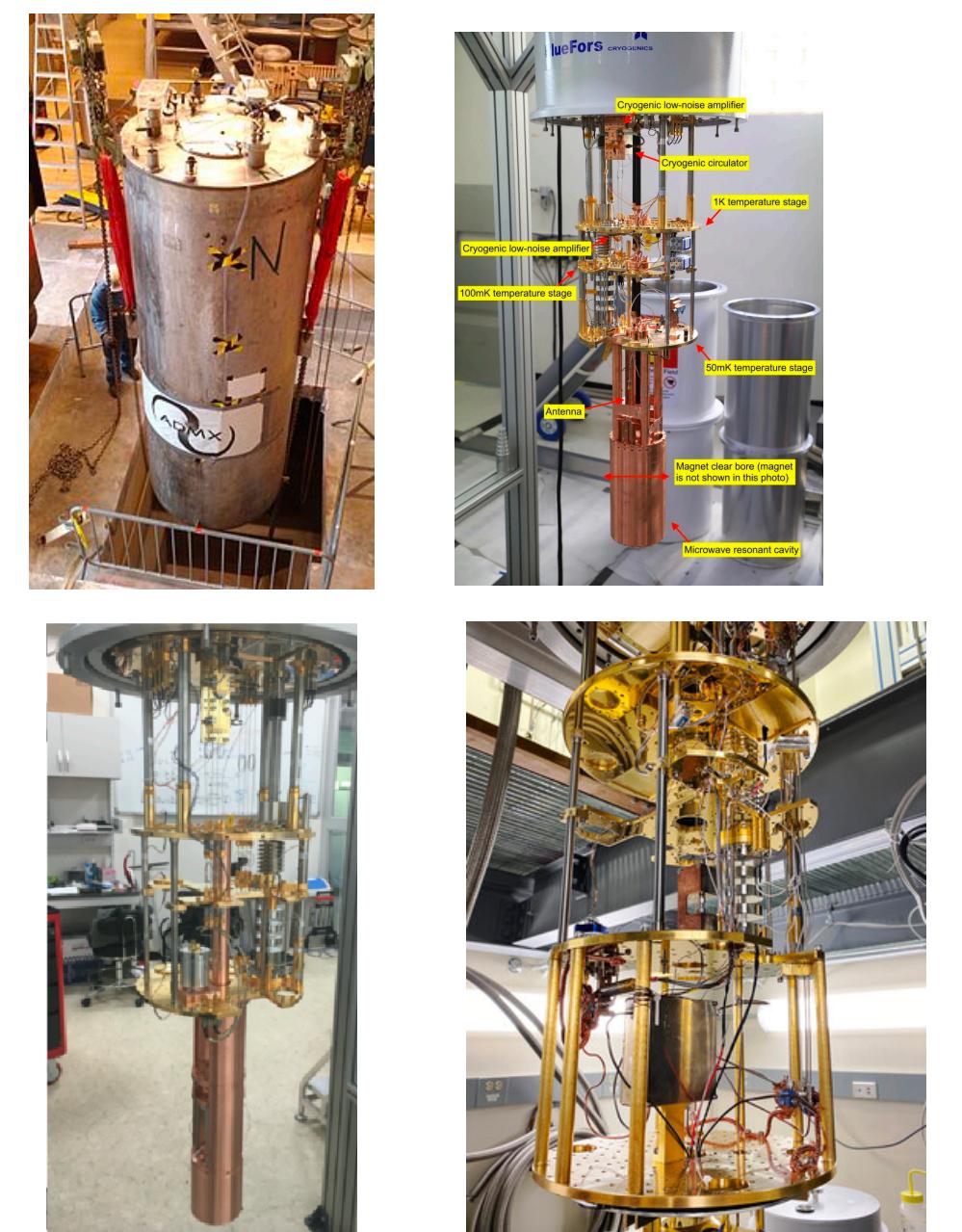
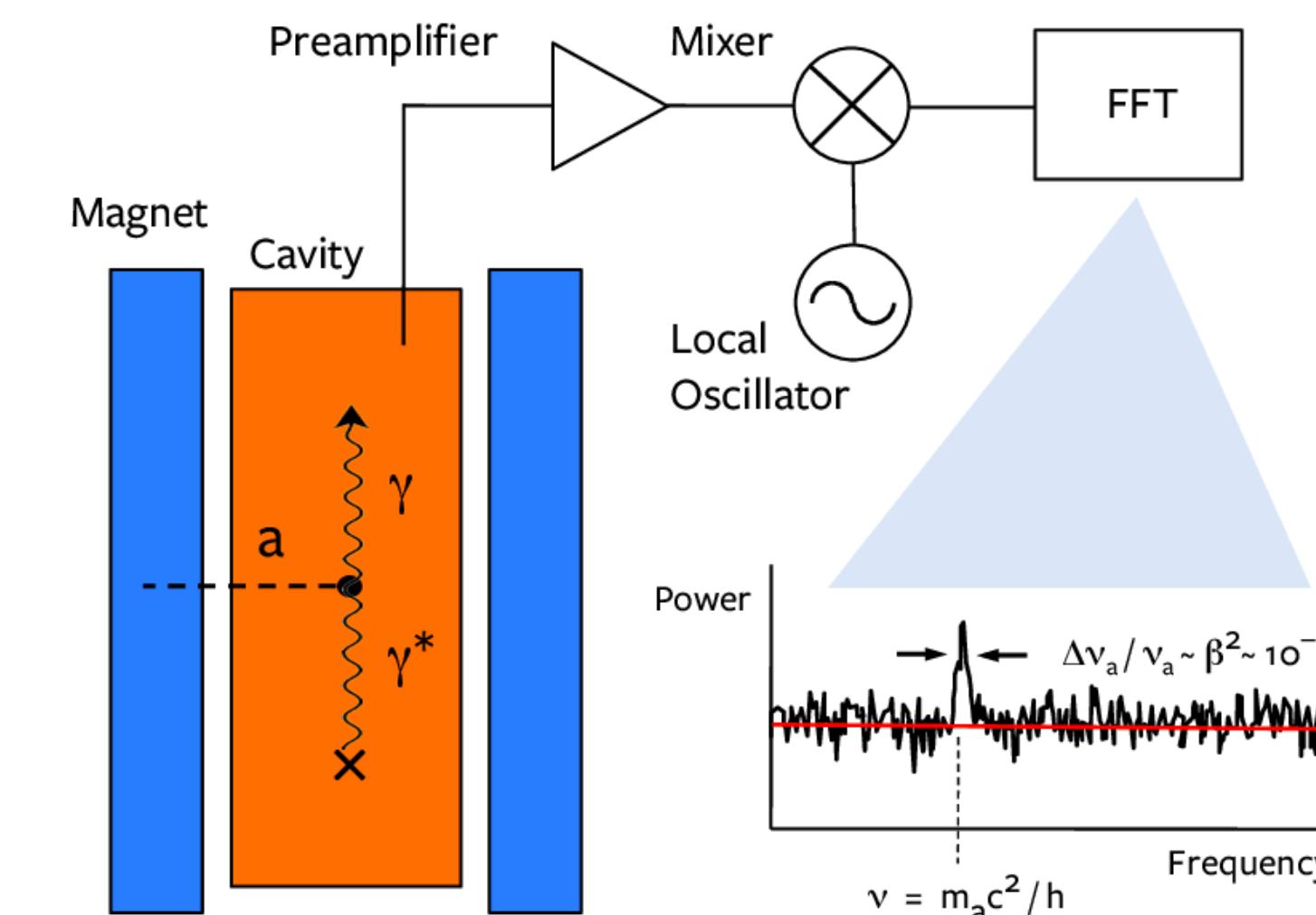
$$\nabla \times \mathbf{B}_a = \frac{\partial \mathbf{E}_a}{\partial t} - g_{a\gamma} \mathbf{B}_0 \frac{\partial a}{\partial t}$$

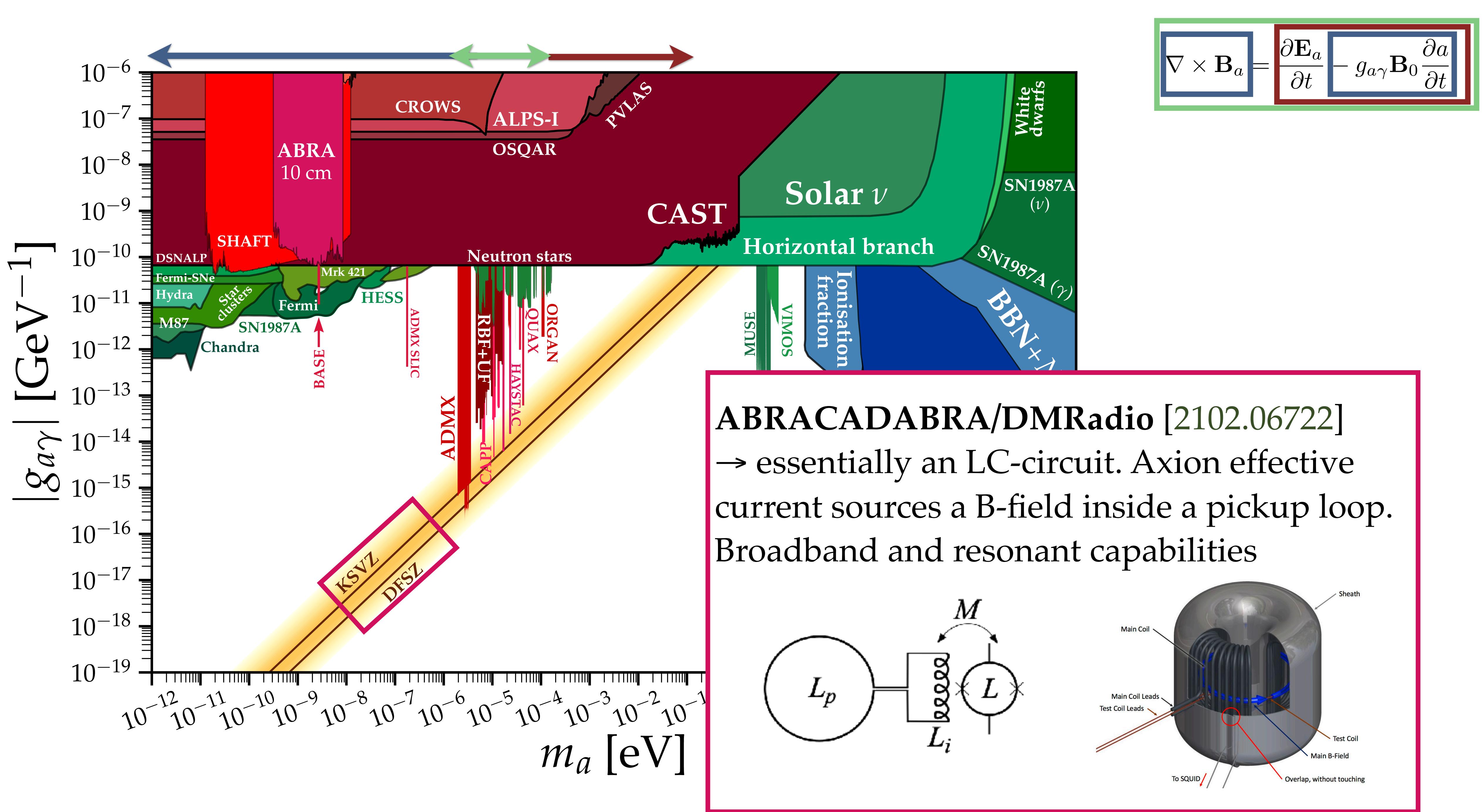


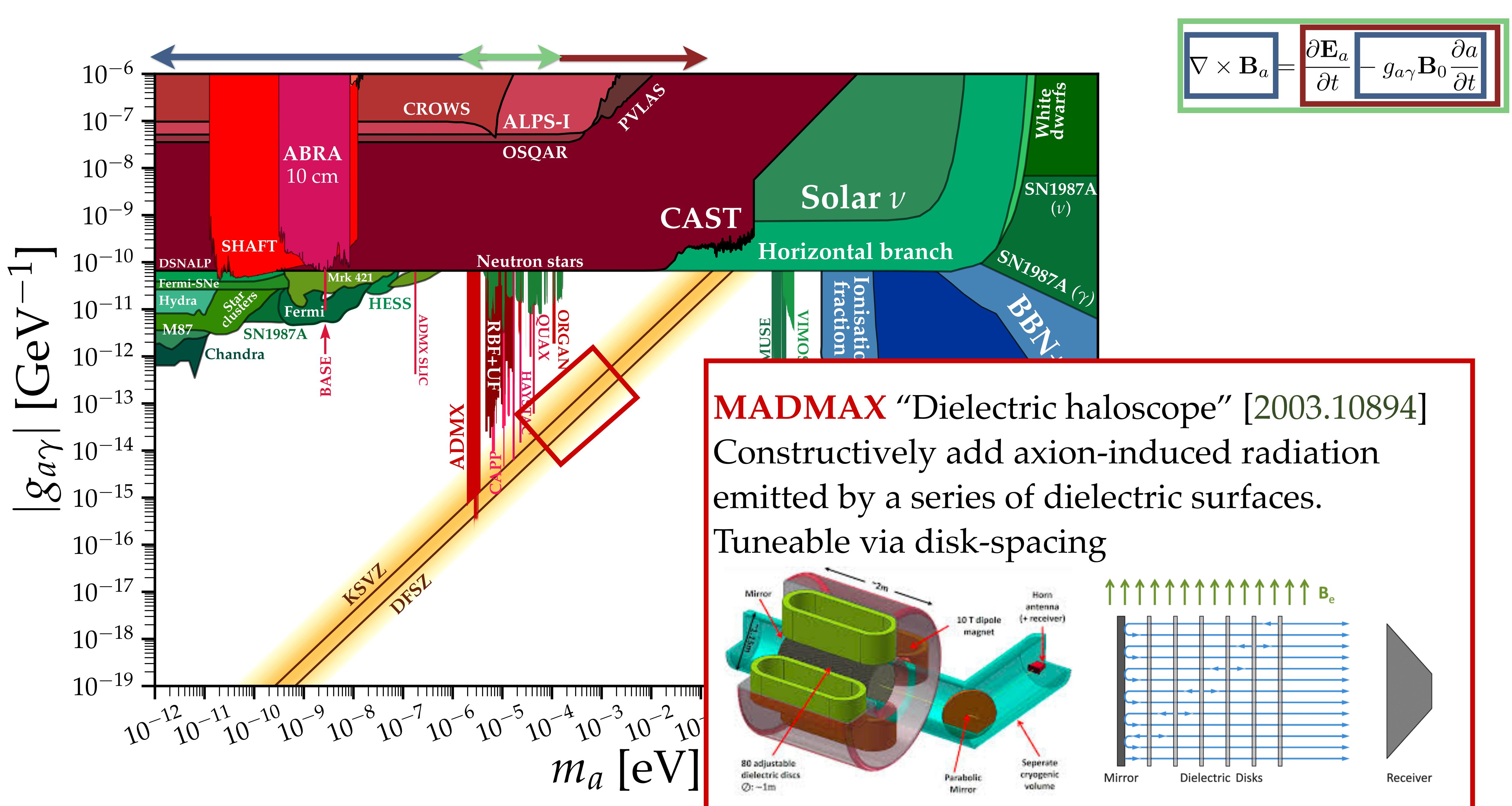
$$\nabla \times \mathbf{B}_a = \frac{\partial \mathbf{E}_a}{\partial t} - g_{a\gamma} \mathbf{B}_0 \frac{\partial a}{\partial t}$$

## Cavity haloscopes

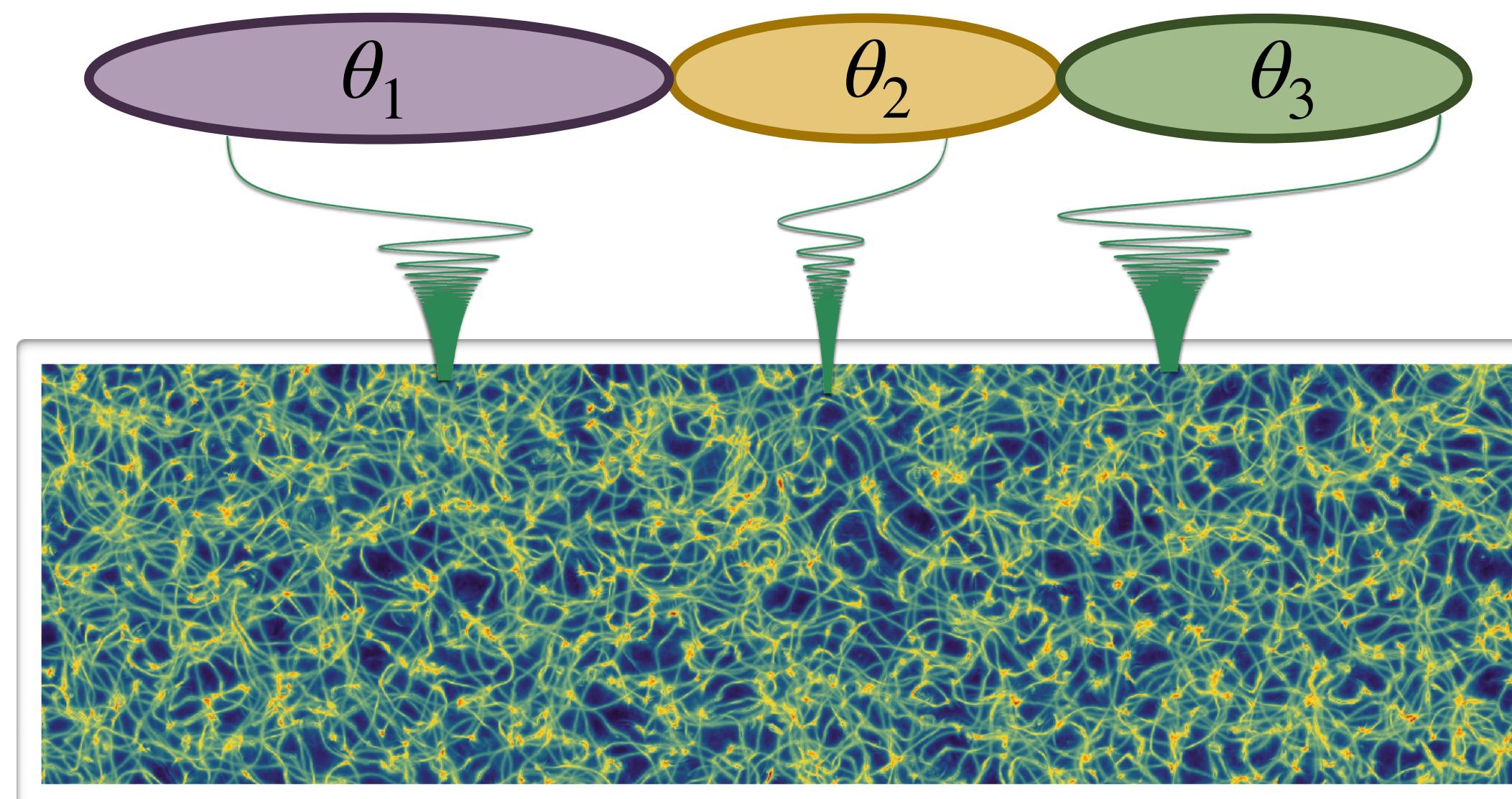
Most well-developed technique. High-Q cavity with modes that couple to the axion giving strong narrow-band signal, a range of axion masses can be scanned using tuning rods







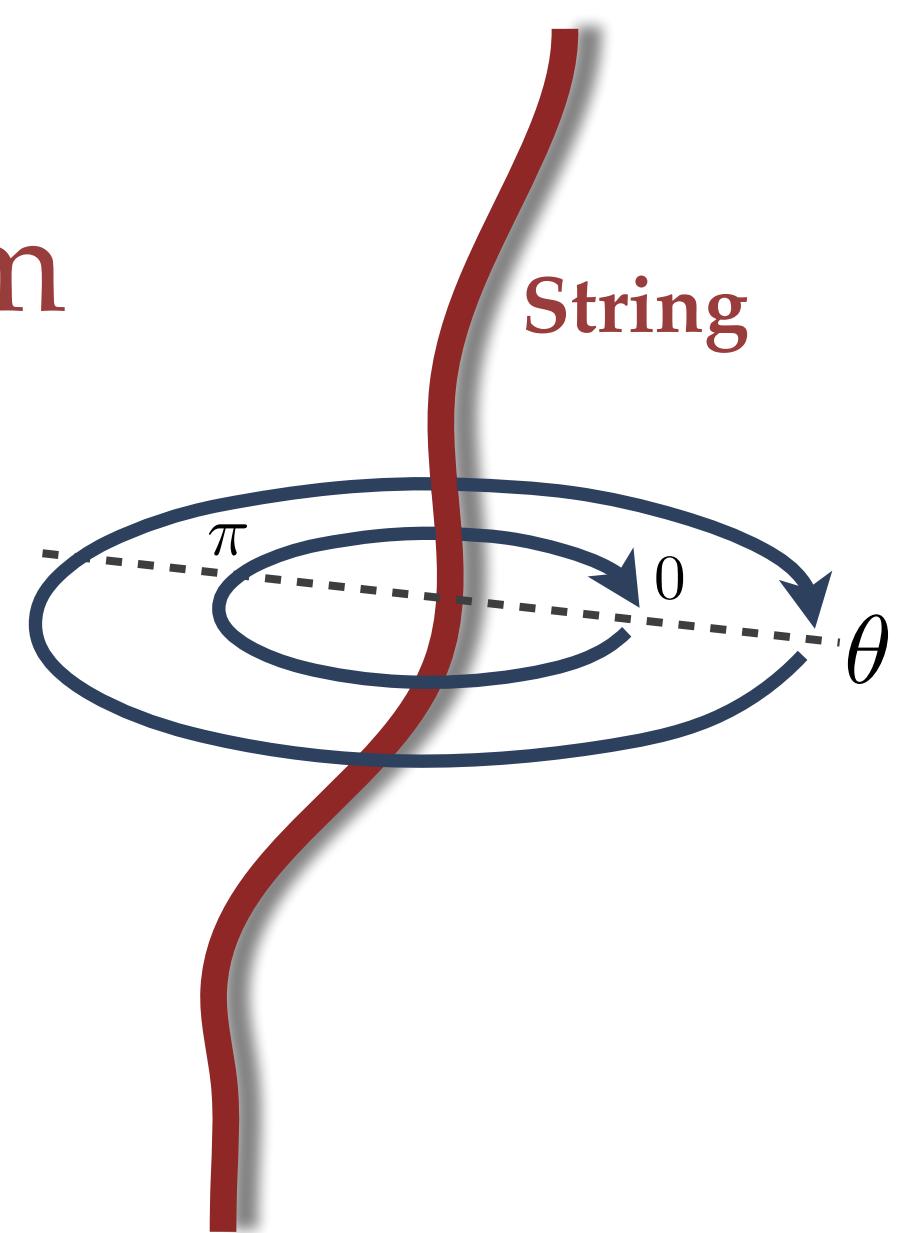
# But there's a complication: what about $\nabla \theta$ ?



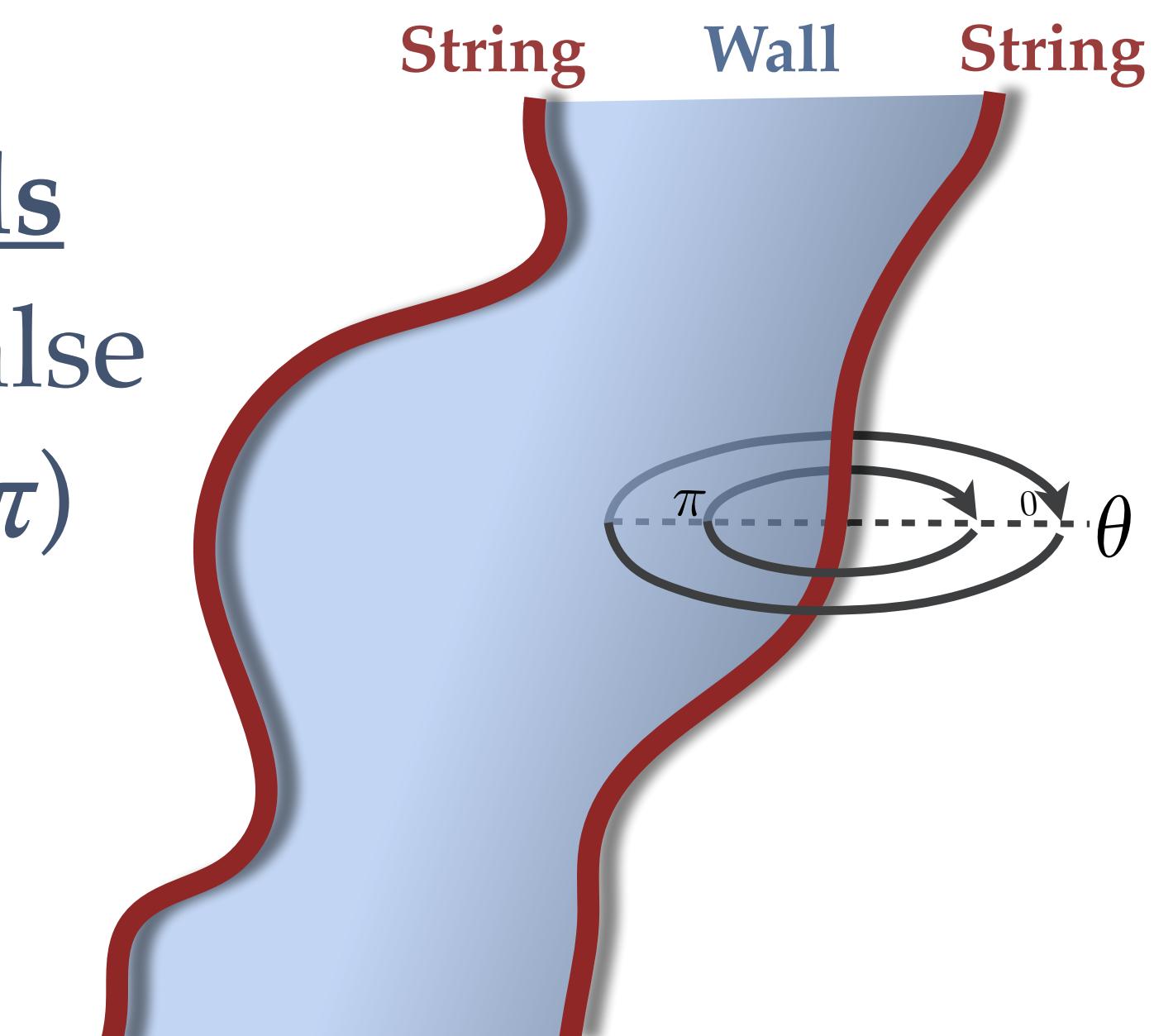
Different causal patches take on  
different initial angles  
→ Field gradients!

$$\leftarrow \ddot{\theta} + 3H\dot{\theta} - \frac{1}{R^2} \nabla^2 \theta + m_a^2 \theta = 0$$

⇒ Cosmic strings from  
axion field winding  
around  $2\pi$

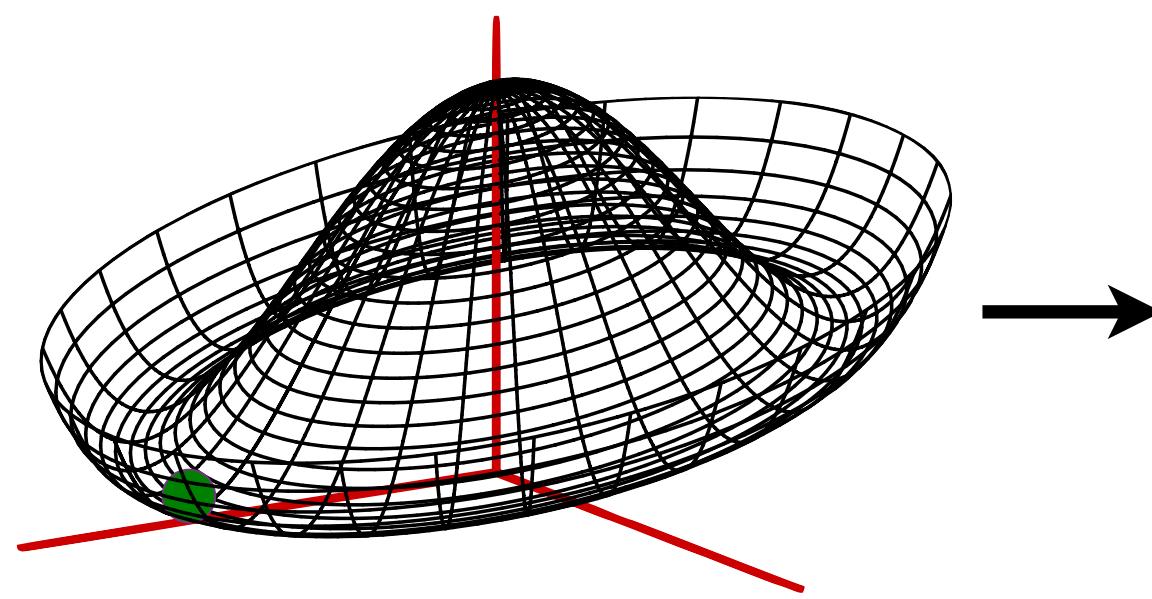


⇒ Domain walls  
between true/false  
vacuum ( $0$  and  $\pi$ )

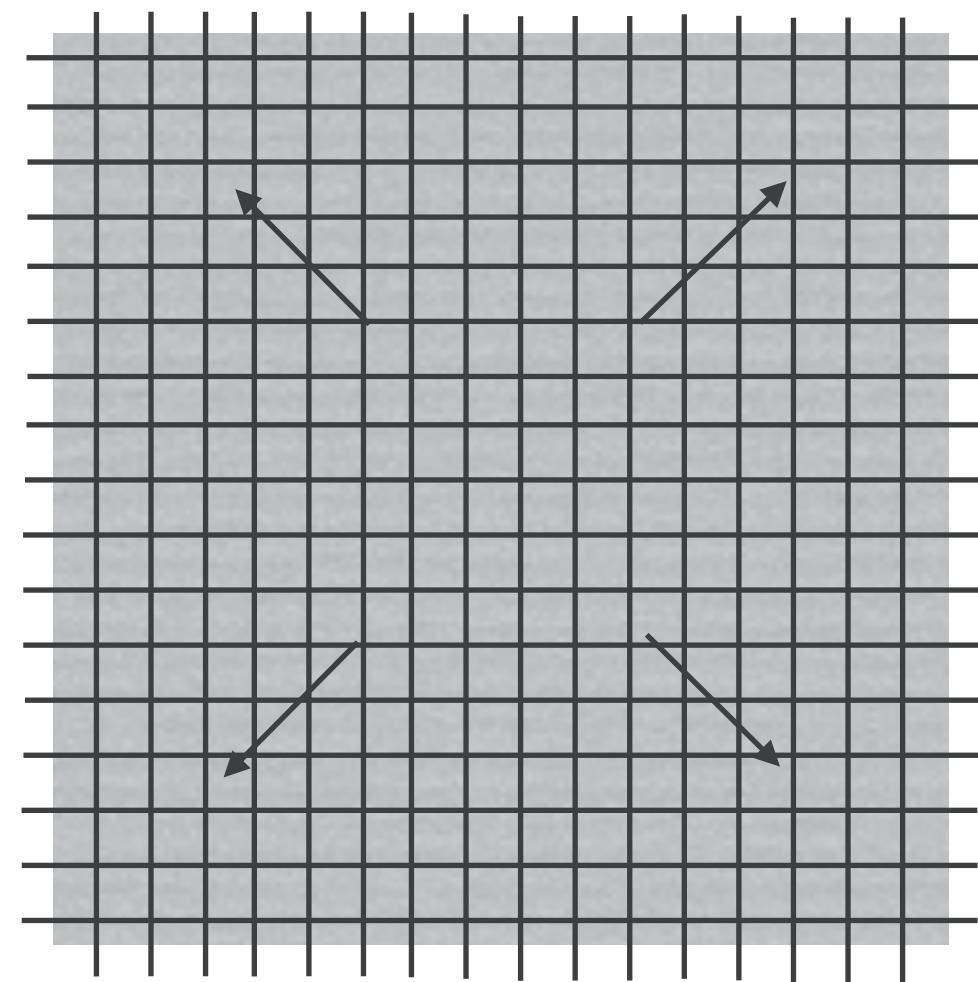


# What do we need to do?

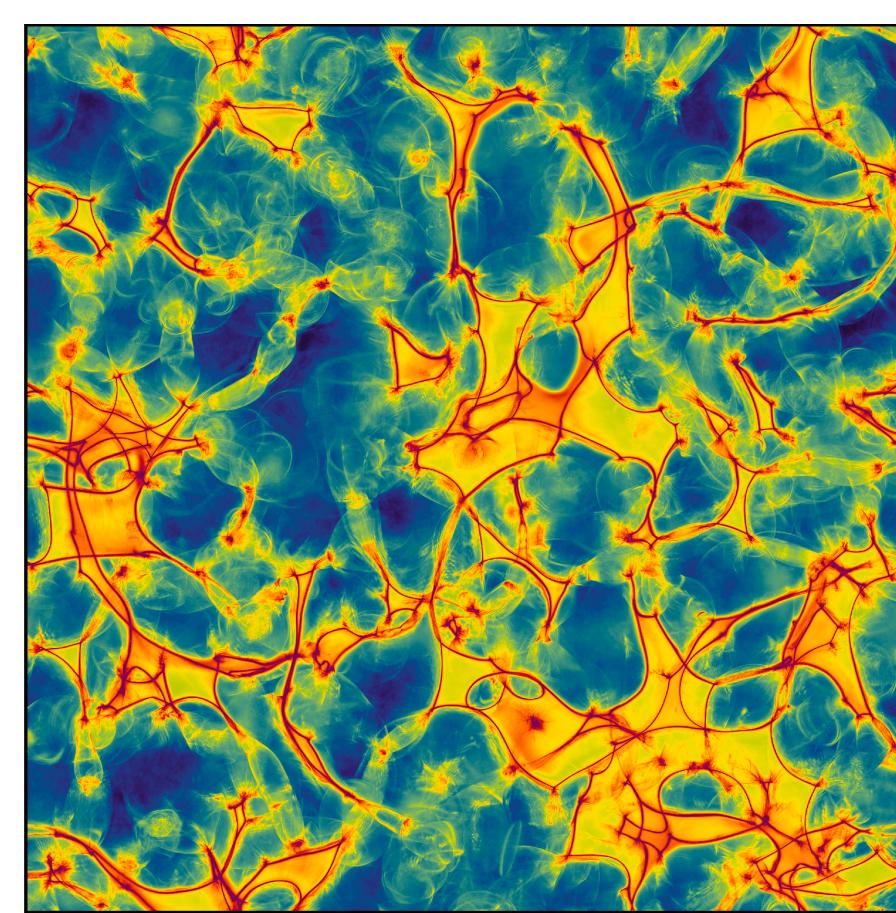
→ simulate



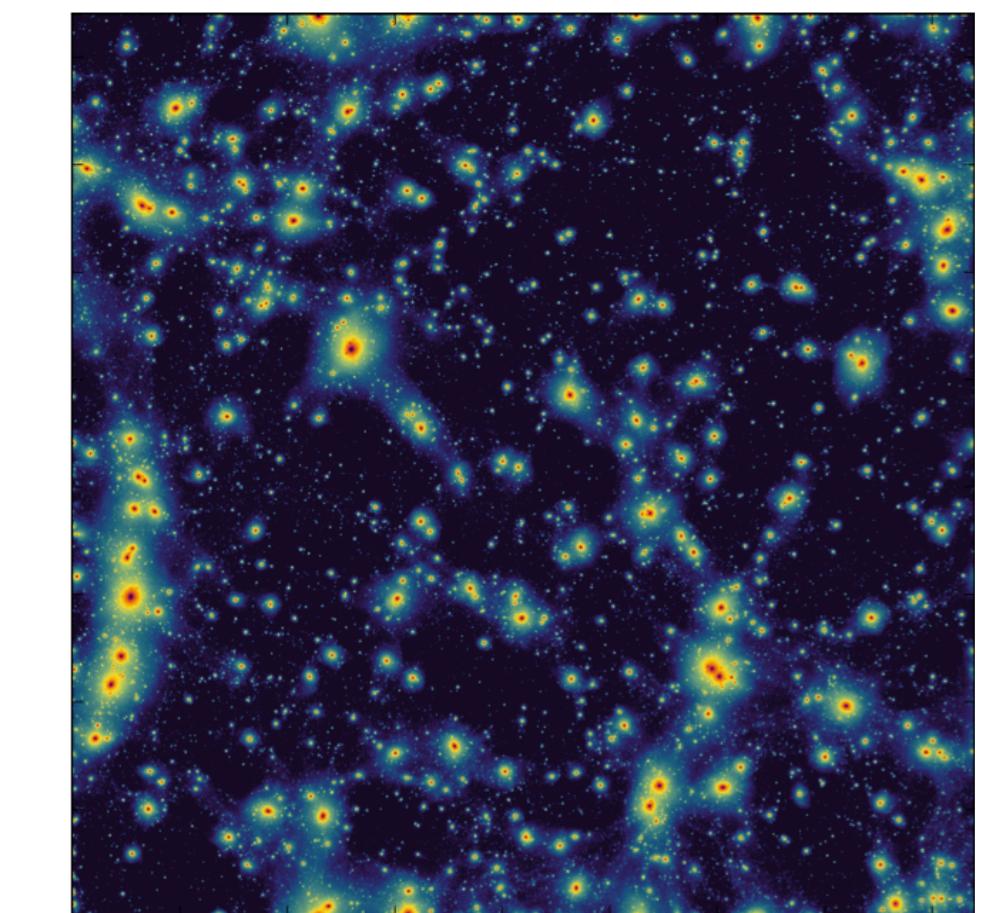
Evolve the  
axion field...



...on an  
expanding  
lattice...



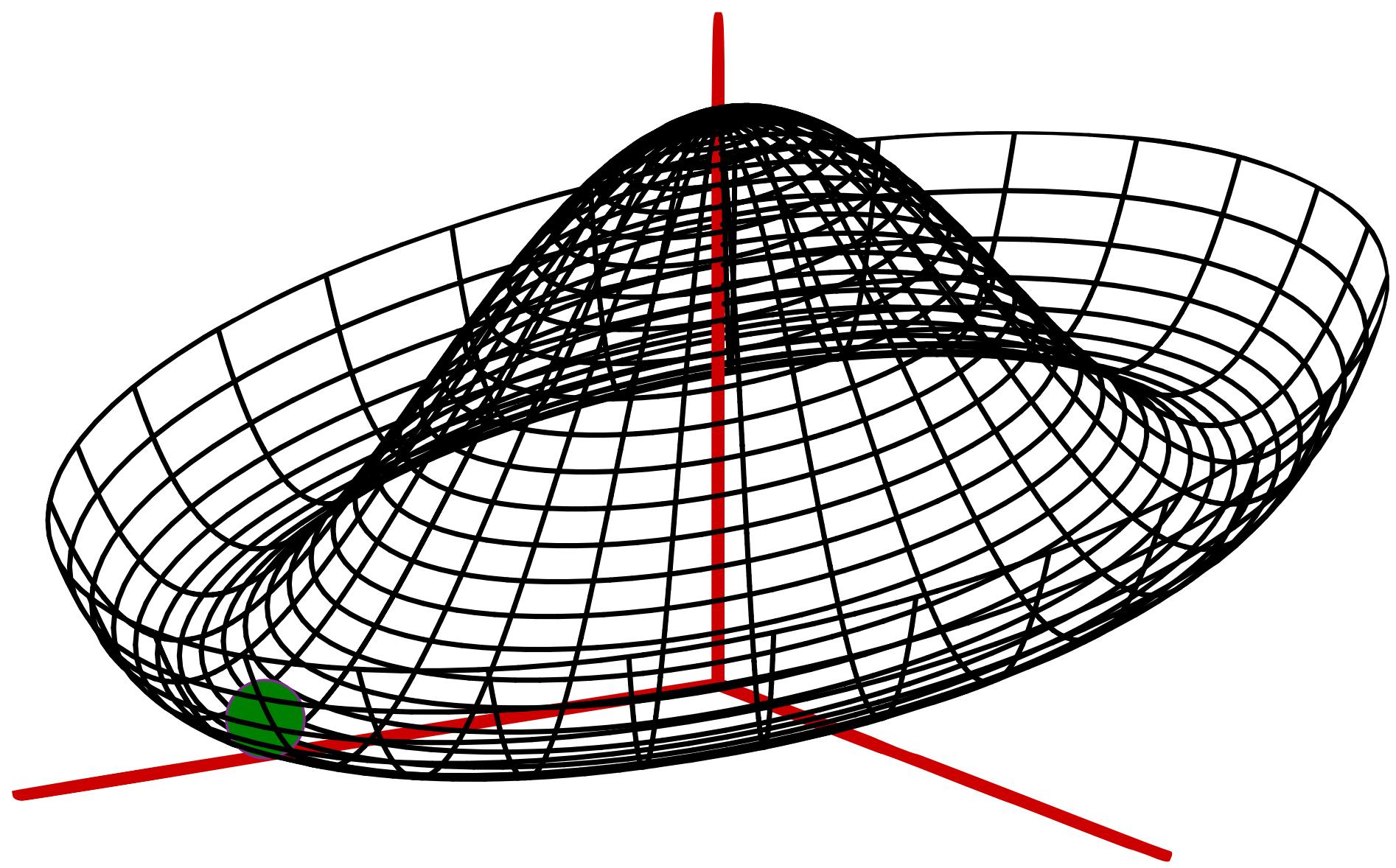
...to measure the  
relic abundance of  
axions...



...and predict its  
present day  
distribution

# The axion potential

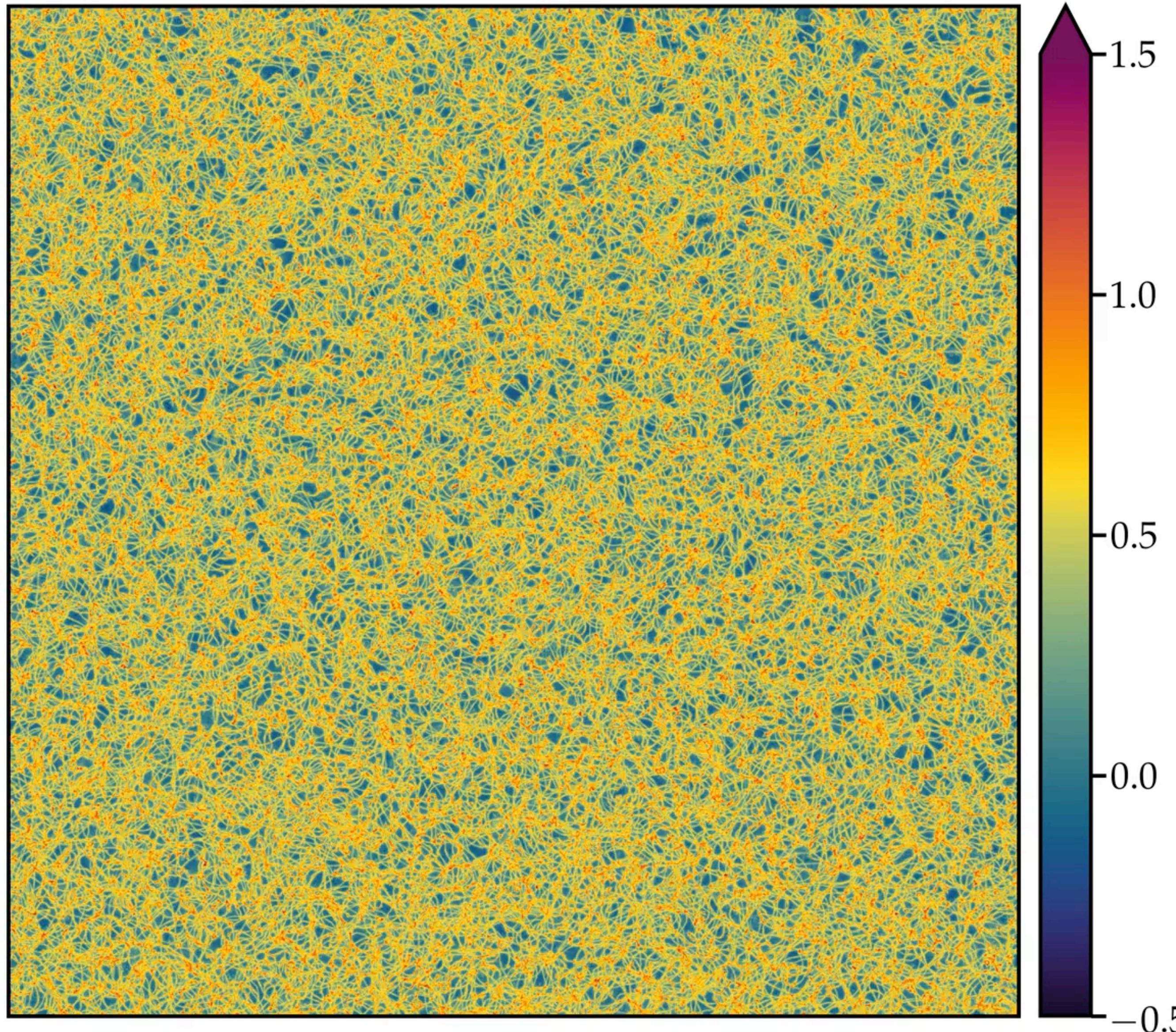
$$V(\phi) = \frac{\lambda_\phi}{8} \left( |\phi|^2 - f_a^2 \right)^2 + \chi(T)(1 - \cos \arg \phi)$$

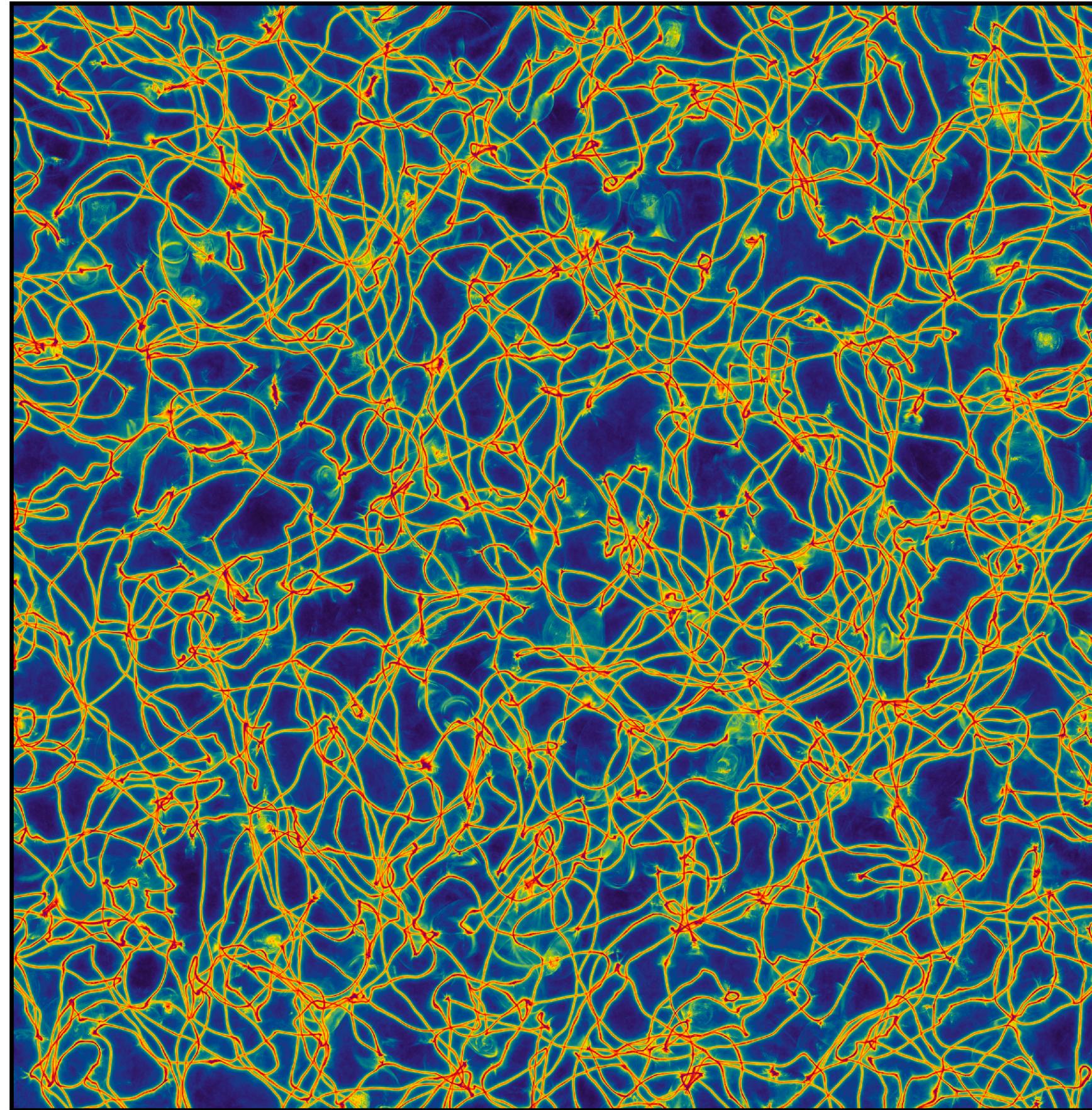


$$\phi(\mathbf{x}) \sim |\phi(\mathbf{x})| e^{i\theta(\mathbf{x})}$$

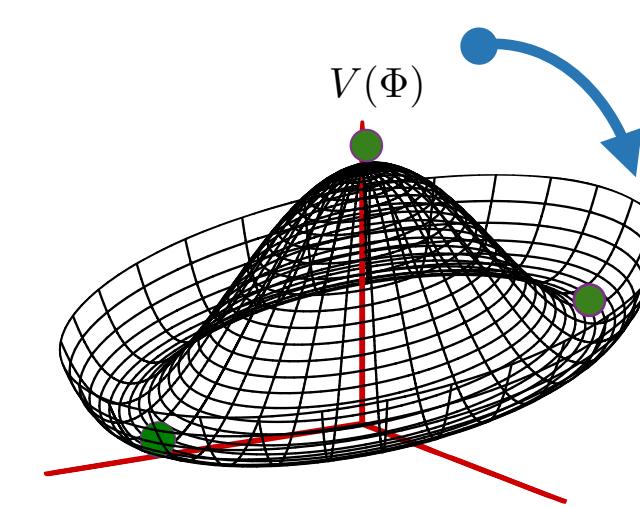
**Wine bottle**  
Governs  
 $|\phi(\mathbf{x})|$   
**Radial dof: “saxion”**  
Sets string width

**Tilt**  
Governs  
 $\theta(\mathbf{x}) = a(\mathbf{x})/f_a$   
**Angular dof: “axion”**  
Sets domain wall width

$\tau = 0.5$  $\log_{10}(\rho_a/\bar{\rho}_a)$ 



# Evolution of the axion field in the post-inflationary scenario



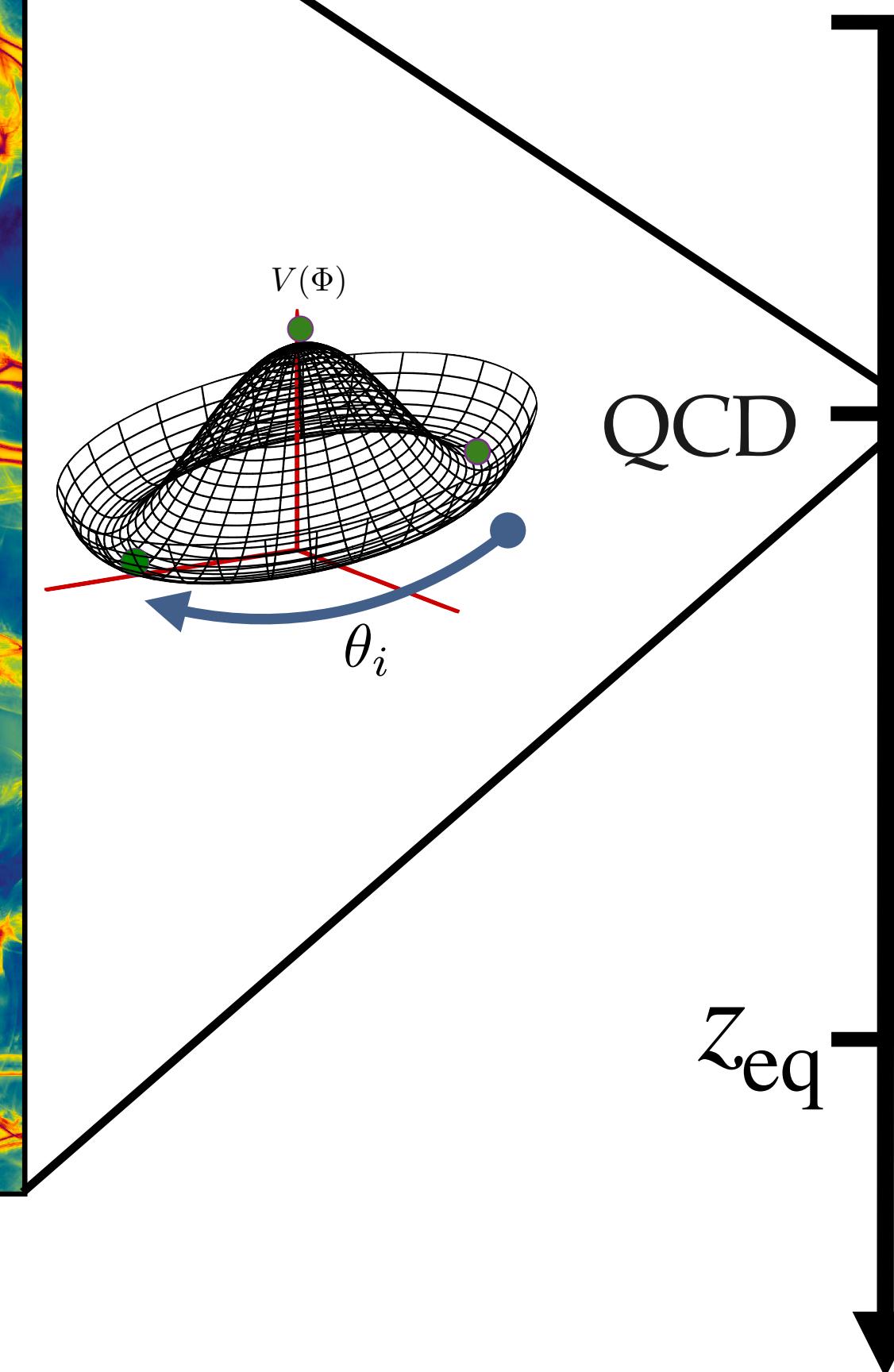
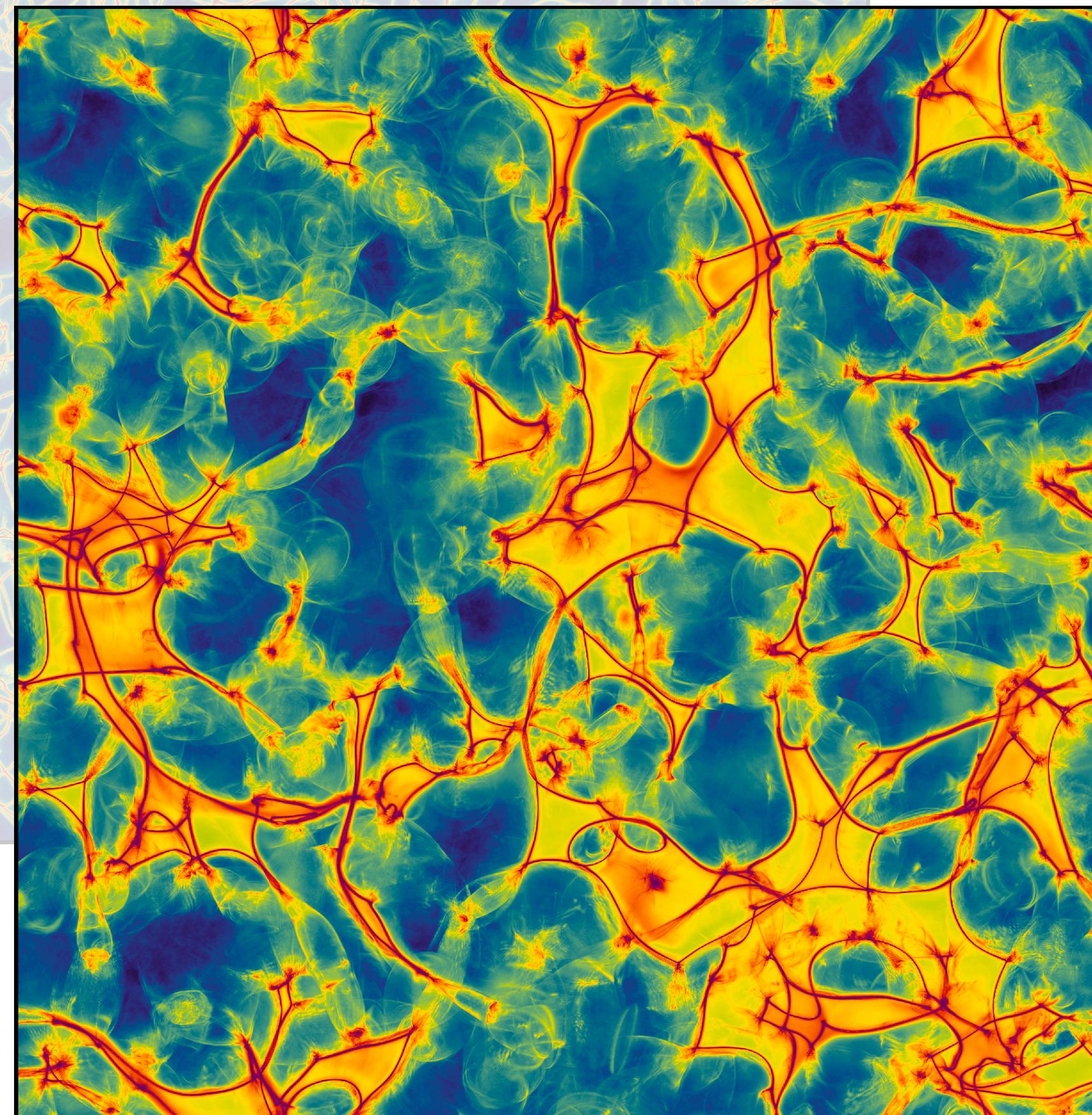
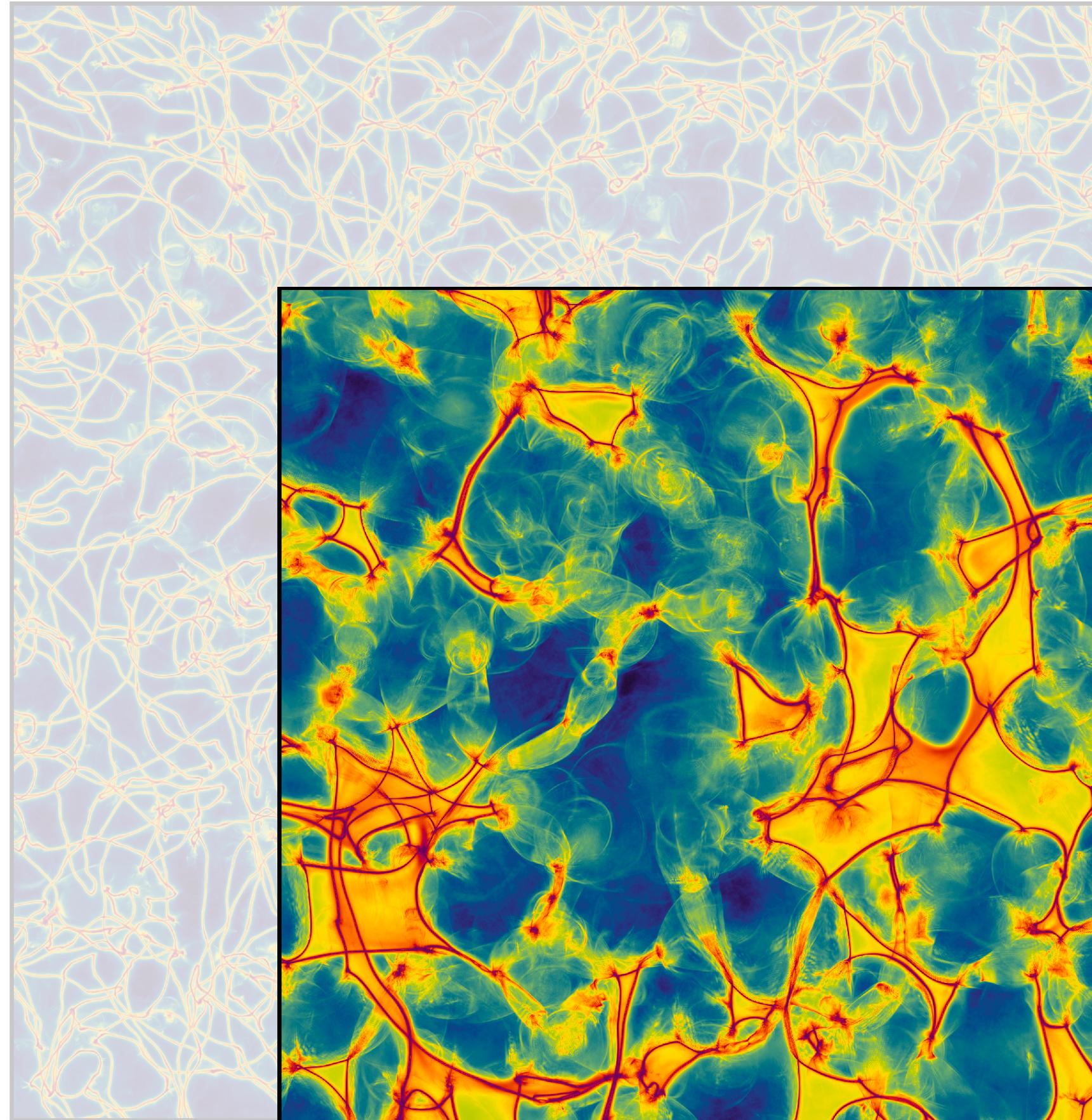
Symmetry  
breaking

QCD

$z_{\text{eq}}$

String network scaling

# Evolution of the axion field in the post-inflationary scenario



Symmetry  
breaking

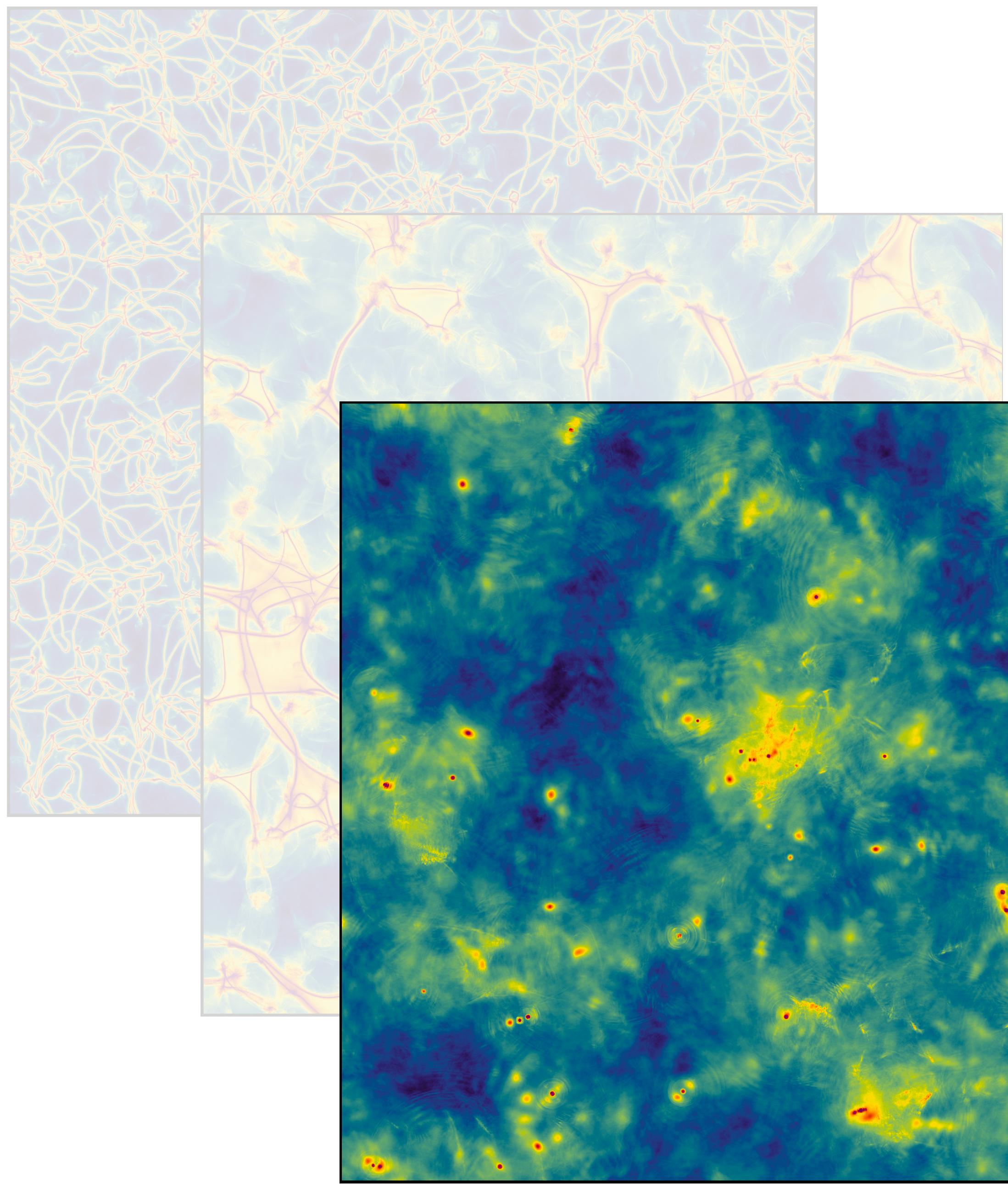
QCD

$z_{\text{eq}}$

String network scaling

Domain walls attached to strings  
→ network collapses

# Evolution of the axion field in the post-inflationary scenario

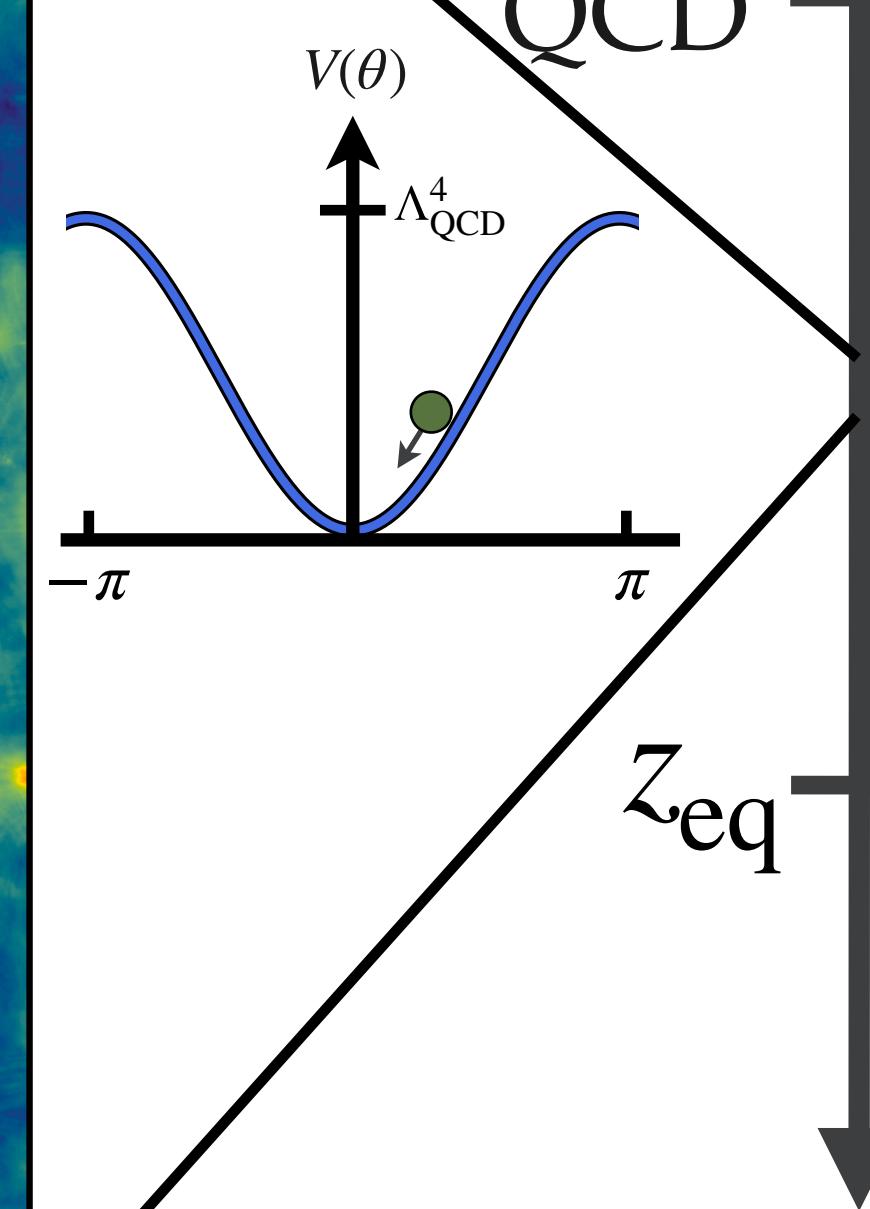


Symmetry  
breaking

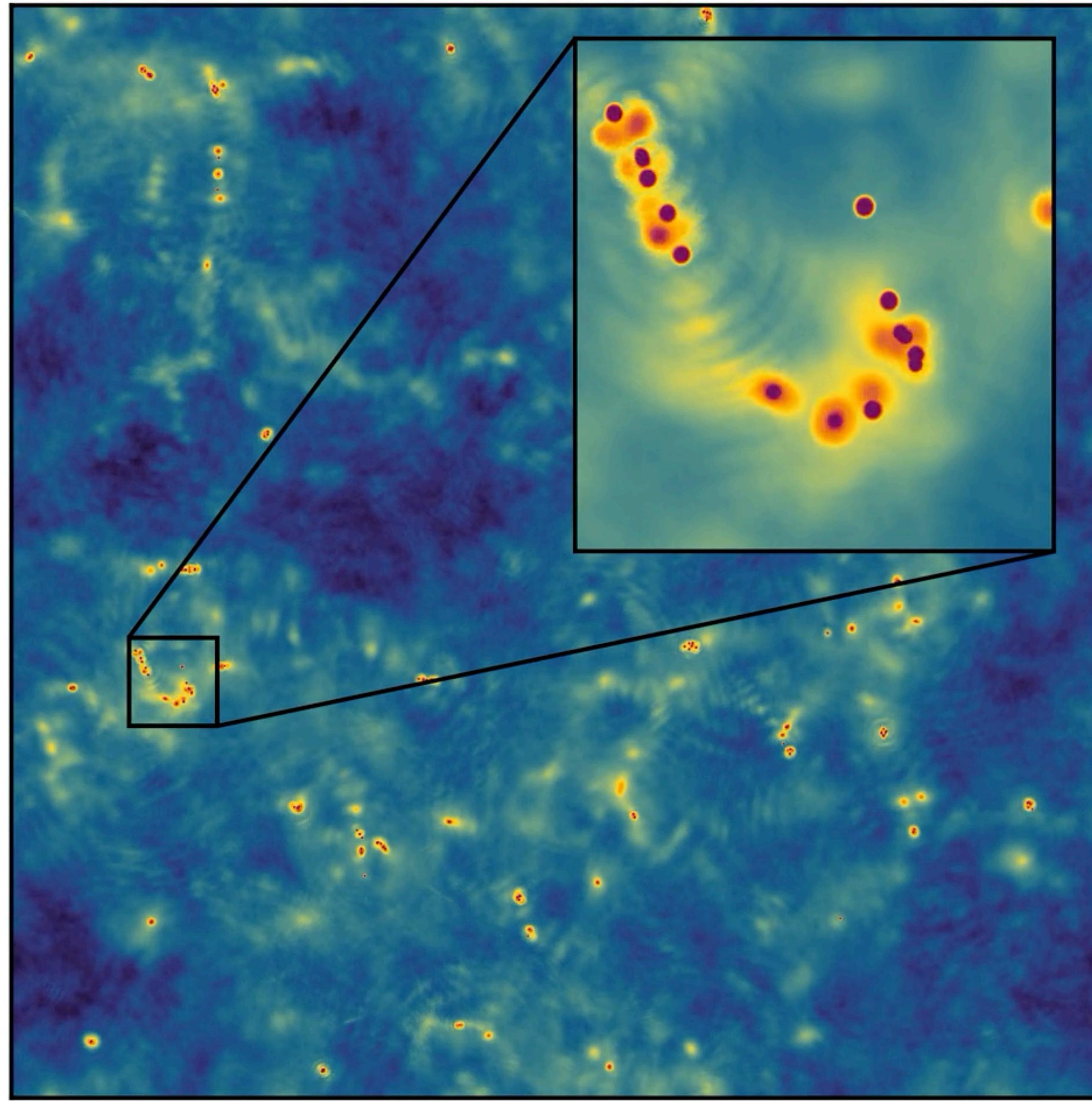
String network scaling

Domain walls attached to strings  
→ network collapses

Inhomogeneous distribution of  
axions free streams until non-  
relativistic



Seeds of structure  
gravitationally collapse  
into halos



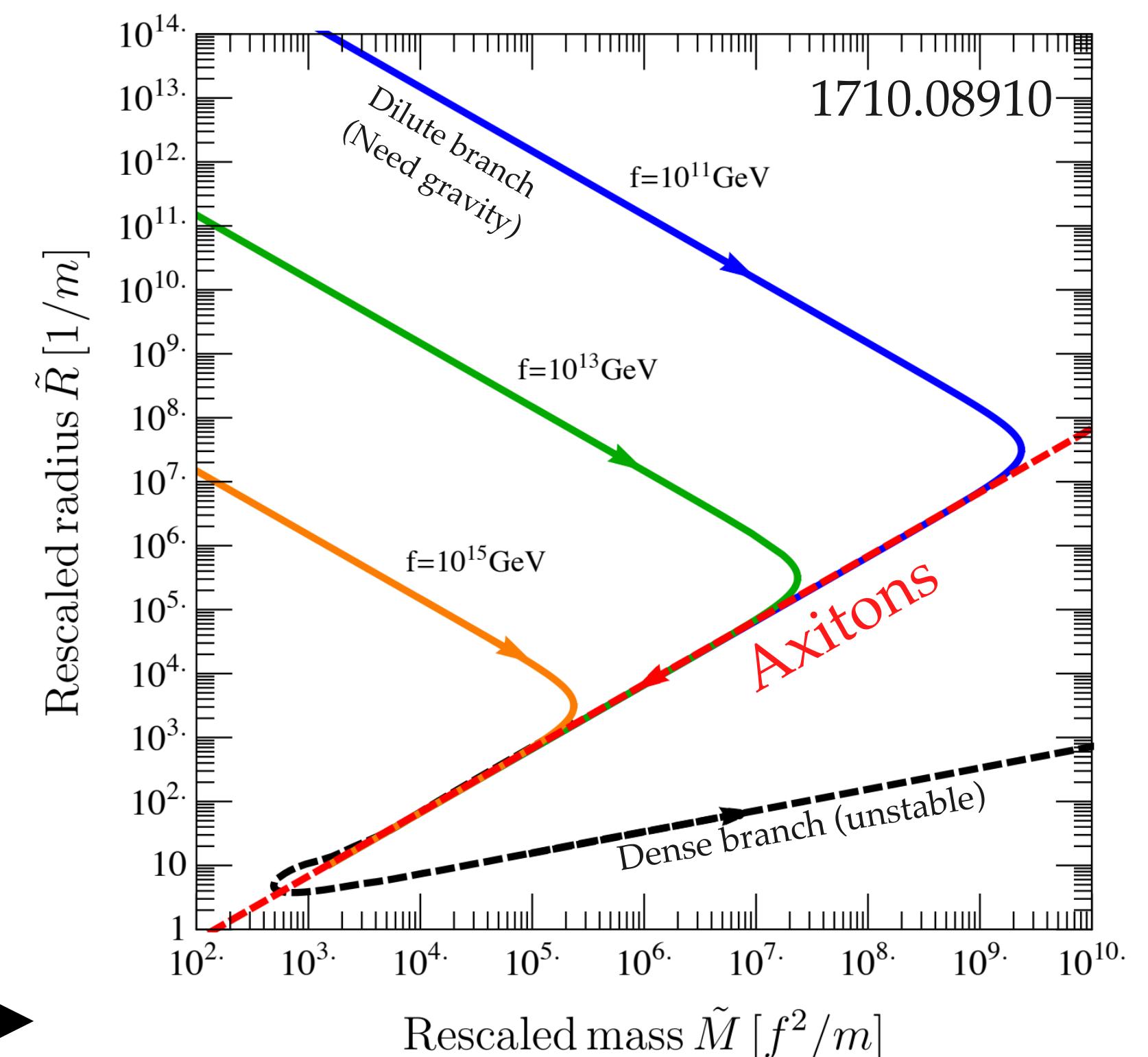
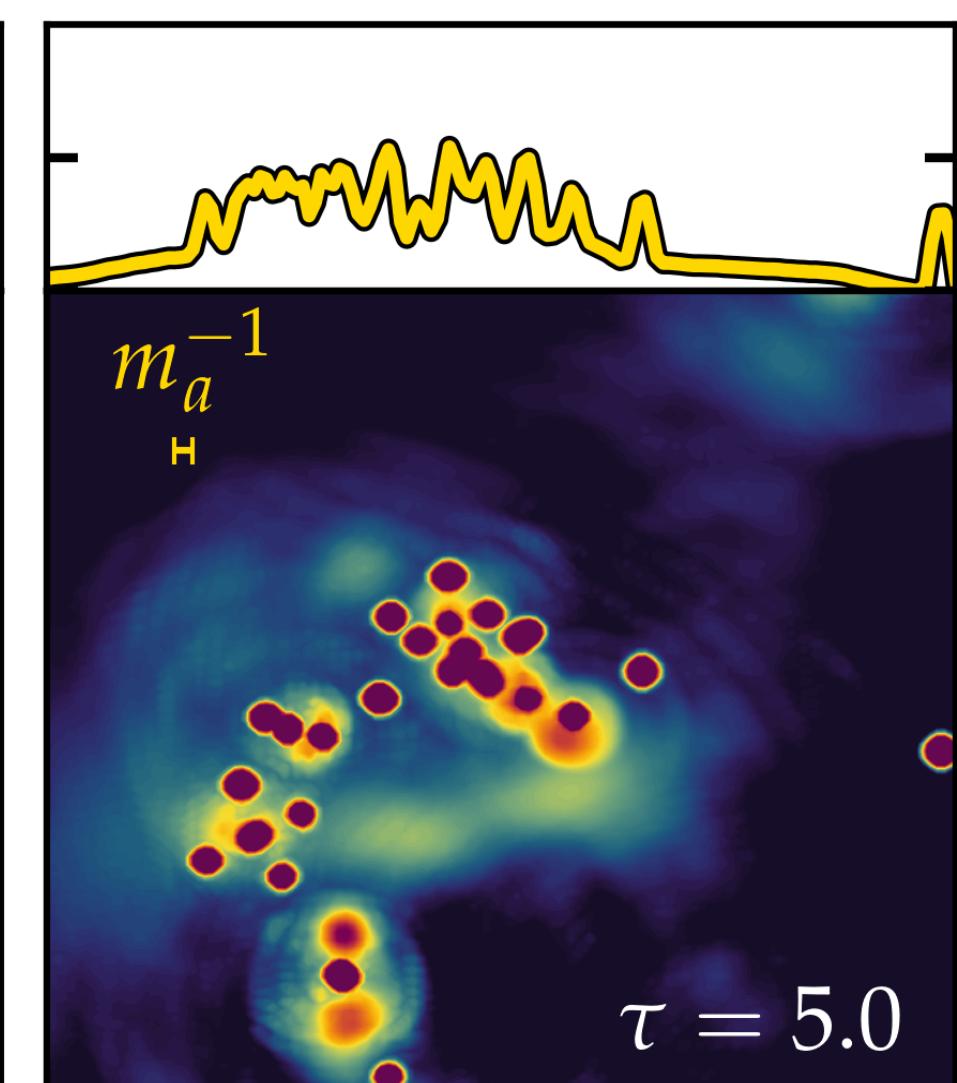
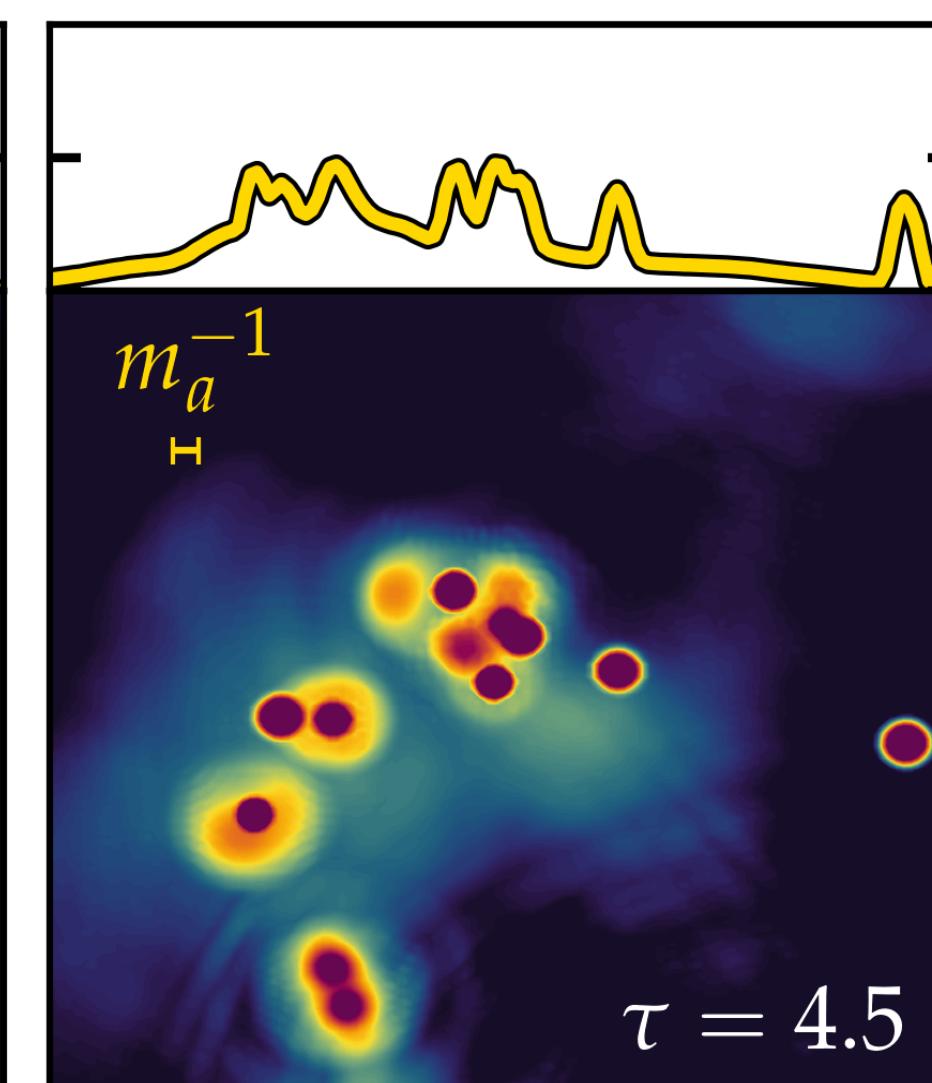
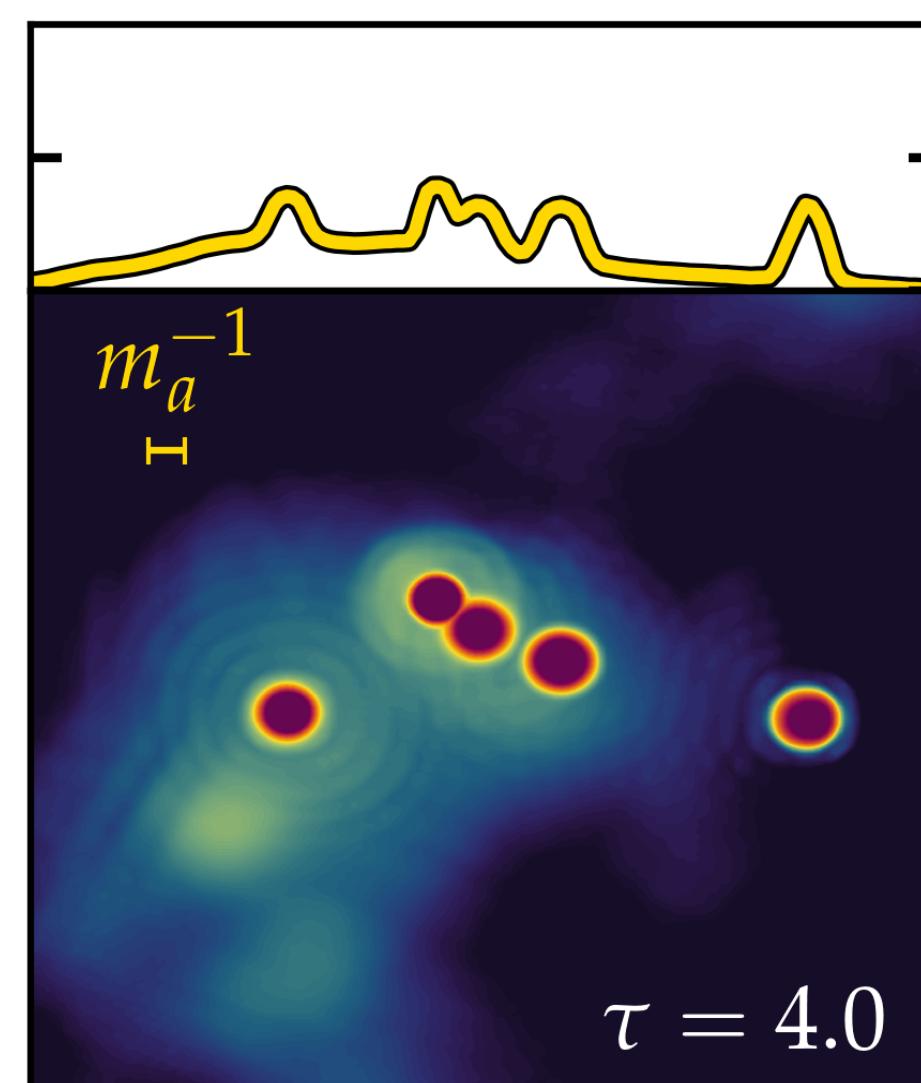
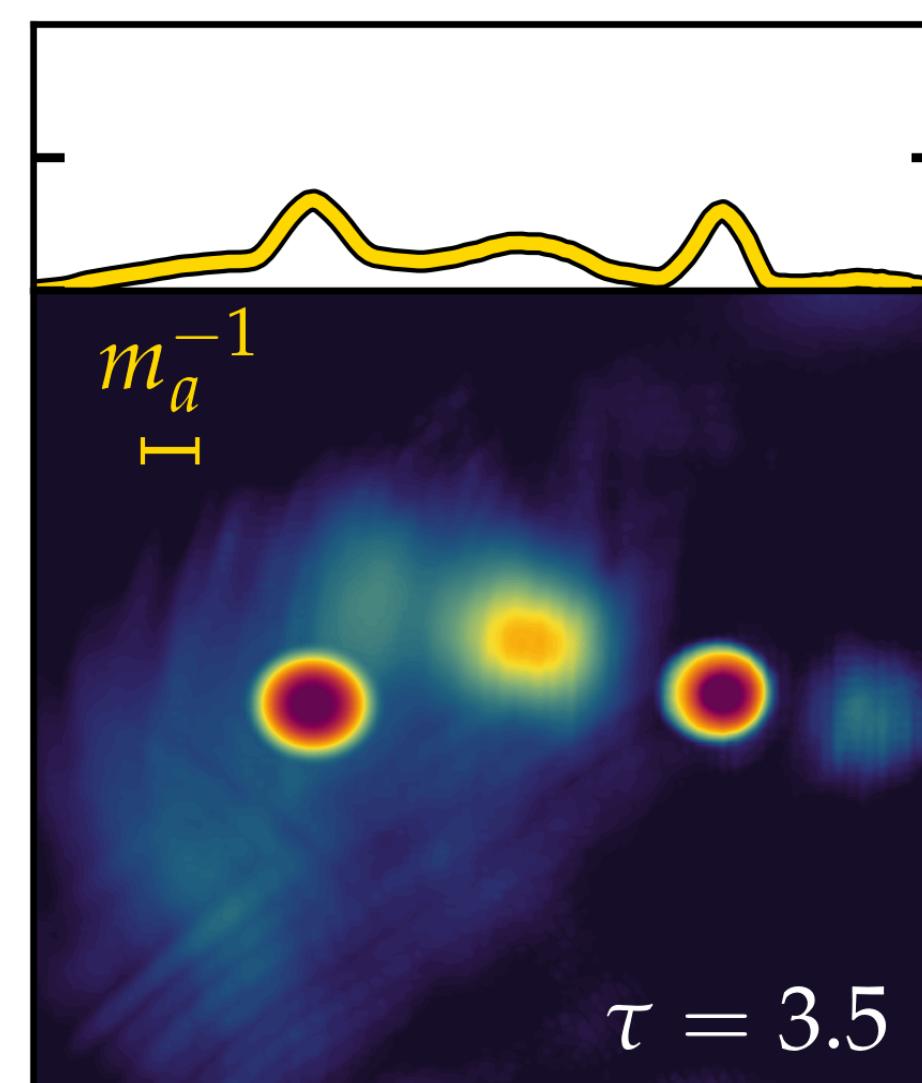
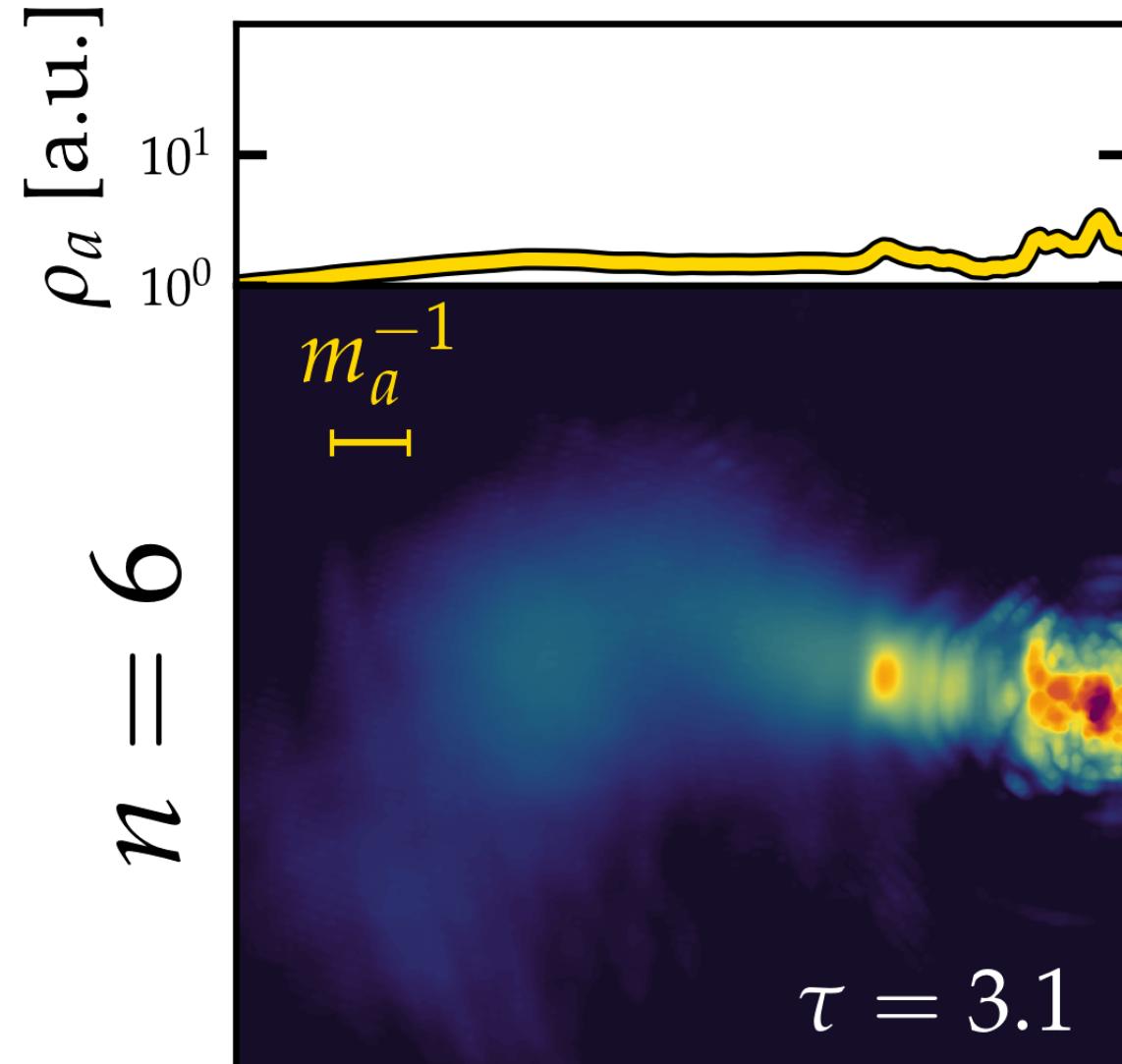
Large numbers of high-density  
oscillating lumps are seeded  
towards the end of the  
simulation

**“Axitons”**

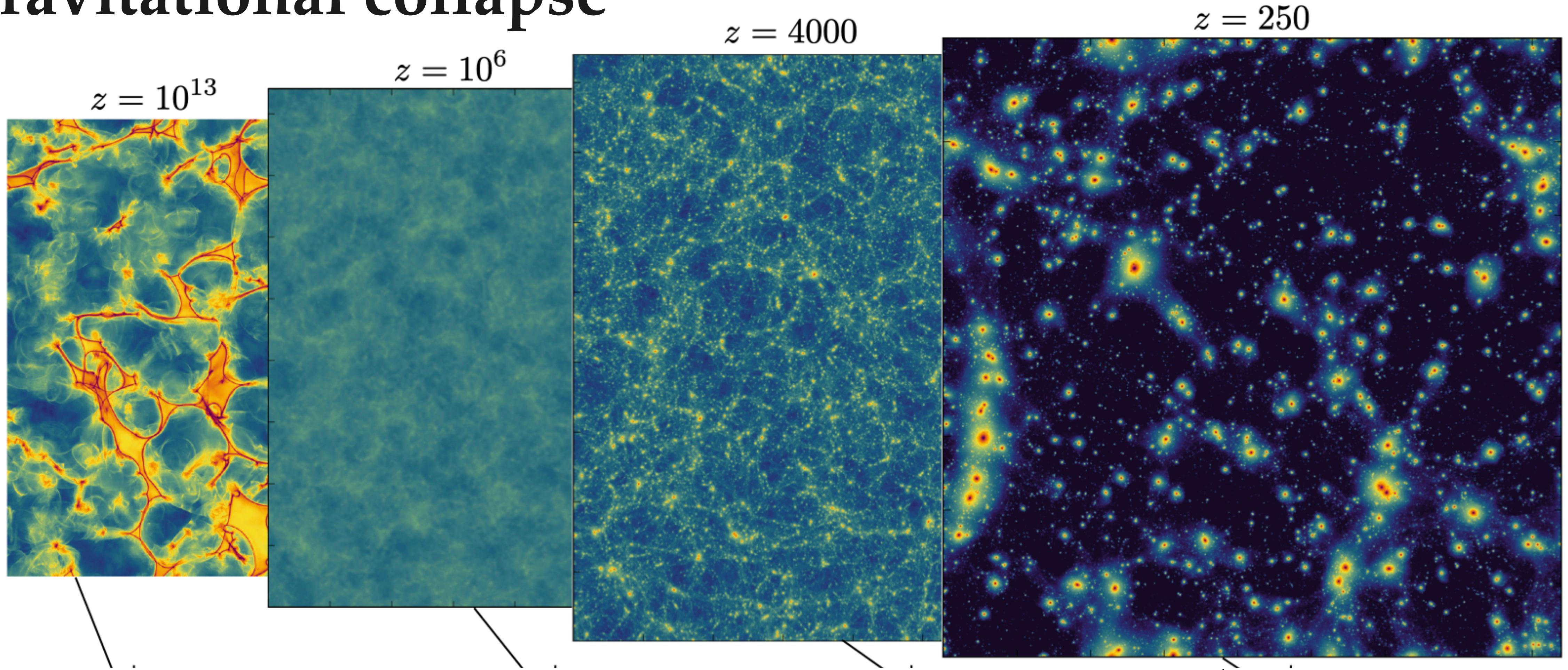
# Axitons (aka oscillons)

- Quasi-stable solutions of the Sine-Gordon equation with a growing mass  $m_a^2(T) \propto T^{-n}$
- Can also be thought of as the unstable transition between dilute and dense axion stars
- Size =  $1/m_a(T)$
- Eventually dissipate once the axion mass stops growing as  $T \rightarrow 0$

Axion mass increasing with time →



# Gravitational collapse



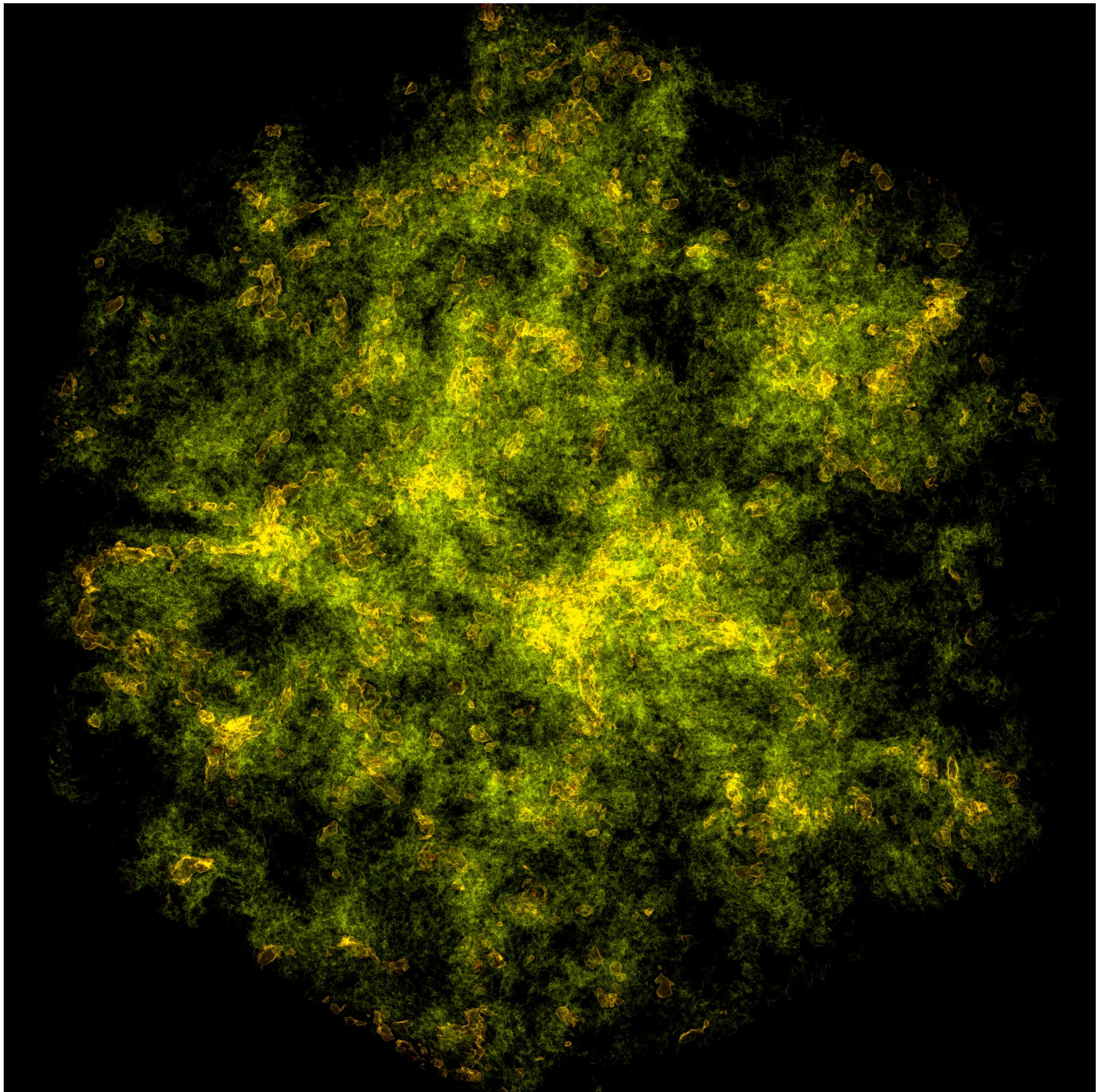
Initial conditions  
from  
field simulation

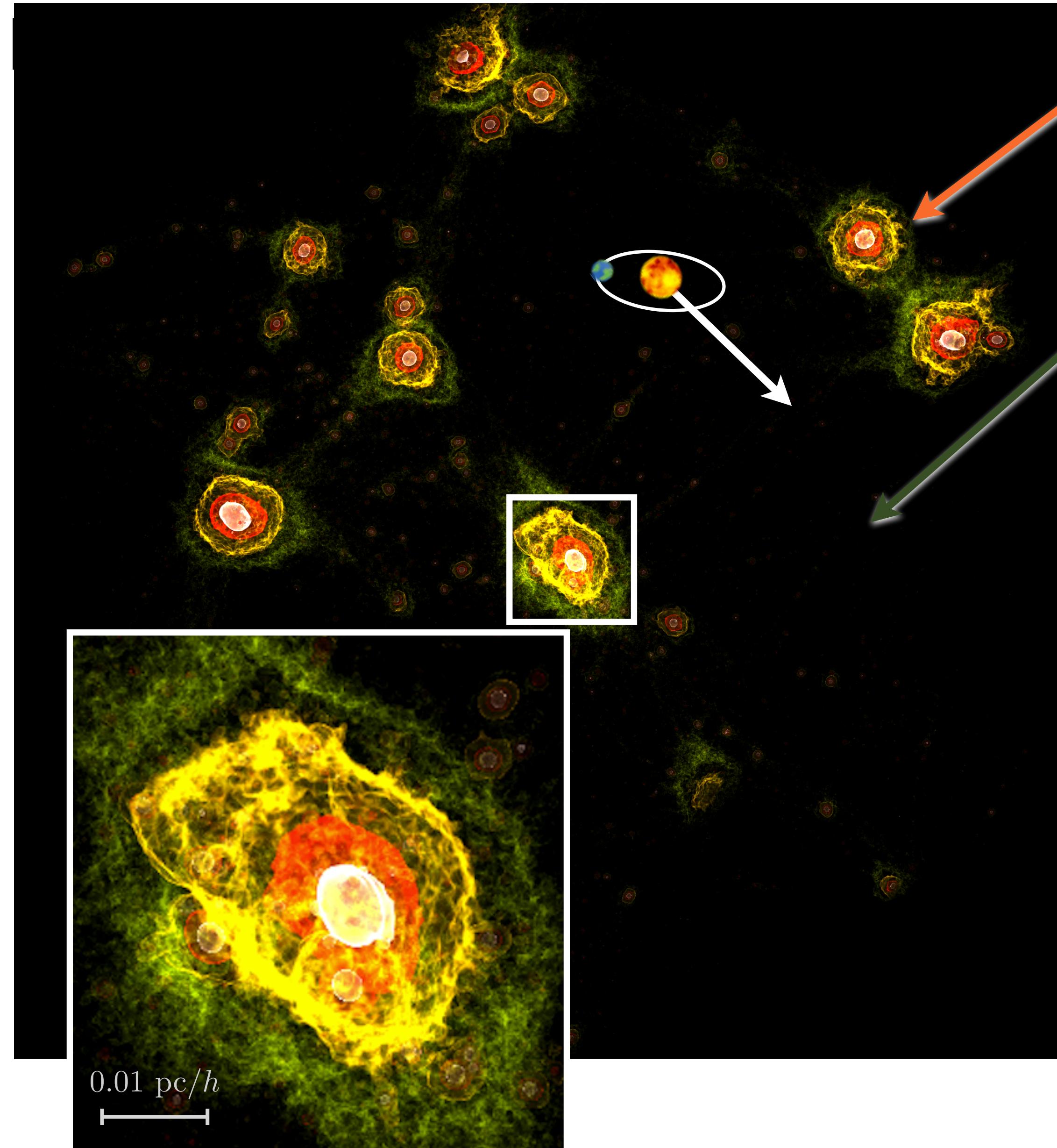
WKB approx. +  
Schrodinger-Poisson  
system for linear growth

N-body methods for  
non-linear gravitational  
collapse

Gravitational collapse of seeds left over by the axitons leads to the formation of  $\sim$ AU sized clumps of axions with masses  $M \in [10^{-15}, 10^{-9}] M_{\odot}$

→ **axion miniclusters**





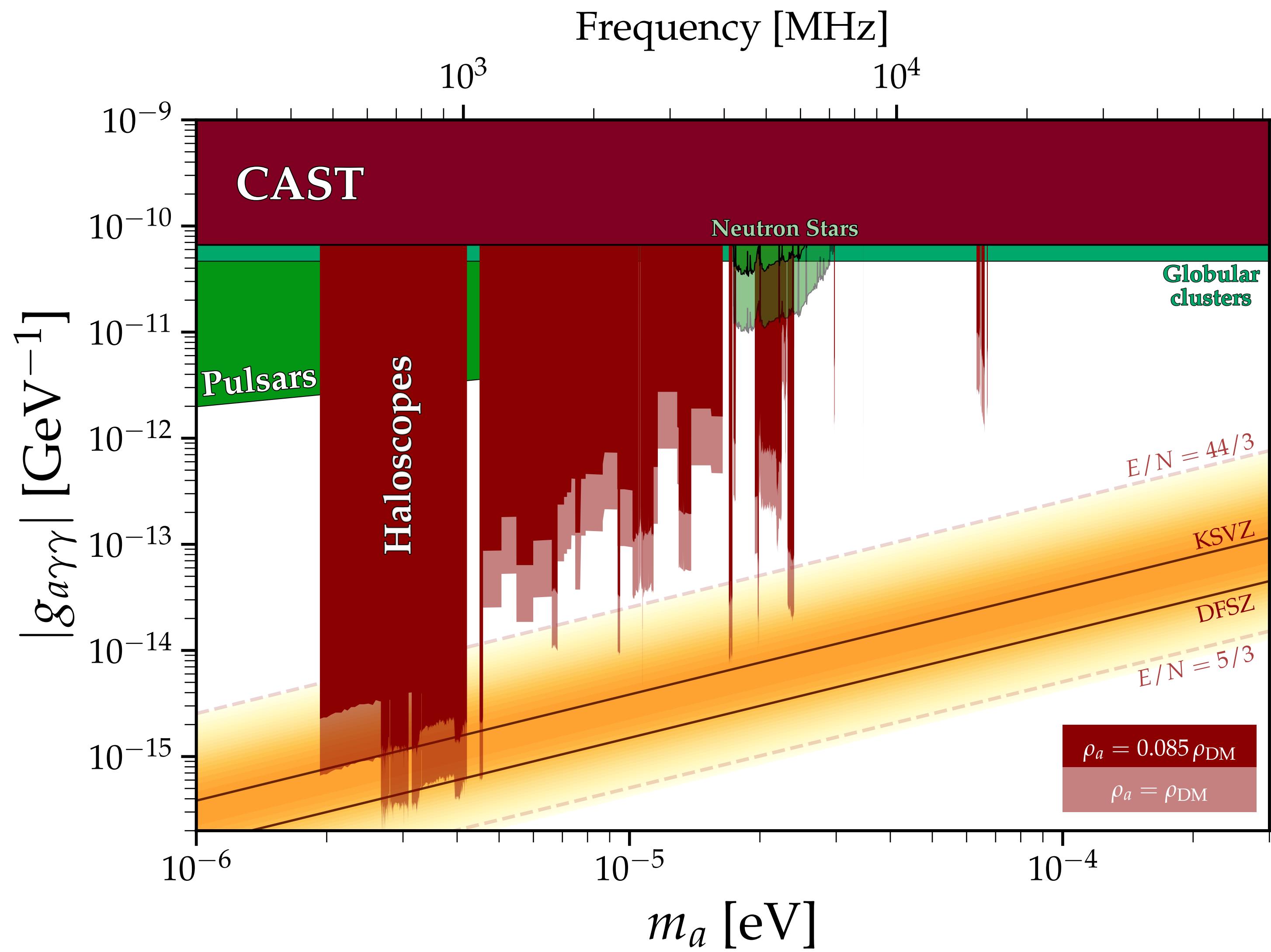
**Miniclusters**

**Minivoids**

Miniclusters contain  $>80\%$  of the axions but make up  $<1\%$  of the volume

Experiments travel about 0.2 mpc per year so are much more likely to sample the minivoids than the miniclusters

# Implications for haloscopes

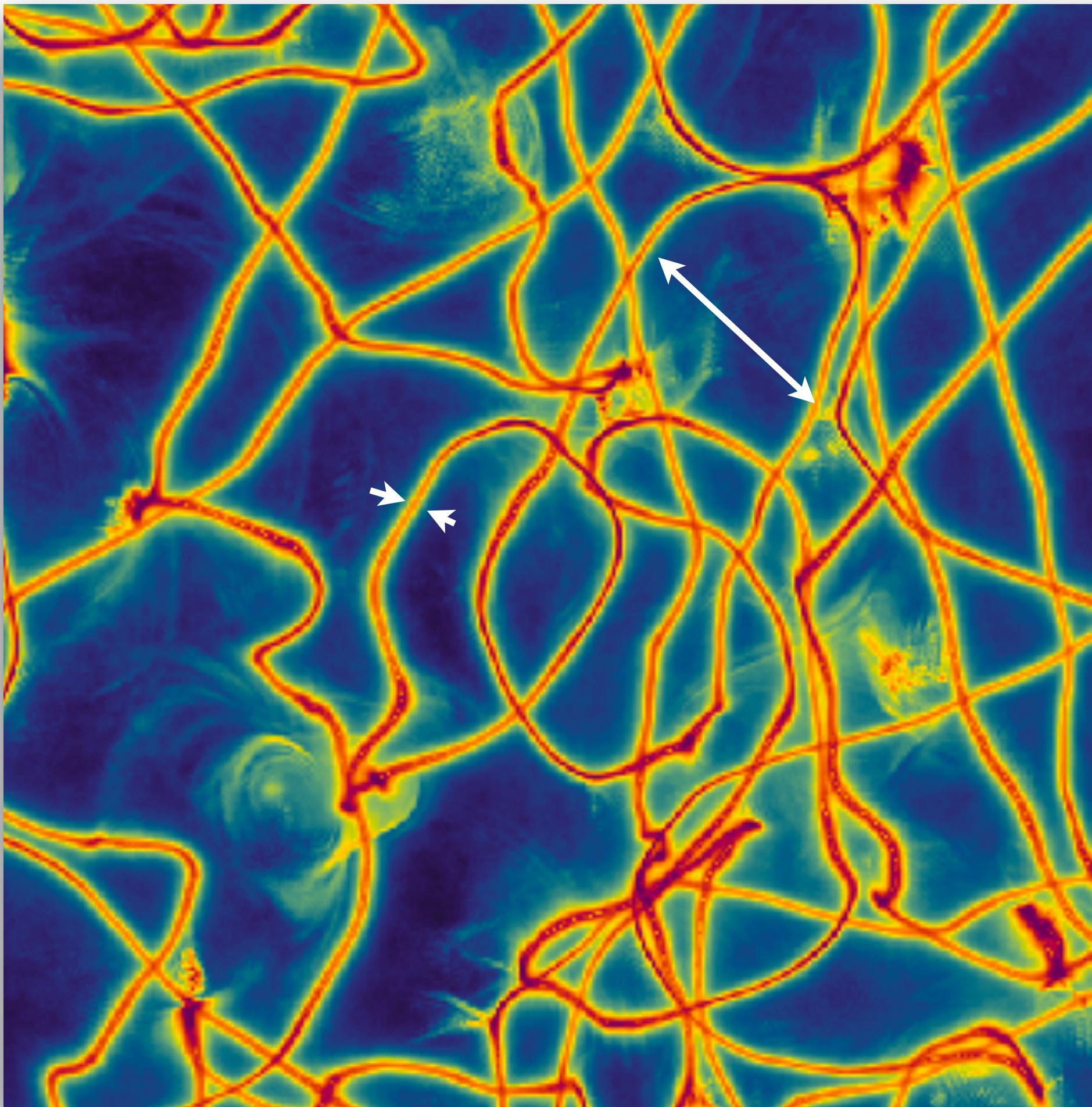


Typical density in the minivoids is  $\sim 0.085$  of the mean density of dark matter

→ the miniclusters are no longer growing at the final redshift of the simulation, therefore this places a **lower bound** on the density of axions

→ Not a nice conclusion, but it could have been much worse!

# Problem 2: String radiation and the dynamical range

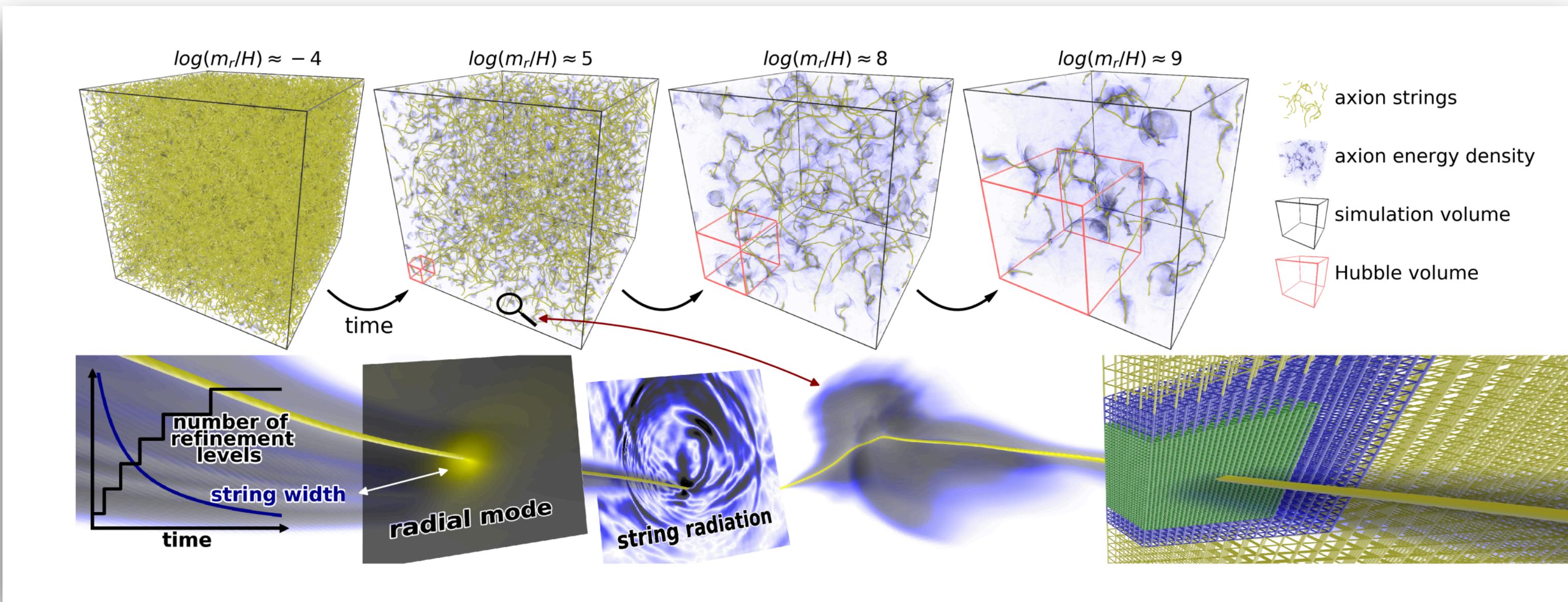


- Want to simulate larger than the causal horizon  
 $L \sim 1/H$
- Whilst also resolving string cores  
 $\Delta x \sim 1/m_s = (\sqrt{\lambda} f_a)^{-1}$
- For a realistic model e.g.  $f_a = 10^{11}$  GeV  
 $\rightarrow m_s/H \sim 10^{28}$
- With current resources  $\lesssim 8192^3$  grid points, can only simulate  $m_s/H \sim 10^{3-4}$
- Might be okay since this parameter enters as  $\log m_s/H$  in the string tension but still needs to be checked, e.g. with AMR [2108.05368]

# Key issues to be resolved

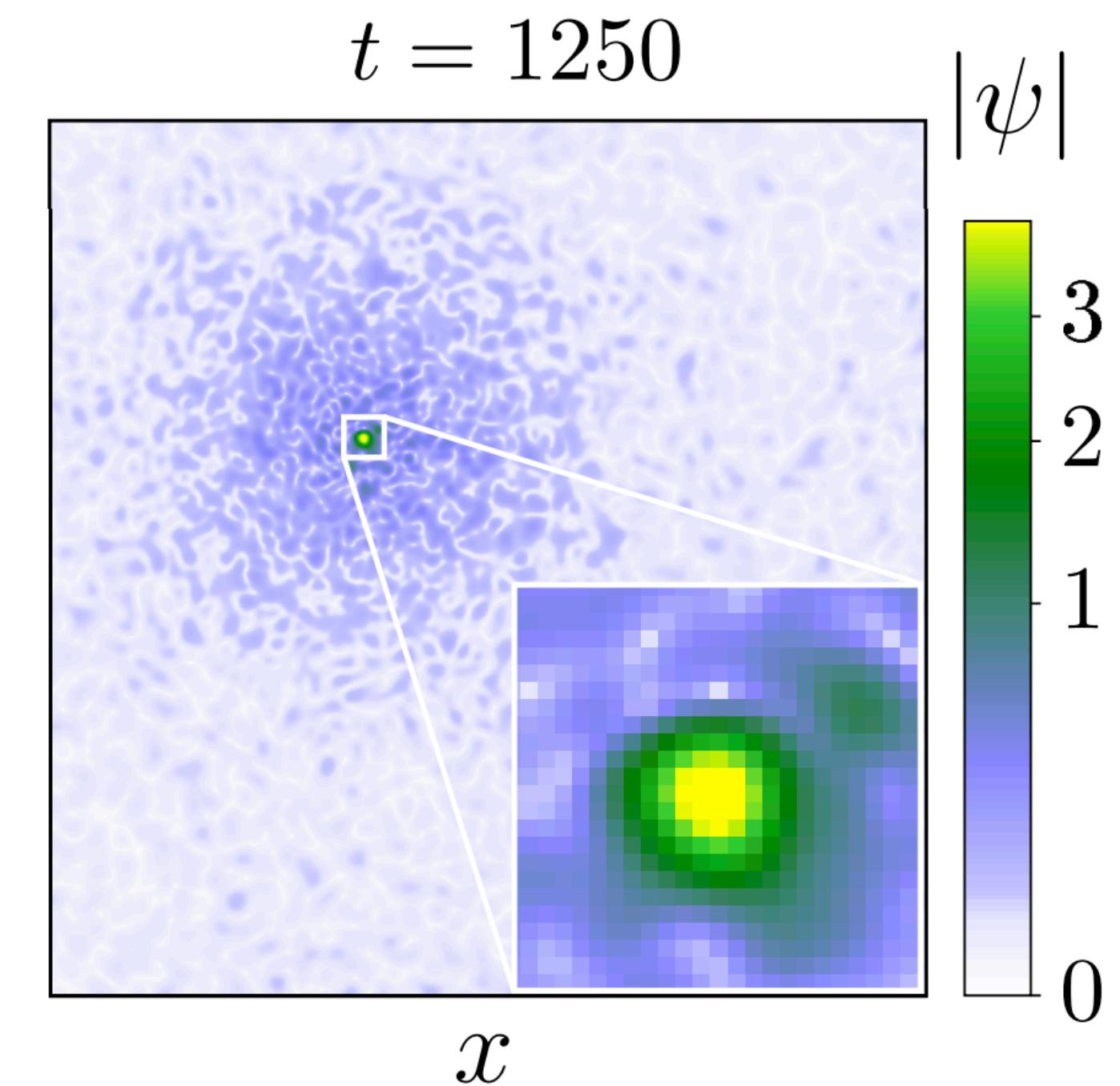
## Axion dark matter abundance/axion mass

- Current simulations cannot study the string scaling regime in full due to high required dynamical range, see e.g. [2007.04990]
- Not clear if the spectrum is dominated by IR modes (meaning large overproduction) or UV modes (Important for predicting the axion mass, see [2108.05368] for recent work with AMR)

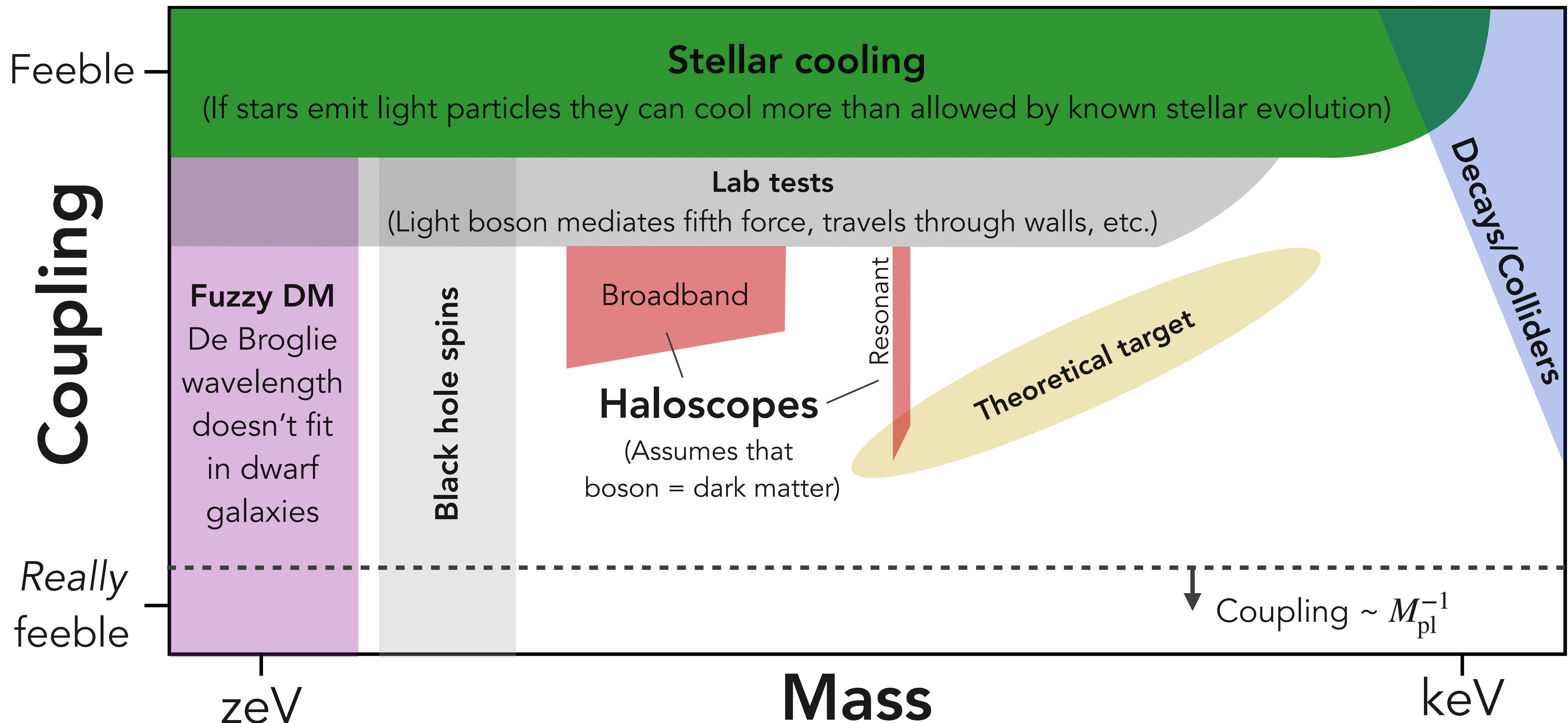


## Implications of an inhomogeneous axion distribution

- Formation of gravitationally bound structures “axion miniclusters”
- Do miniclusters survive to present day? Could be important for direct and indirect searches
- Can stable axion stars form inside miniclusters? Could there be signals of these e.g. fast radio bursts? May need to solve full SP system to understand this. [1804.05857]



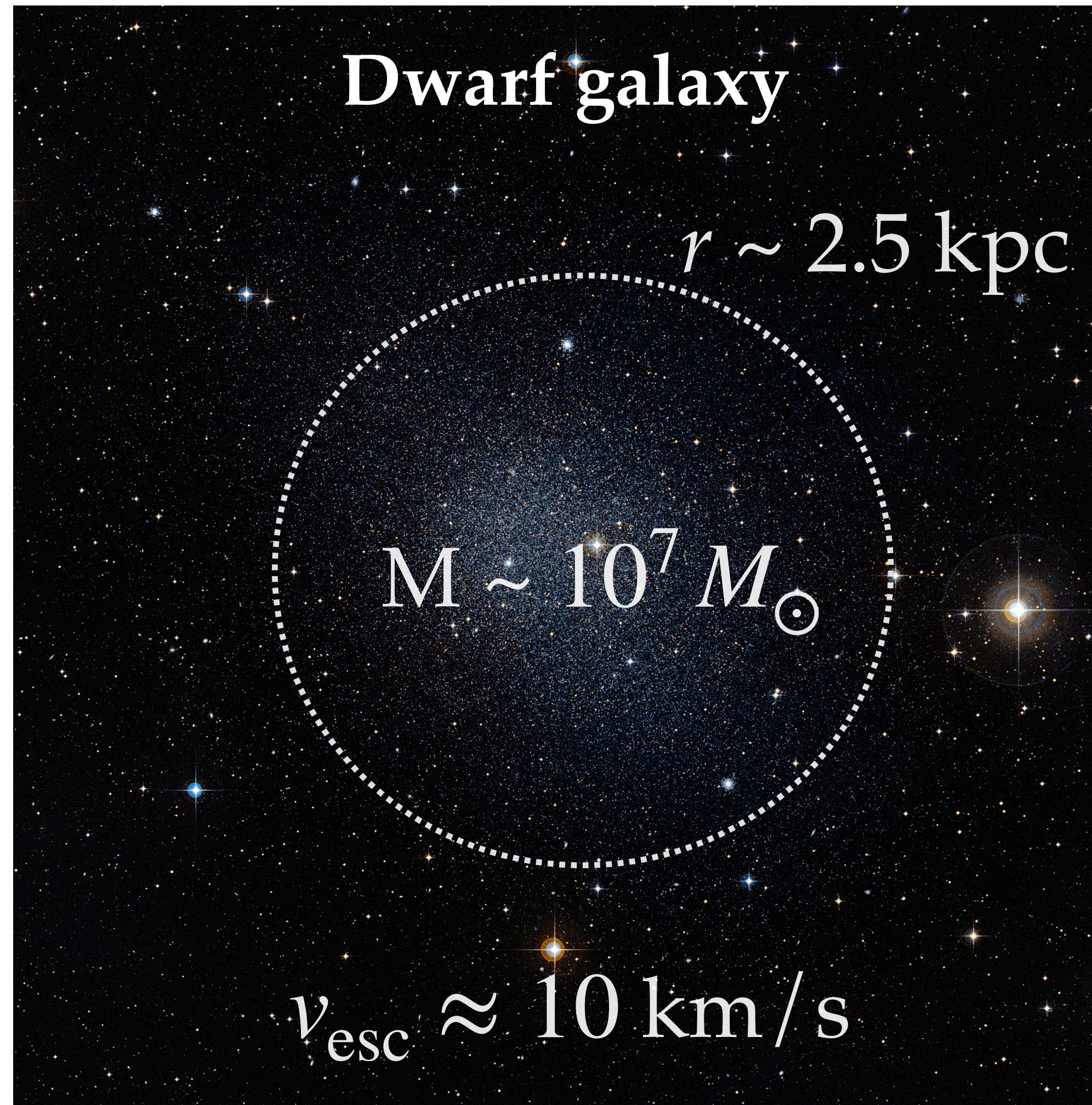
# Schematic of a parameter space for an ultralight boson



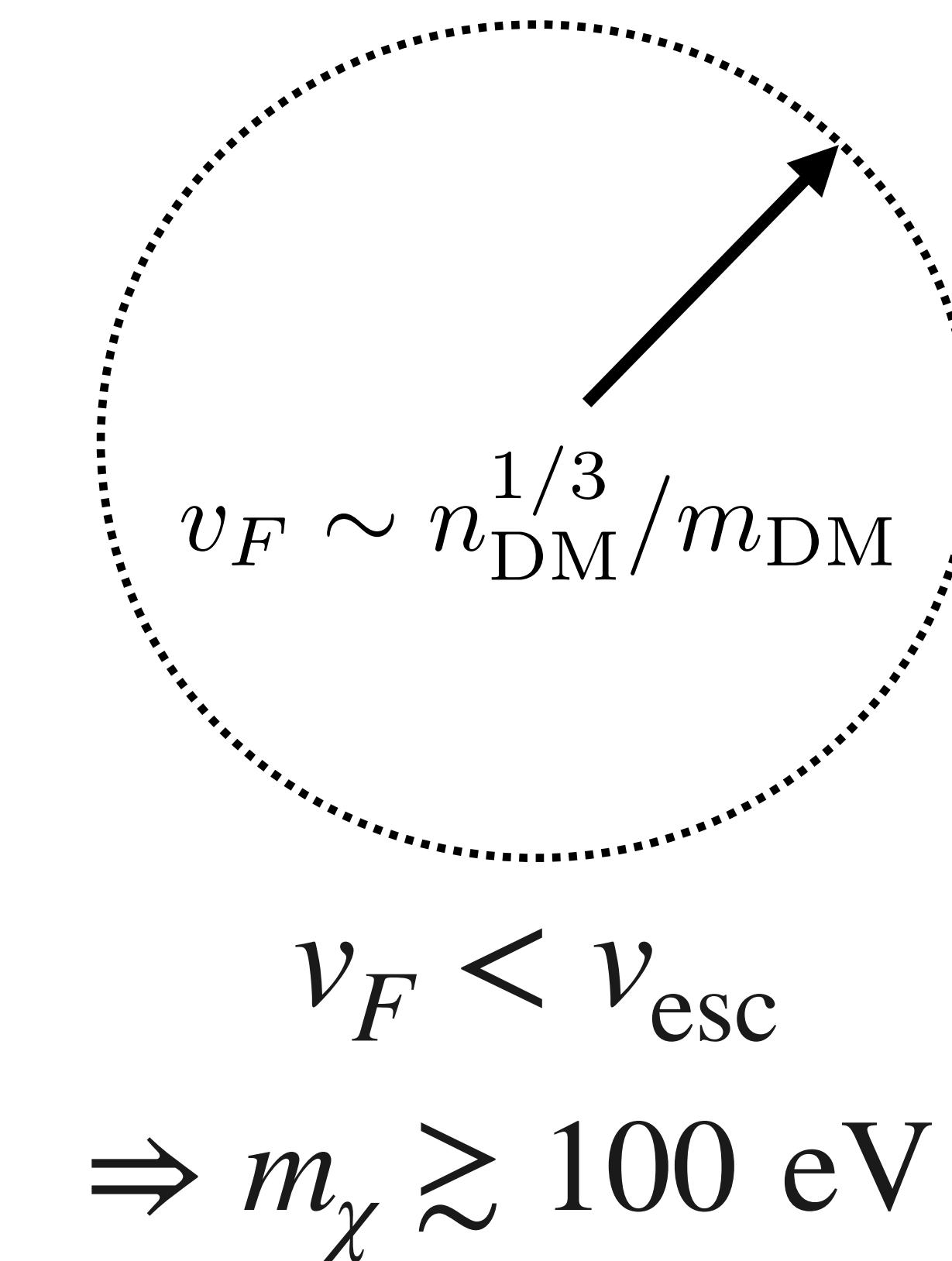
# Wave-like dark matter experiments



# How light can dark matter be if it is made of fermions?



Sphere of degenerate fermions



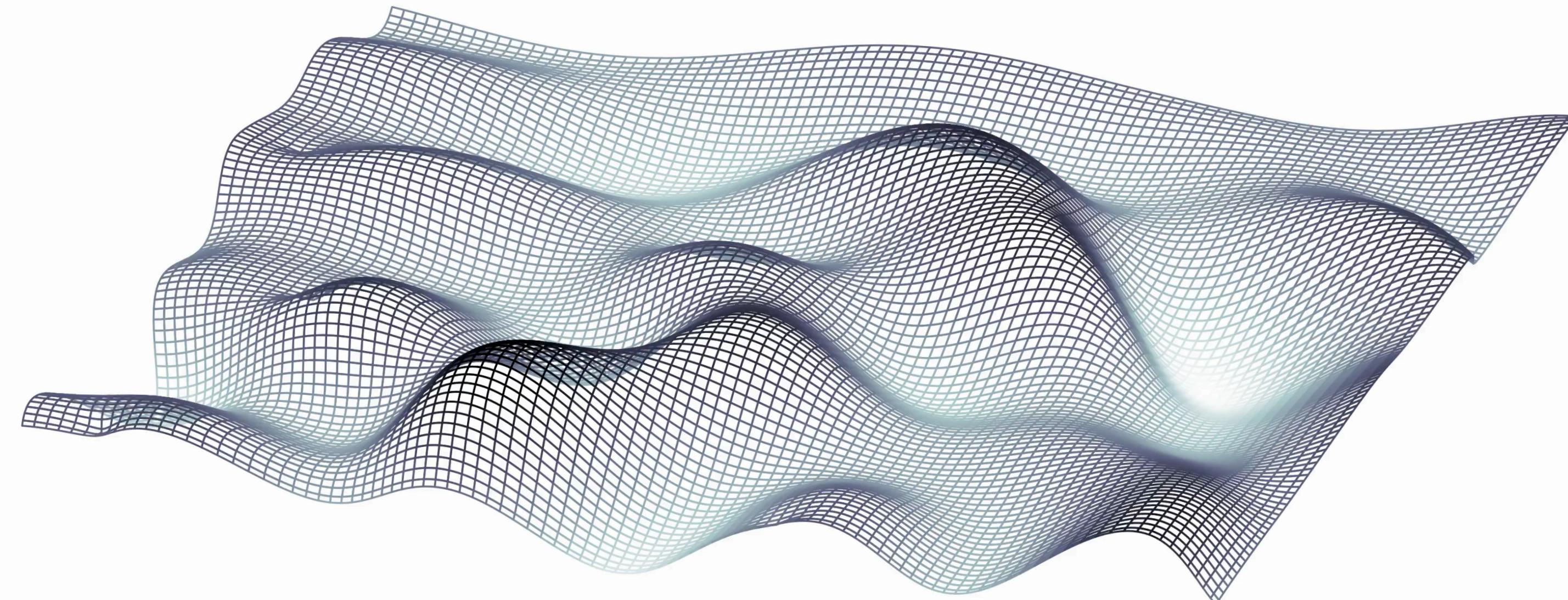
**“Tremaine-Gunn bound”:** Pauli exclusion principle prevents you from cramming fermions lighter than  $\sim 100$  eV into dark matter halos

# Bosonic “wave-like” dark matter

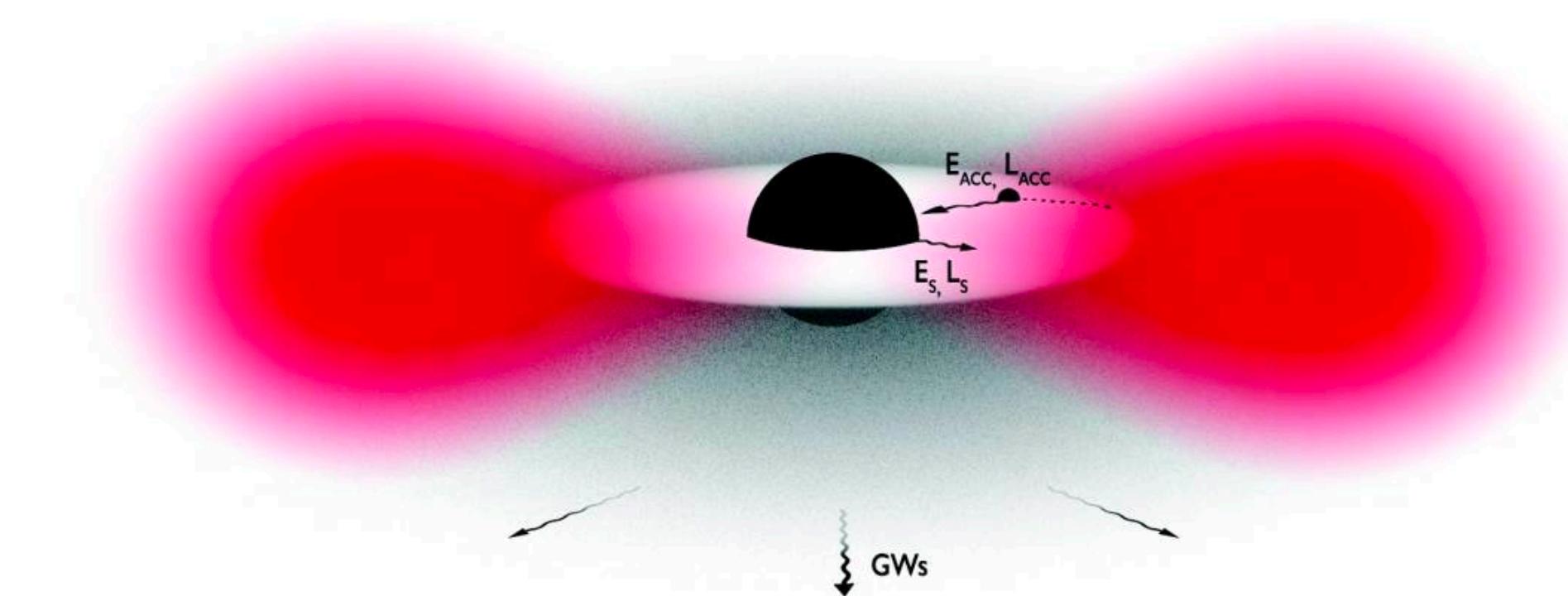
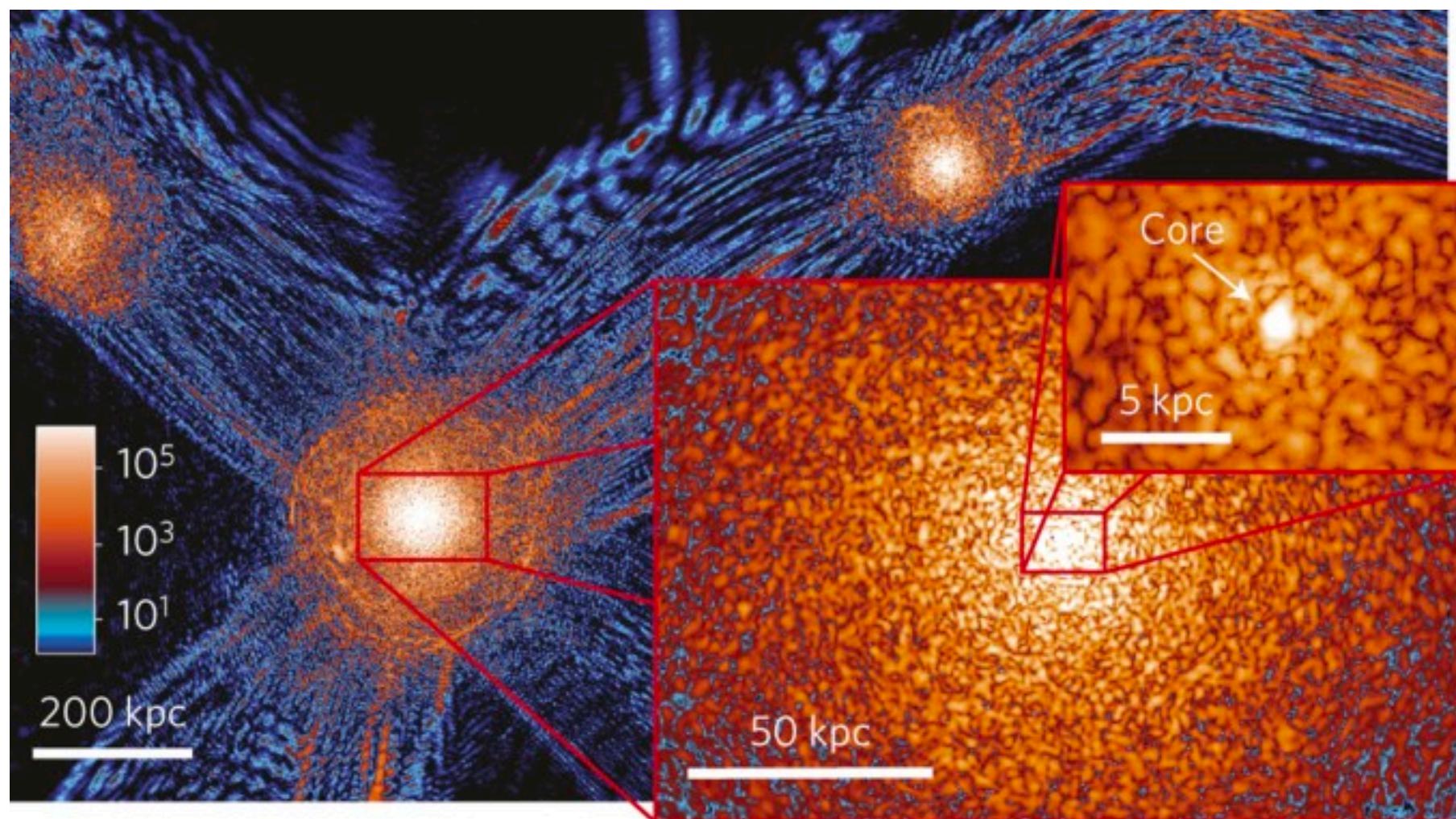
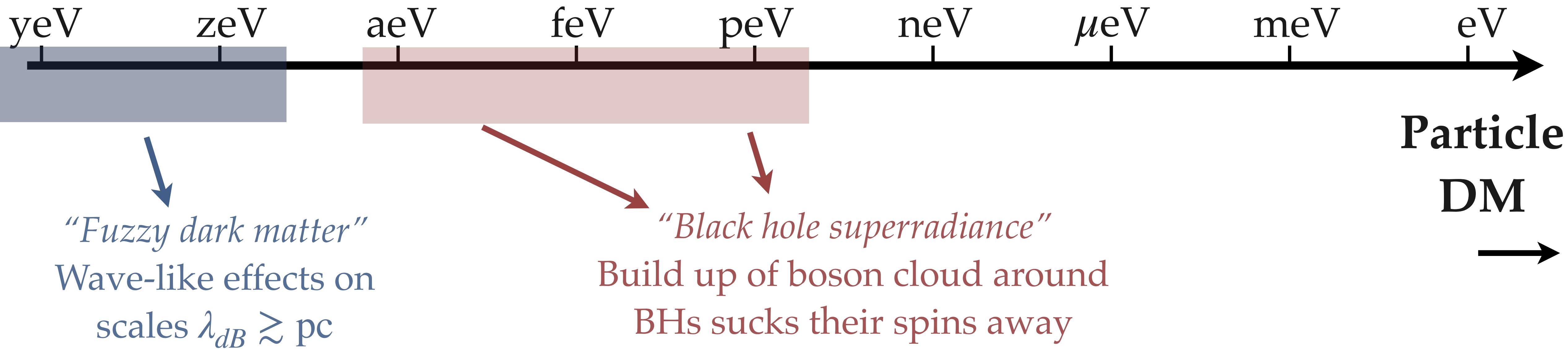
Occupation number:

$$\mathcal{N} \approx (\rho_{\text{DM}}/m) \times \lambda_{\text{dB}}^3$$

$m = 1 \mu\text{eV} \longrightarrow \mathcal{N} \approx 10^{32} \longrightarrow$  Macroscopically occupied ground state  $\longrightarrow$  Classical field



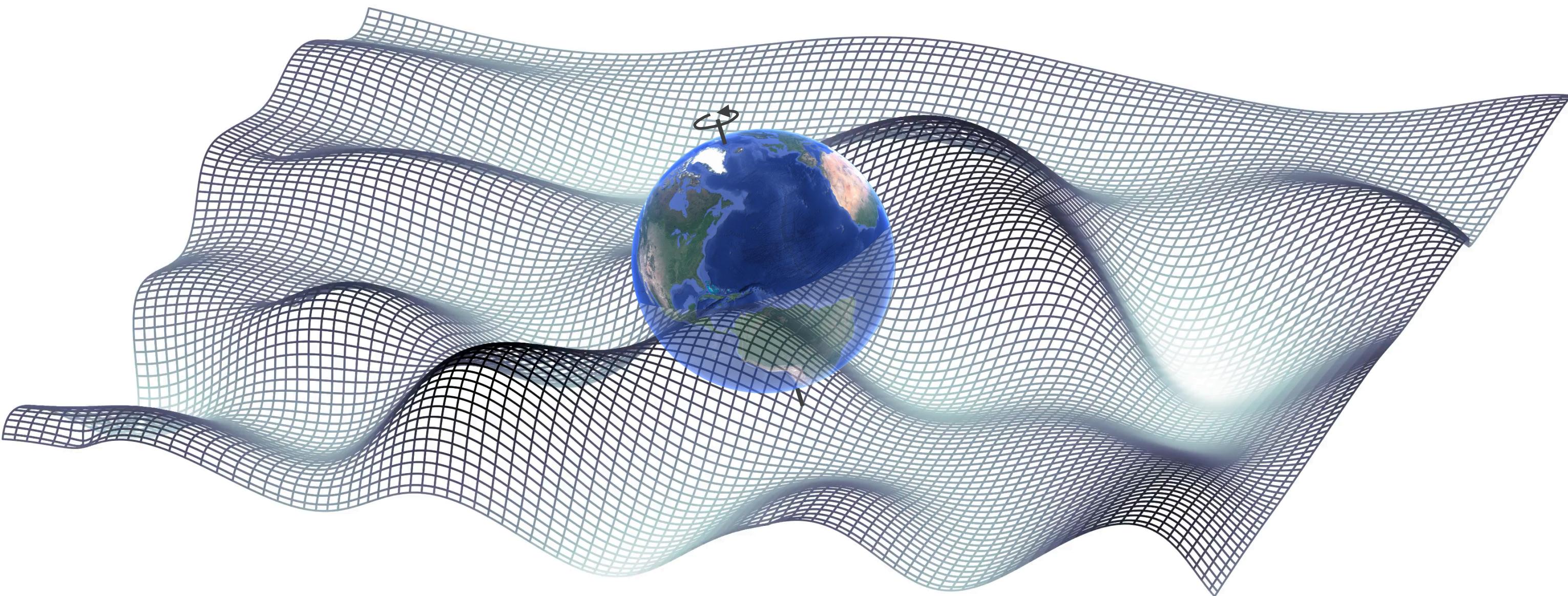
# Wave-like dark matter mass range



# Direct detection

To calculate *any* experimental signal of dark matter we need to know

1. How much dark matter there is around the Earth,  $\rho$
2. How fast it's moving,  $v$



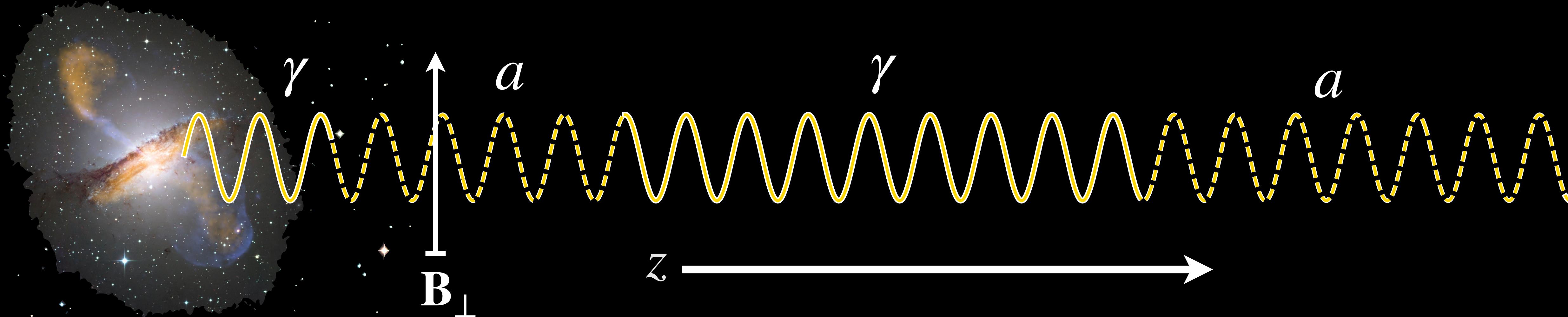
For wave-like dark matter  
these correspond to:

$$\text{Amplitude: } A = \frac{\sqrt{2\rho_a}}{m_a}$$

$$\begin{aligned} \text{Frequency: } \omega &= m_a + \frac{1}{2}m_a v^2 \\ &\approx m_a(1 + 10^{-6}) \end{aligned}$$

# Photon-axion mixing in a B-field

$$\mathcal{L}_{a\gamma} = \frac{1}{2}\partial_\mu a\partial^\mu a - \frac{1}{2}m_a^2 a^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + g_{a\gamma}a\mathbf{E} \cdot \mathbf{B}$$



Linearised wave  
equation for  
photon-axion mixing

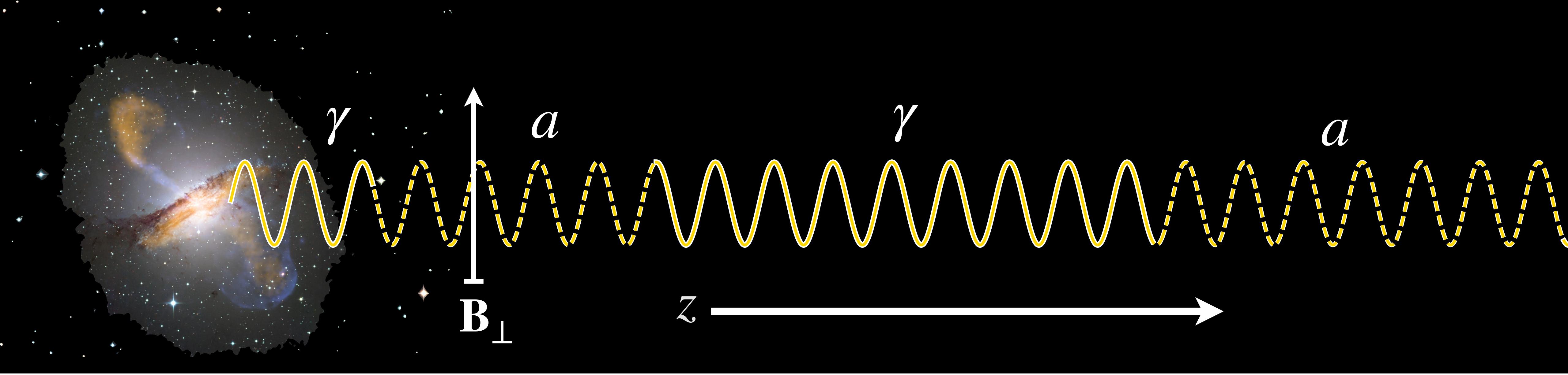
$$\left( \omega + \begin{pmatrix} \Delta_{\text{pl}} & 0 & 0 \\ 0 & \Delta_{\text{pl}} & \Delta_{\gamma a} \\ 0 & \Delta_{\gamma a} & \Delta_a \end{pmatrix} - i\partial_z \right) \begin{pmatrix} |A_\perp\rangle \\ |A_\parallel\rangle \\ |a\rangle \end{pmatrix} = 0$$

$$\Delta_{\gamma a} = \frac{g_{a\gamma} B_\perp}{2} \quad \Delta_a = -\frac{m_a^2}{2\omega} \quad \Delta_{\text{pl}} = -\frac{\omega_{\text{pl}}^2}{2\omega}$$

Mixing element

Axion mass element

Photon mass element



Axion only mixes with  $A_{||}$  so rotate into new basis and solve like neutrino oscillations:

Probability for photon to convert to axion after travelling distance  $z$ :

Where,

$$\begin{pmatrix} |A'_{||}\rangle \\ |a'\rangle \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} |A_{||}\rangle \\ |a\rangle \end{pmatrix}$$

$$|\langle \gamma(0) | a(L) \rangle|^2 = P_{a \rightarrow \gamma} = \sin^2(2\theta) \sin^2 \left( \frac{\Delta}{\cos(2\theta)} \right)$$

$$\theta = \frac{1}{2} \arctan \left( \frac{2\Delta_{\gamma a}}{\Delta_{\text{pl}} - \Delta_a} \right) \quad \Delta = \frac{|m_a^2 - \omega_{\text{pl}}^2|L}{4\omega}$$

## Photon-axion conversion probability

For very light axions,  $m_a$  is negligible compared to  $\omega_{\text{pl}}$   
 $\rightarrow m_a \lesssim 10^{-12} \text{ eV}$  for typical astrophysical plasmas  $n_e = 10^{-3} \text{ cm}^{-3}$

$$\theta \sim g_{a\gamma} \frac{B_\perp \omega}{n_e}$$

$$\Delta \sim \frac{n_e L}{\omega}$$

