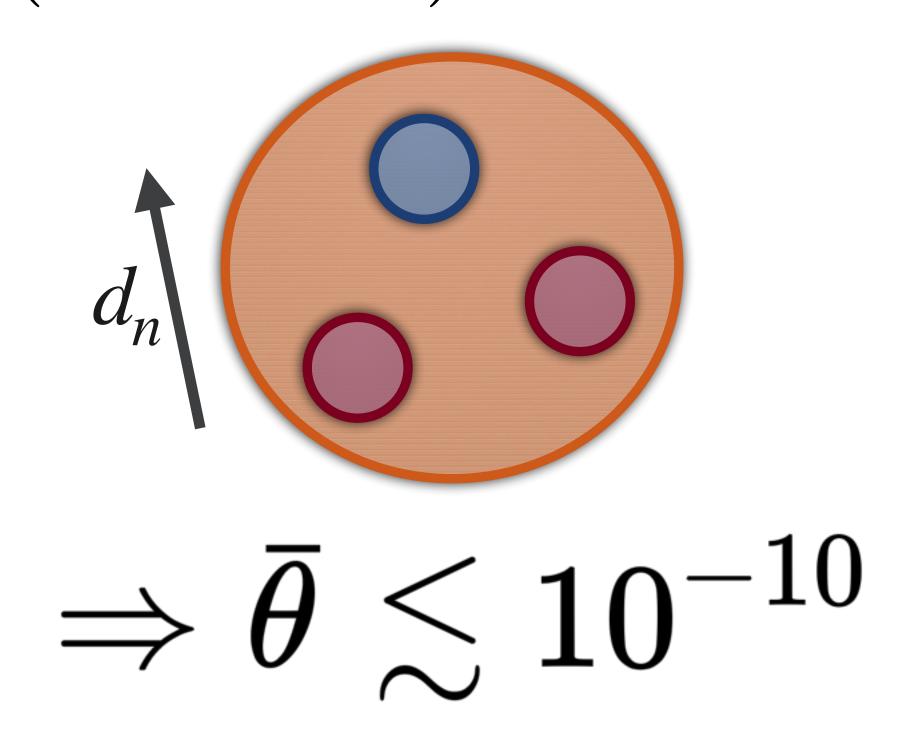


10⁻¹⁹ 10⁻²⁰ 10⁻²¹ 10-22 EDW limits / e Cm 10⁻²³ 10⁻²⁴ 10⁻²⁵ 10-2 10⁻²⁸ 10⁻²⁹ 1960 1970 1980 1990 2000 2010 202 year

Strong CP problem

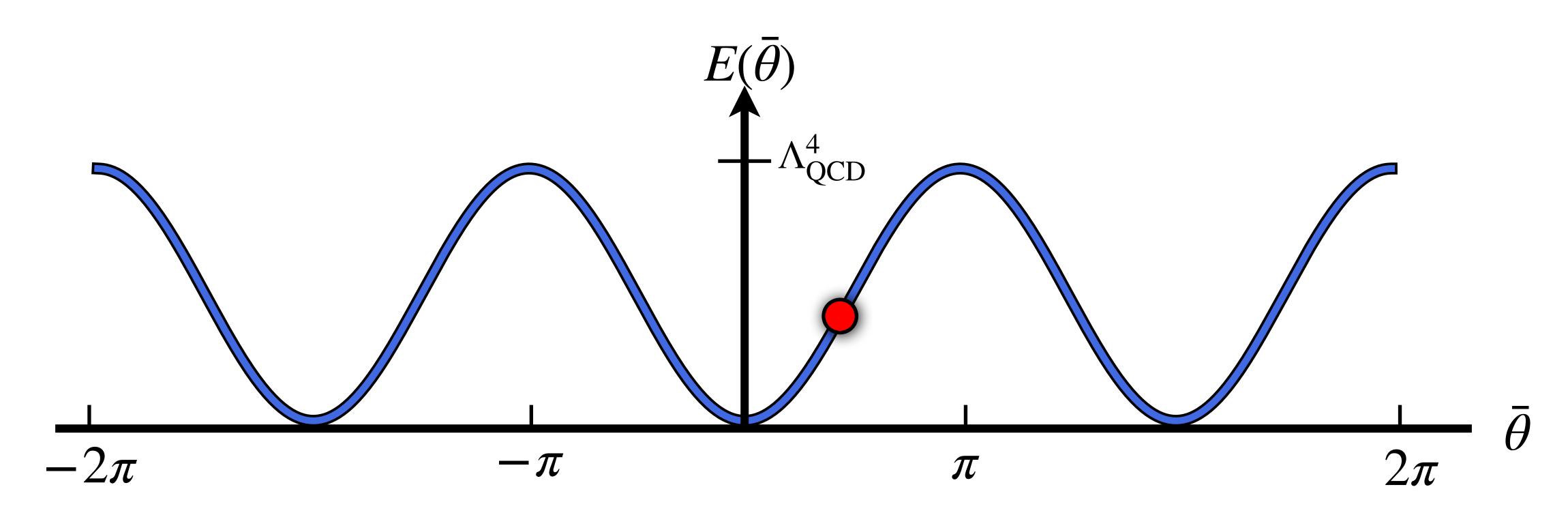
Neutron EDM set by fundamental constant of SM (QCD theta)

$$d_n = (2.4 \pm 1.0)\bar{\theta} \times 10^{-3} e \,\text{fm}$$



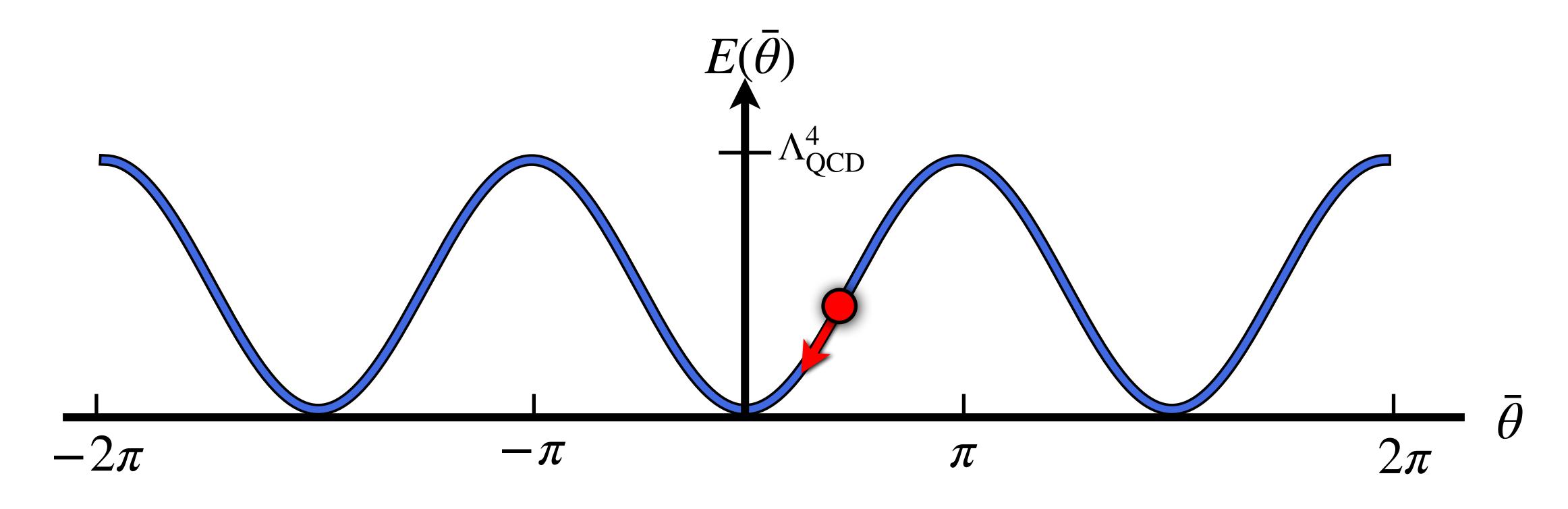
The solution, a la Peccei-Quinn

QCD vacuum energy already minimised at $\bar{\theta}=0$ (Vafa-Witten theorem). However $\bar{\theta}$ is just a parameter, there is no mechanism to cause it to want to minimise energy



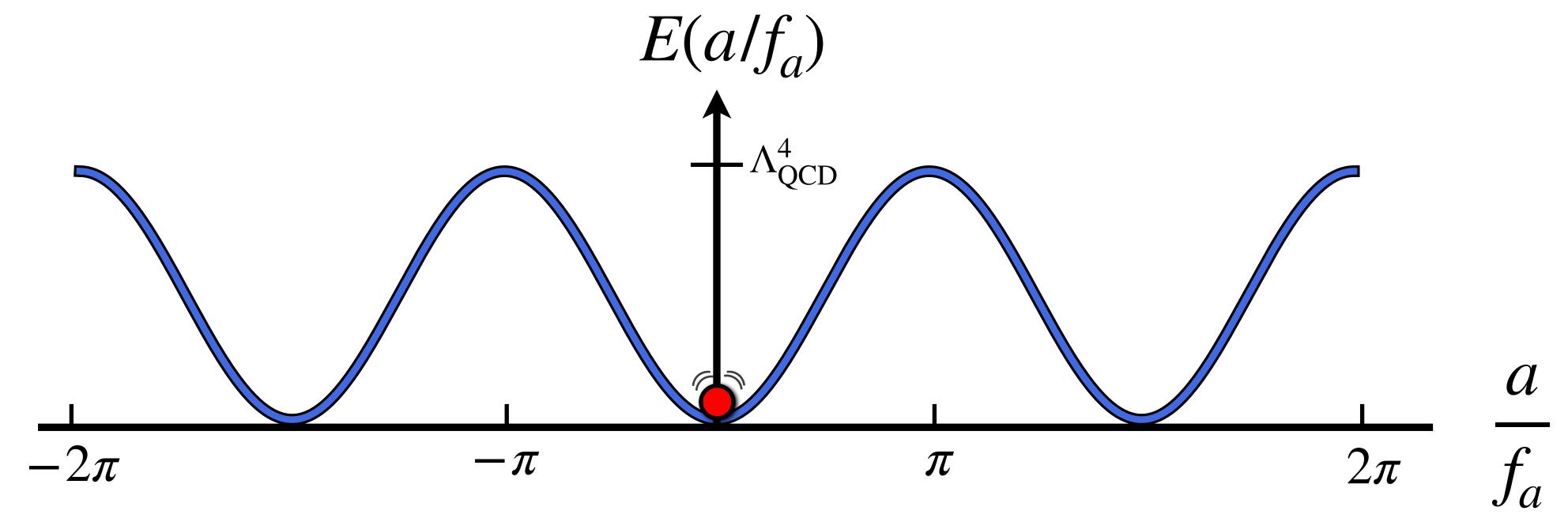
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PQ mechanism: what if there was?

The axion



- Introduce Goldstone boson, a, that couples to gluons $\propto (alf_a) G\tilde{G}$. It will have an (approximate) shift symmetry that can be used to cancel off any unwanted CP violation while VW theorem ensures $\langle a \rangle = 0$
- In the process the field acquires a potential and thus a small mass

$$V(a) \approx \Lambda_{\rm QCD}^4 \left[1 - \cos \left(\bar{\theta} + \frac{a}{f_a} \right) \right] \longrightarrow m_a \simeq \frac{\Lambda_{\rm QCD}^2}{f_a} \simeq 6 \,\mathrm{meV} \left(\frac{10^9 \,\mathrm{GeV}}{f_a} \right)$$

The axion effective theory

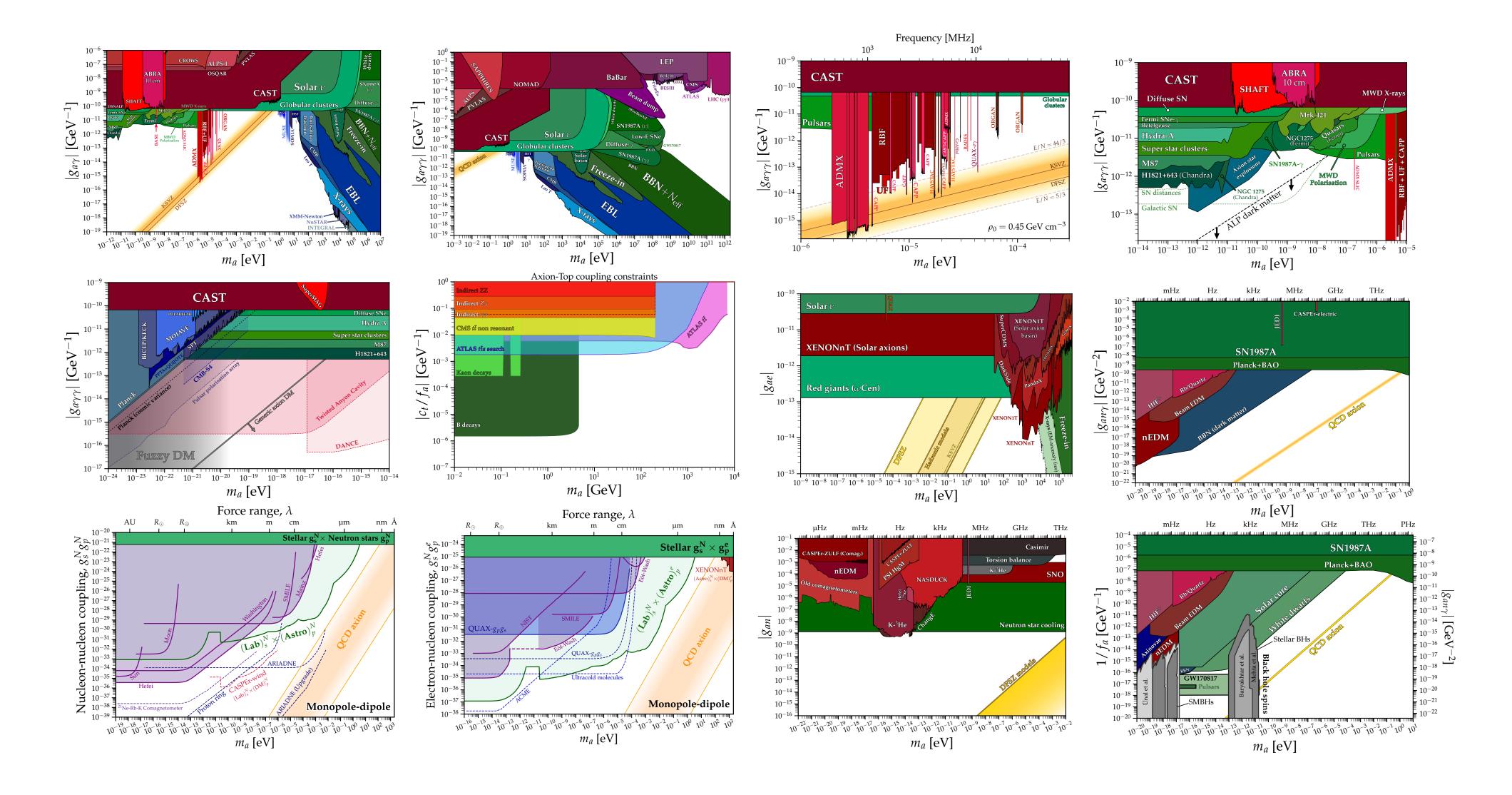
Introduce axion as the pseudo-Goldstone boson of a new global U(1), spontaneously broken at scale f_a . May also couple to the photon and fermions

$$\mathcal{L} = \frac{1}{2} \left(\partial_{\mu} a \right) \left(\partial^{\mu} a \right) - \frac{1}{2} m_a^2 a^2 - \frac{g_{a\gamma}}{4} a F_{\mu\nu} \widetilde{F}^{\mu\nu} + \partial_{\mu} a \sum_{\psi} \frac{g_{a\psi}}{2m_{\psi}} \left(\bar{\psi} \gamma^{\mu} \gamma^5 \psi \right)$$

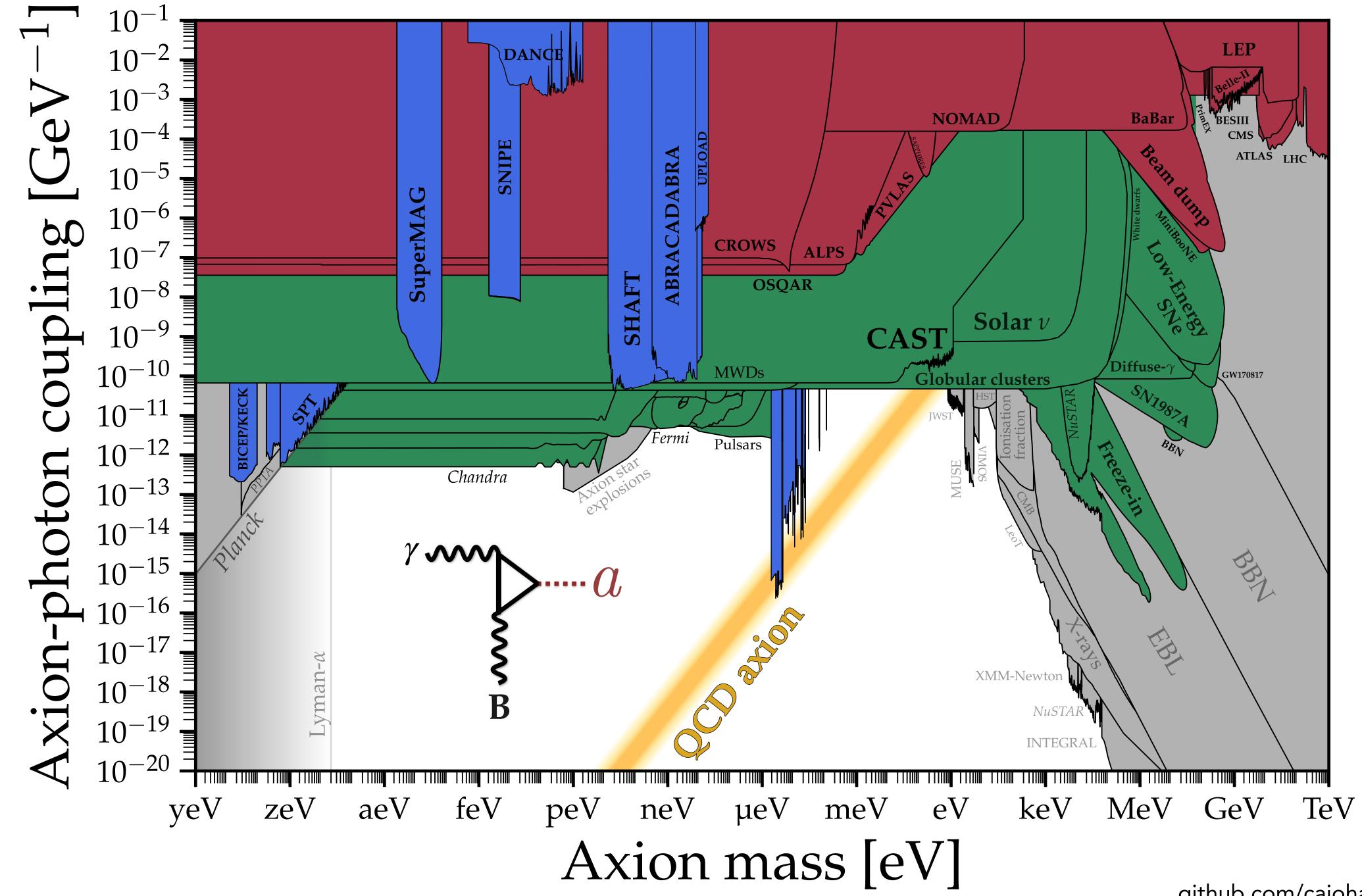
Importantly, all couplings suppressed by $g \sim f_a^{-1}$. So set symmetry breaking scale as high as you like to evade observational constraints

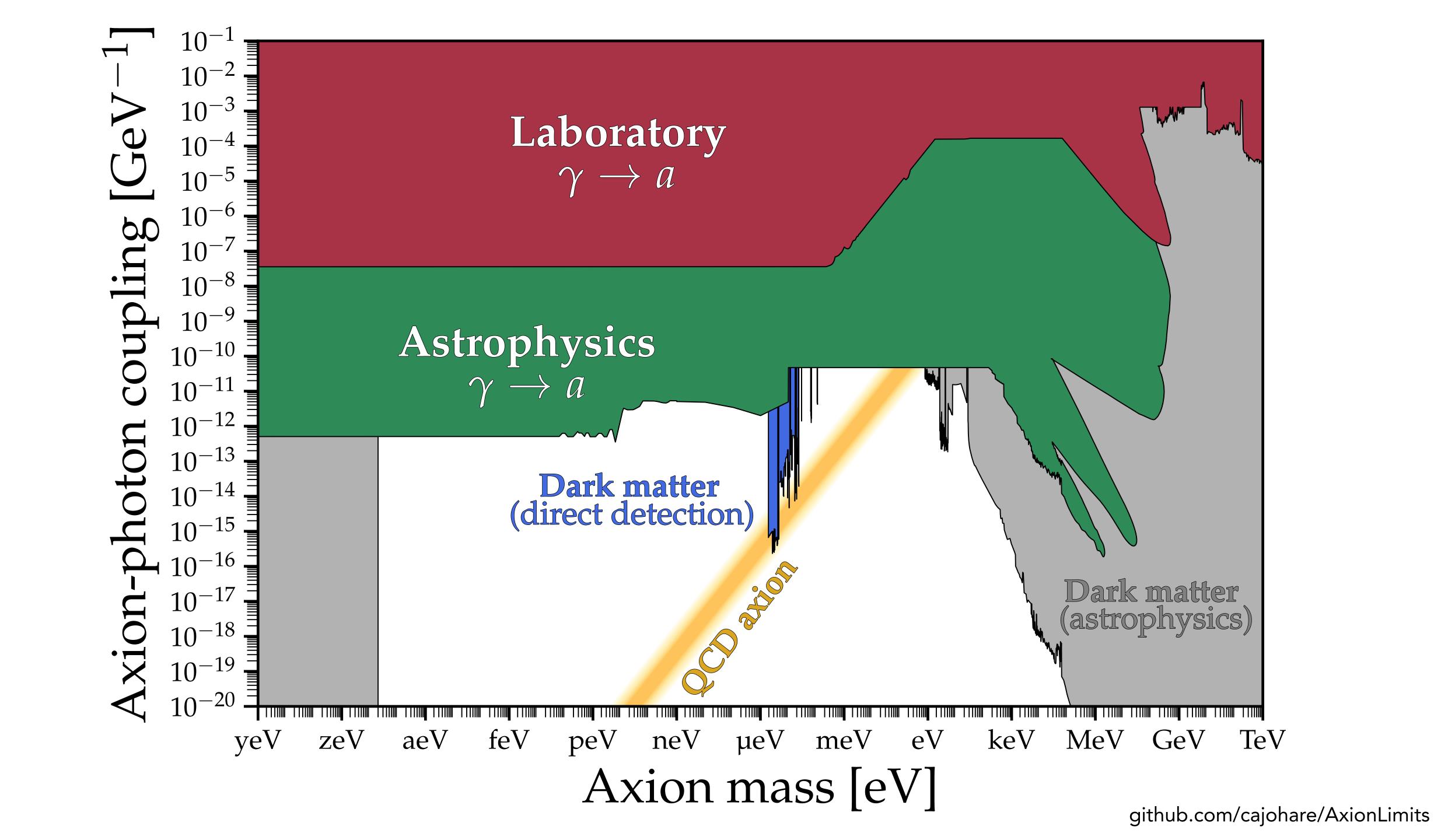
→ ideal candidate for dark matter

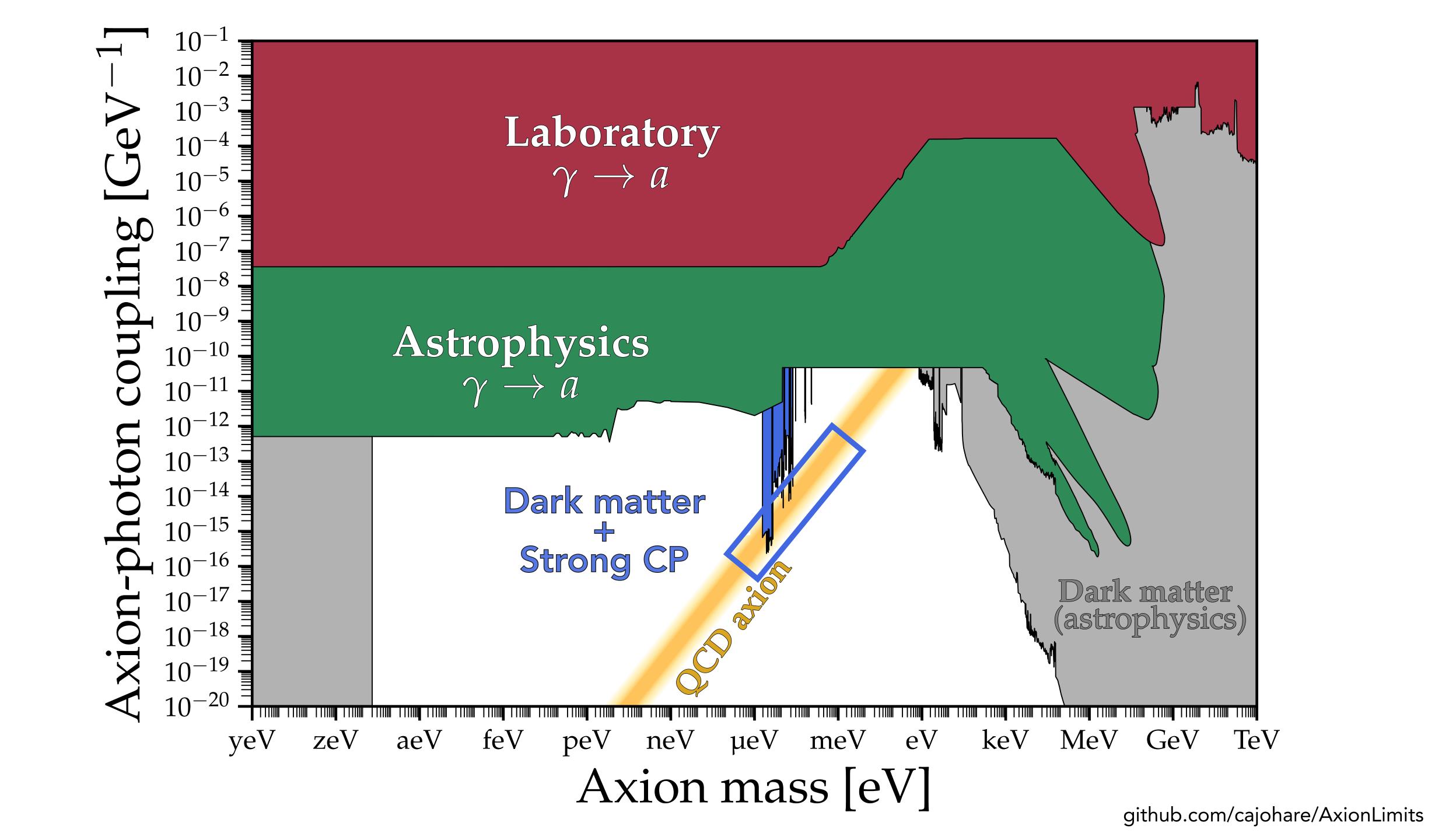
Lots of activity, but still potentially many years away from a discovery



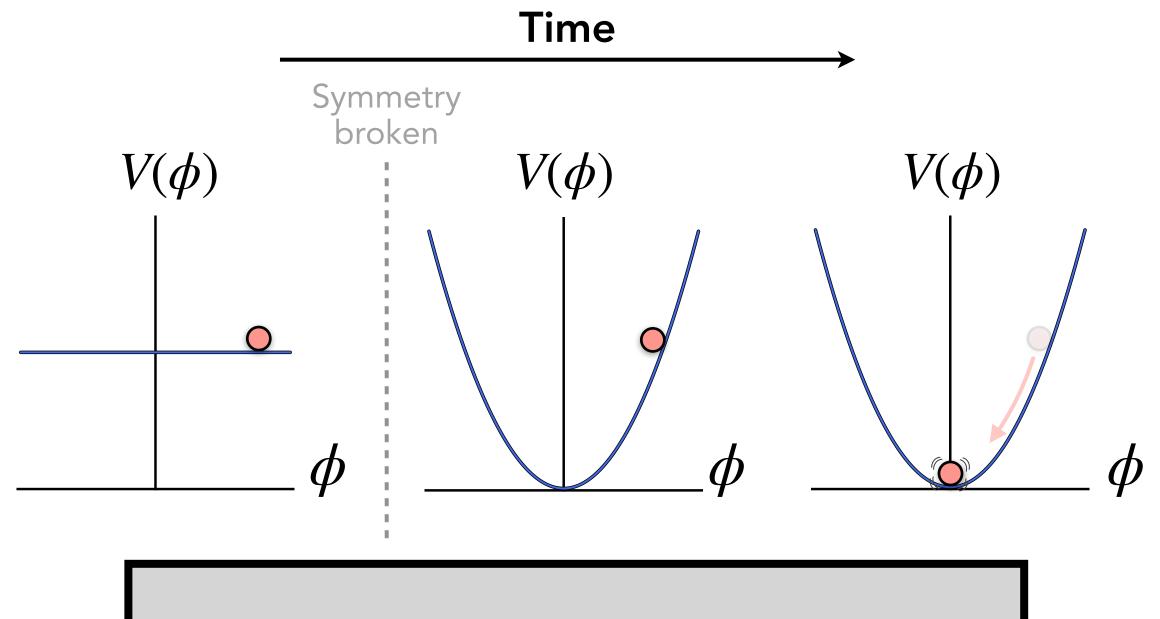
For more, see <u>cajohare.github.io/AxionLimits/</u> \rightarrow Now hosts results from >300 publications!



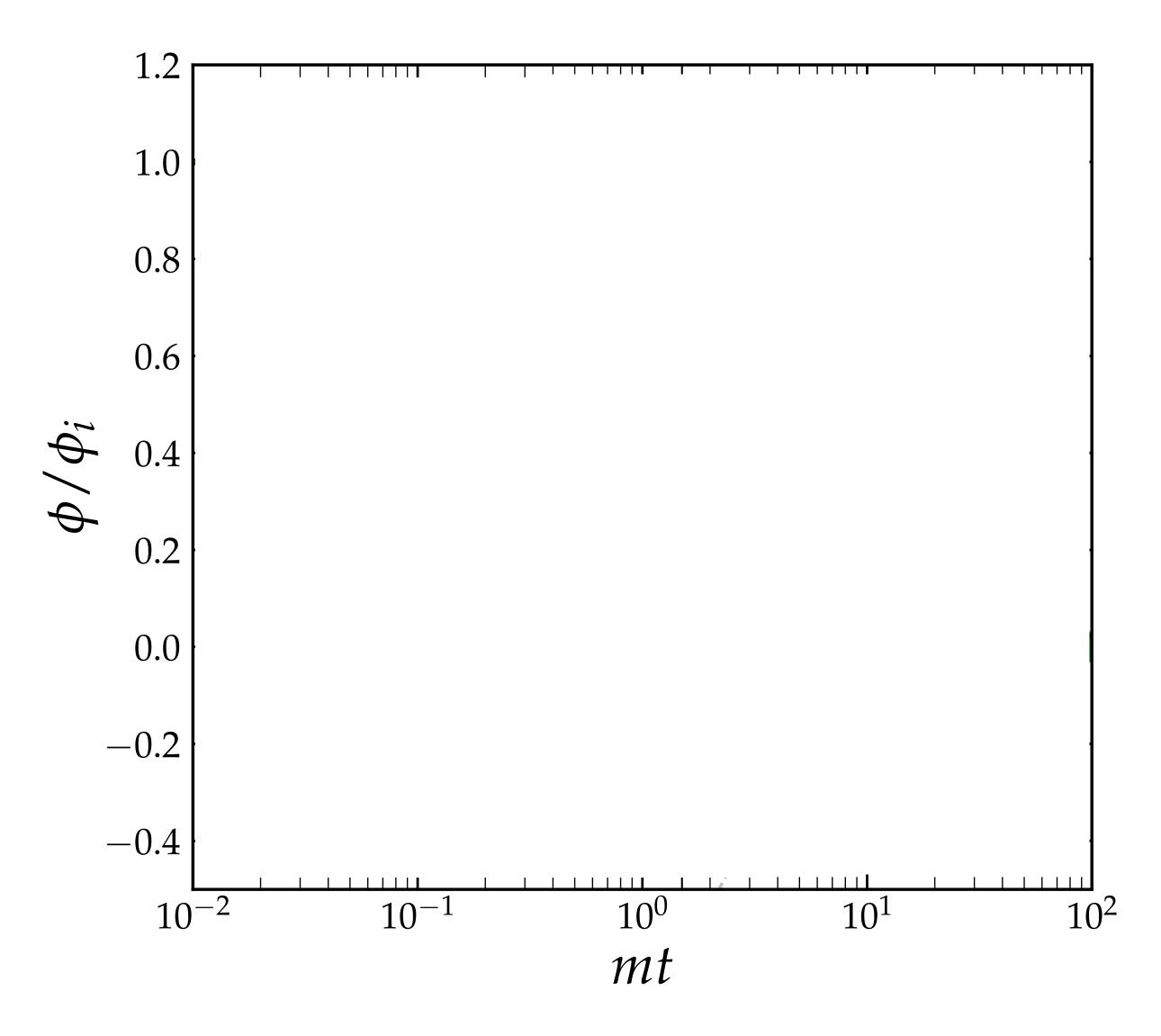


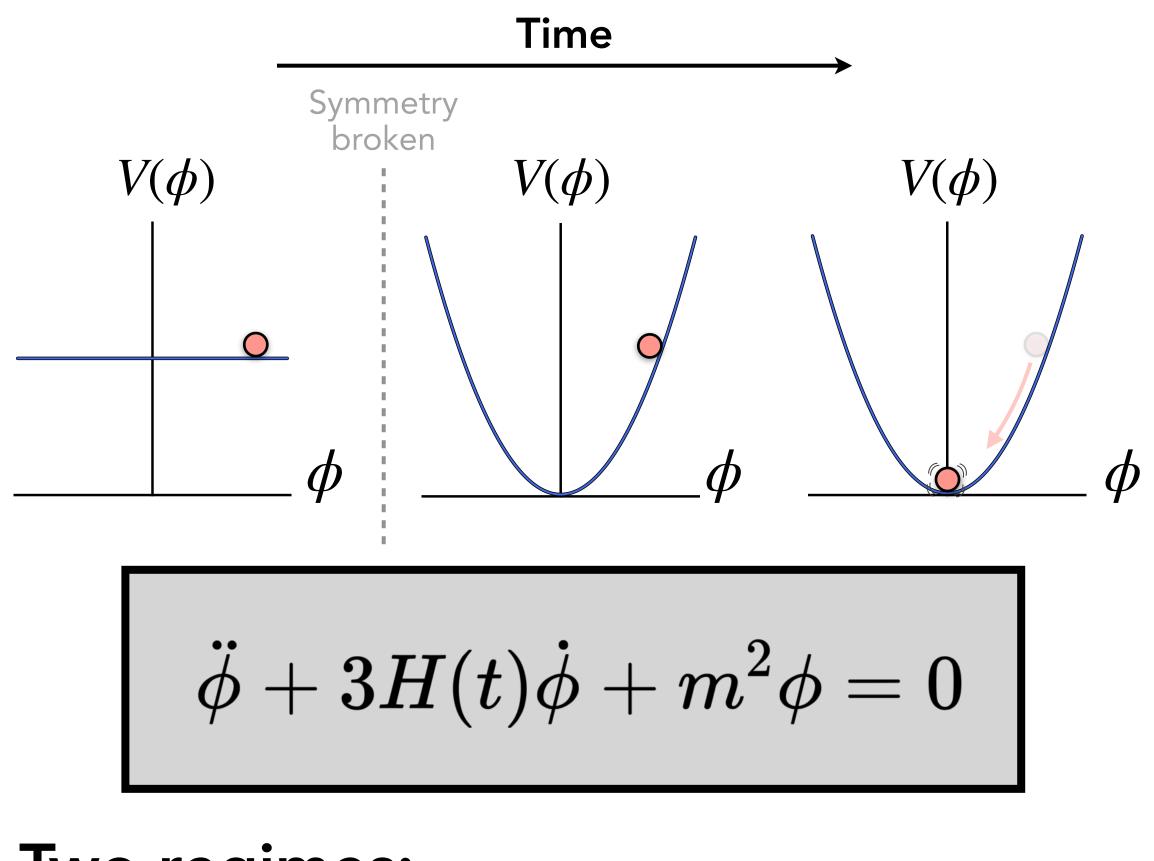


How can a scalar field be the dark matter? → the misalignment mechanism

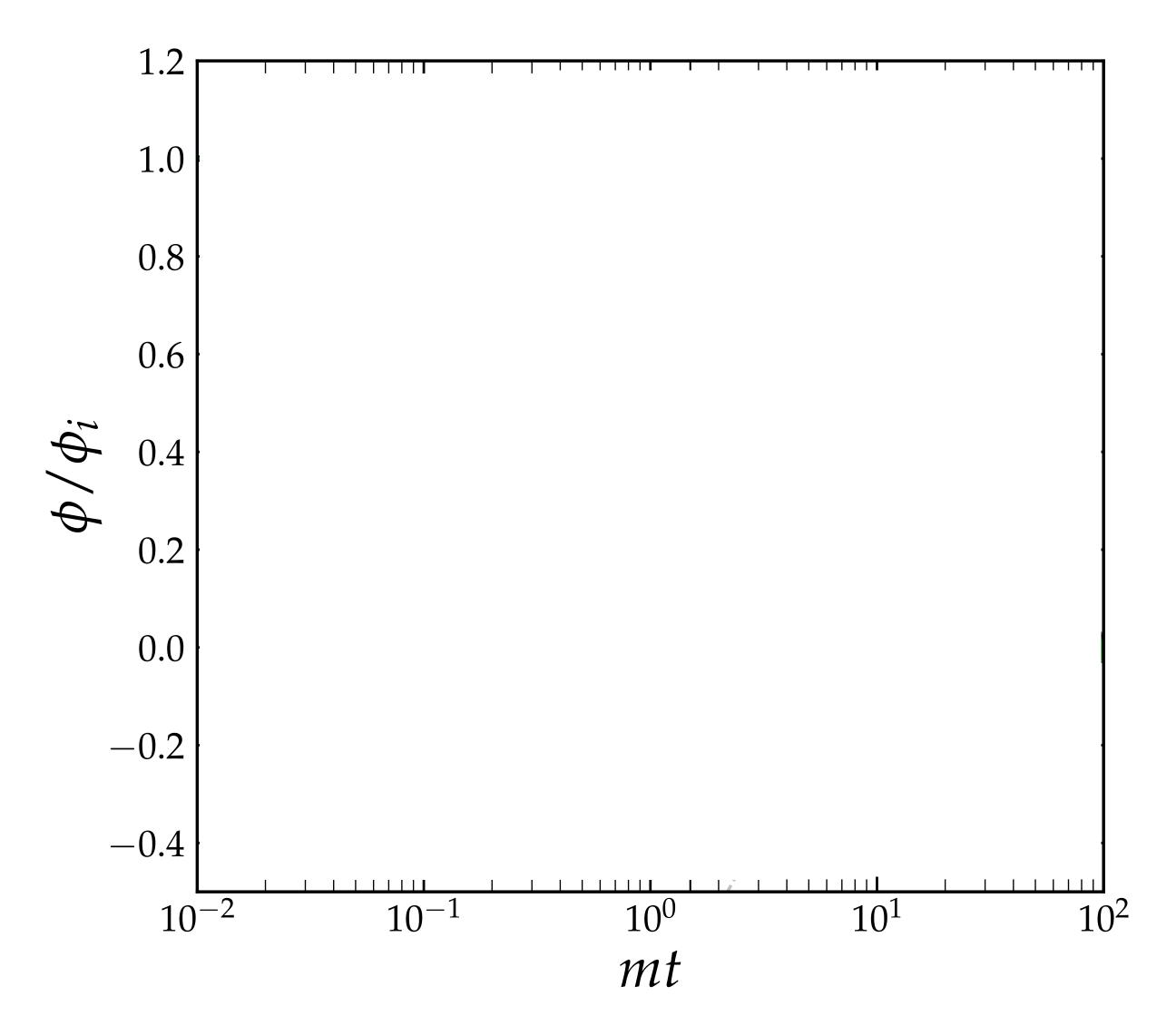


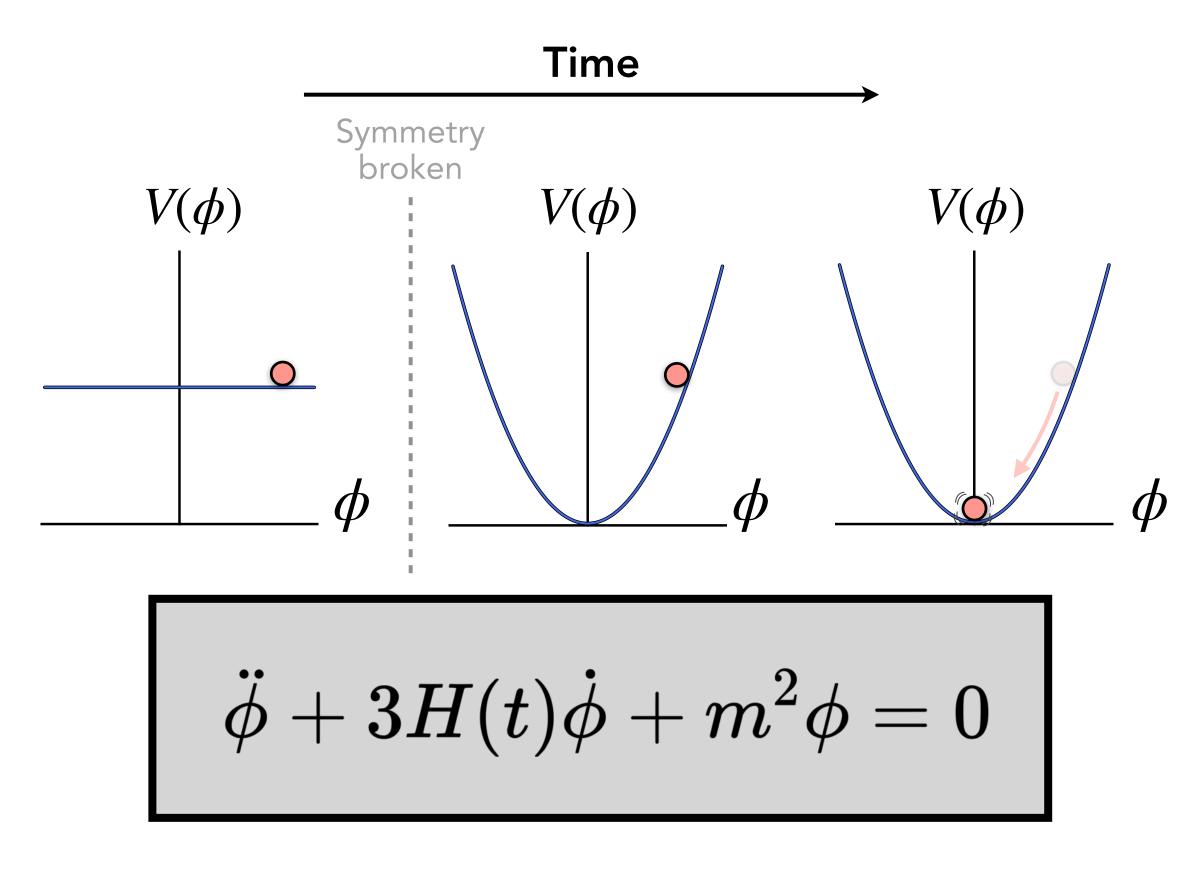
$$\ddot{\phi} + 3H(t)\dot{\phi} + m^2\phi = 0$$





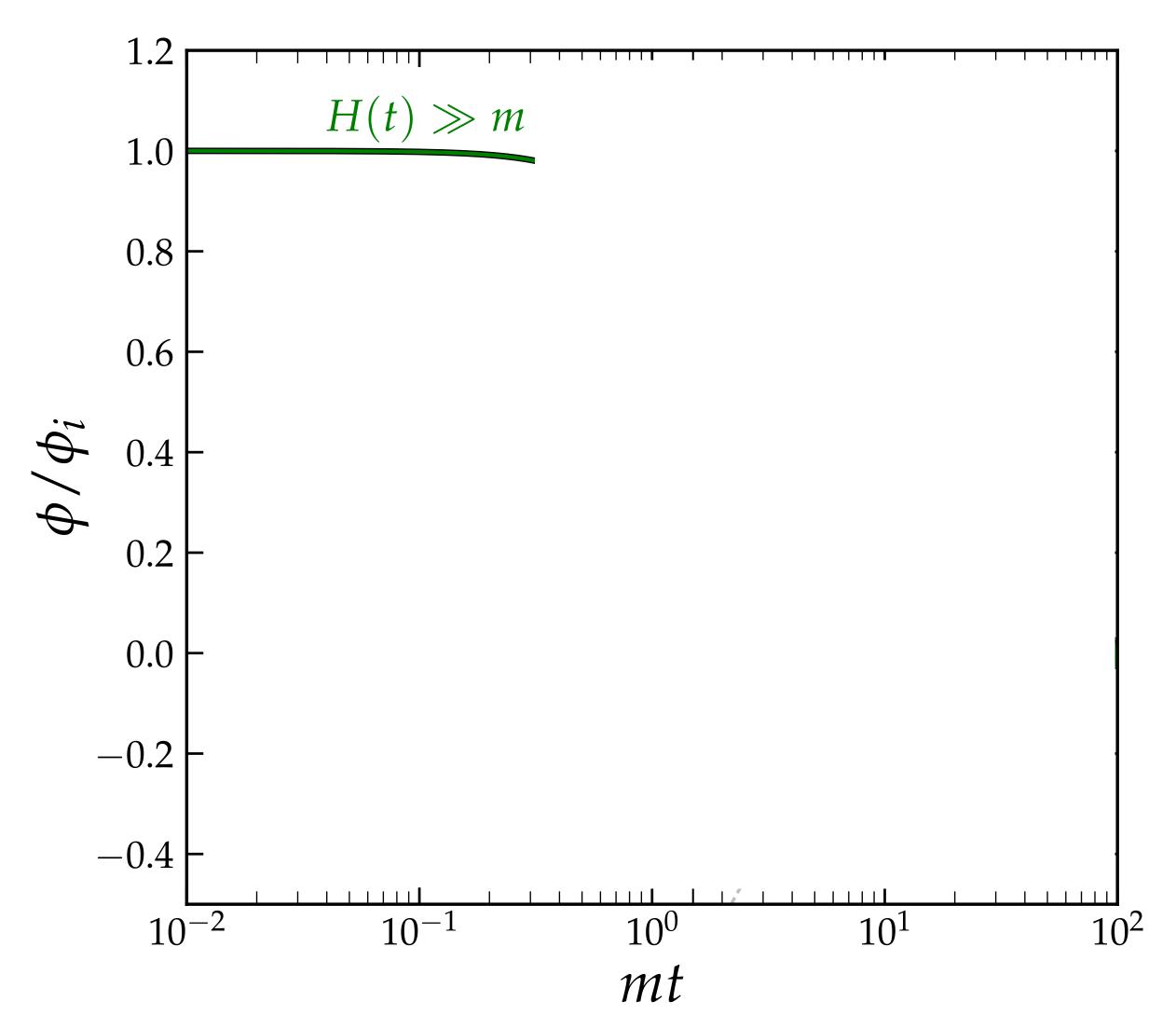
Two regimes:

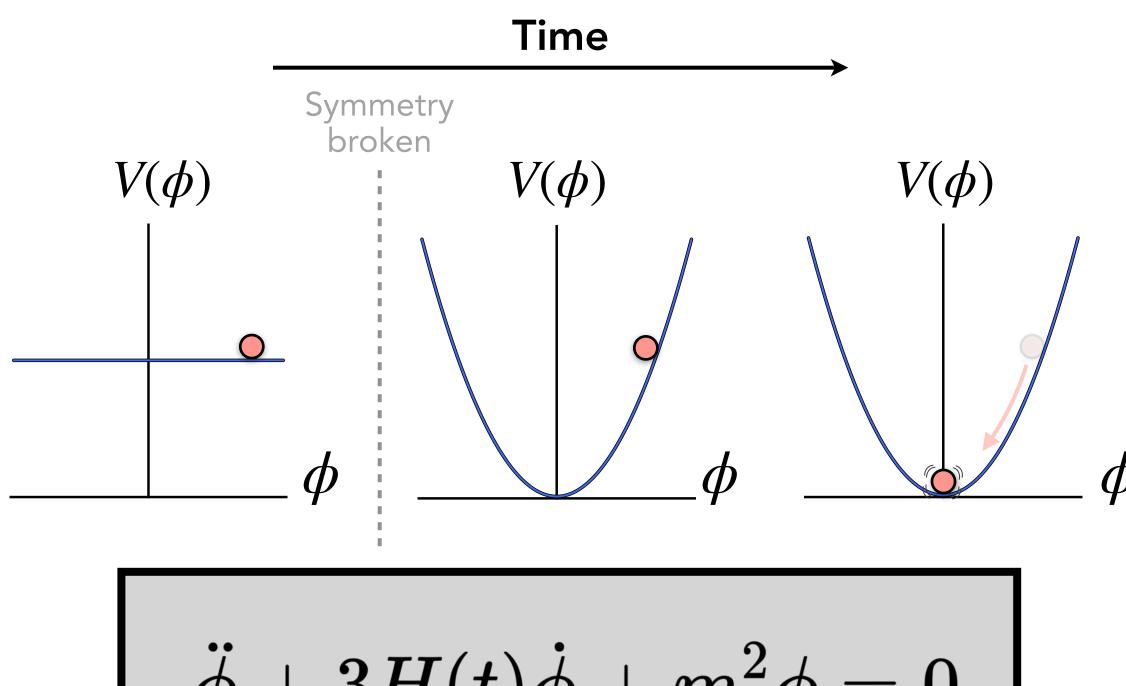




Two regimes:

• $3H(t) \gg m \rightarrow \text{overdamped}$

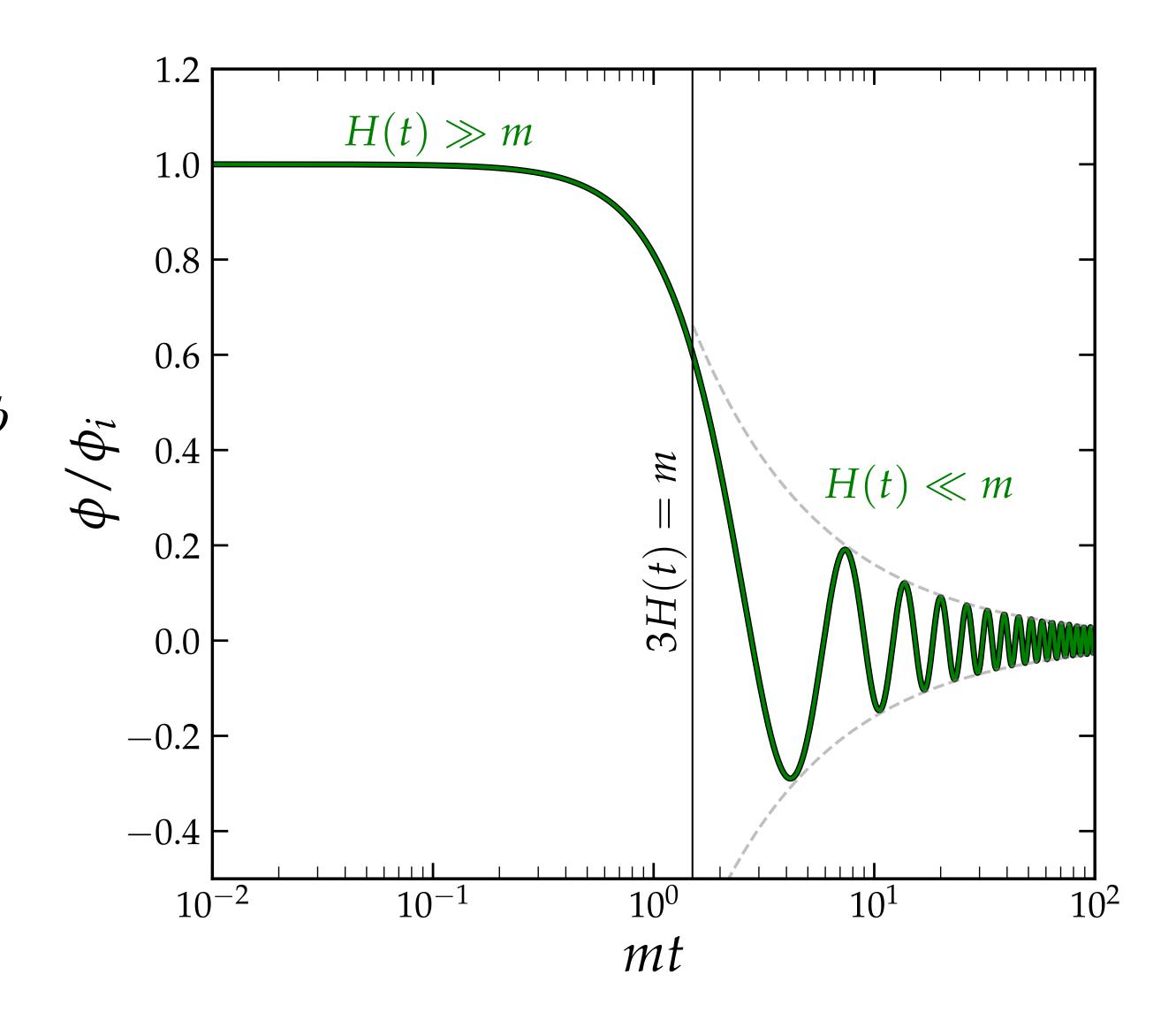




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Two regimes:

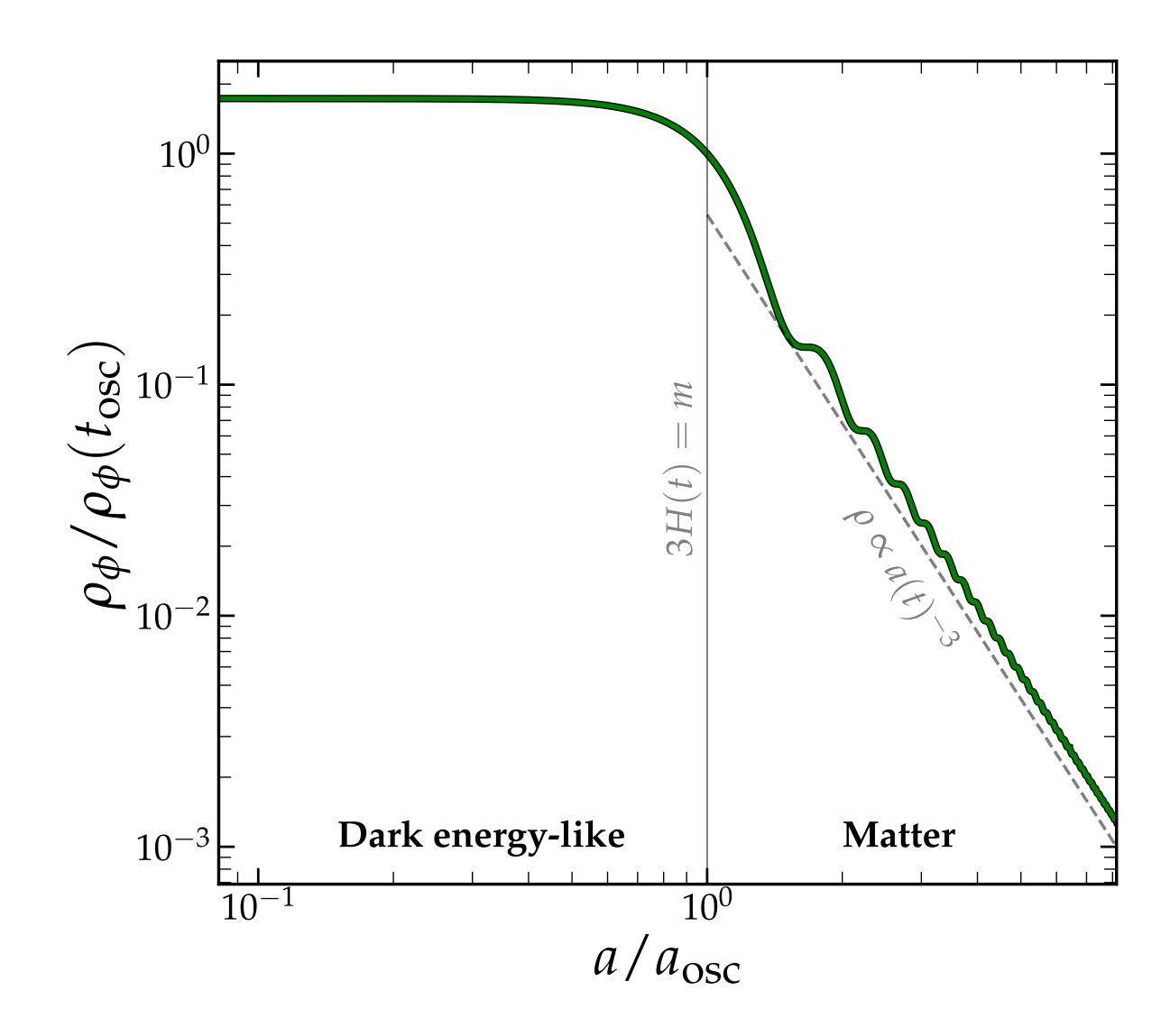
- $3H(t) \gg m \rightarrow \text{overdamped}$
- $3H(t) \ll m \rightarrow \text{damped harmonic oscillator}$



Misalignment mechanism for a generic scalar

Consider energy density in the scalar field

$$ho_{\phi} = rac{1}{2}\dot{\phi}^2 + rac{1}{2}m^2\phi^2 \ \Longrightarrow
ho_{\phi} \propto a^{-3}$$



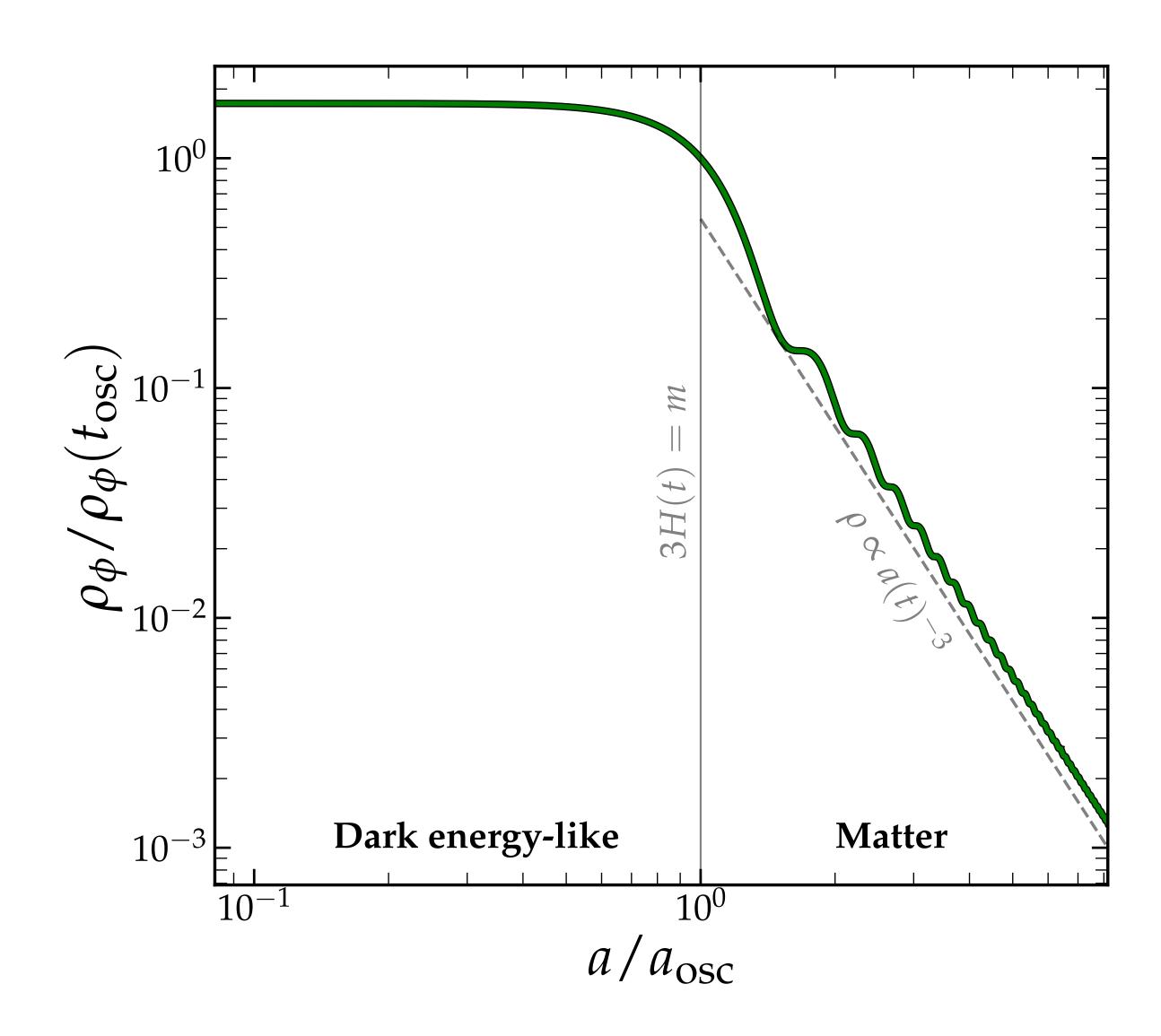
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ho_{\phi} \propto a^{-3}$$

Redshifts like dark matter, with abundance today:

$$\Omega_{
m DM} h^2 \propto \phi_i^2 m^{1/2}$$



Axion misalignment

Axion is the Goldstone (θ) appearing after the U(1)_{PQ} is broken at scale f_a .

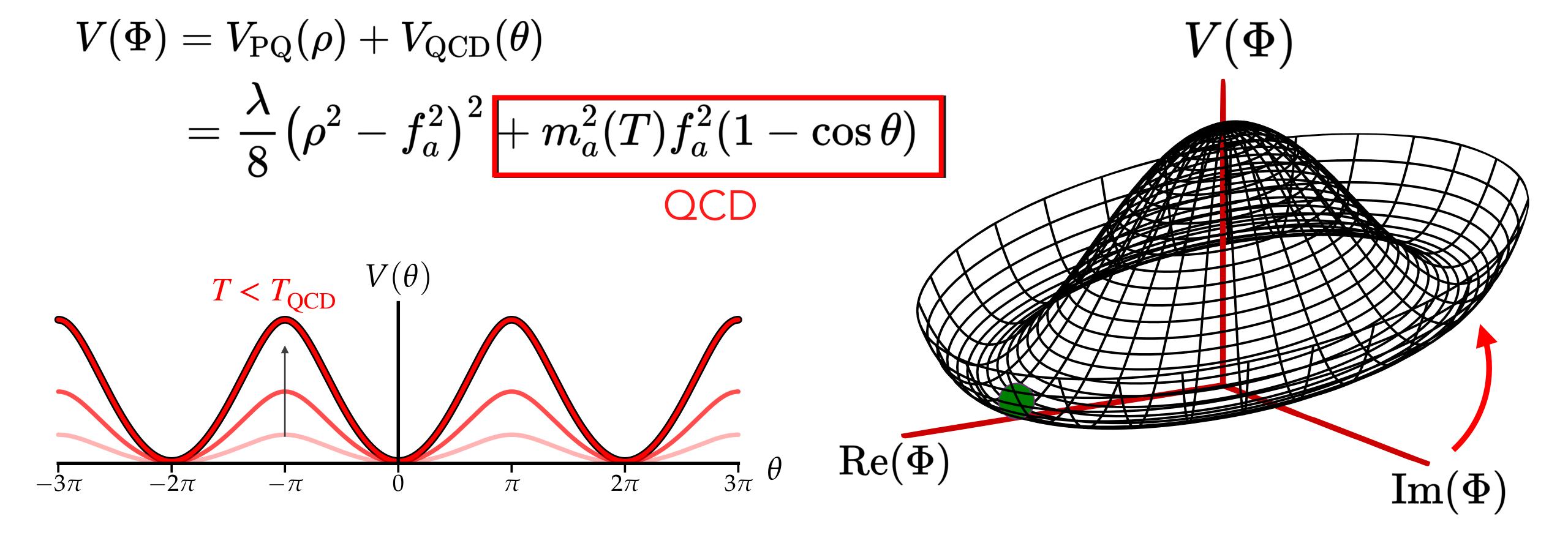
We write it as the phase of a complex scalar field: $\Phi(t) = \rho \, e^{i heta}$

$$V(\Phi) = V_{ ext{PQ}}(
ho) + V_{ ext{QCD}}(heta) \qquad V(\Phi) \ = rac{\lambda}{8} \left(
ho^2 - f_a^2
ight)^2 + m_a^2(T) f_a^2 (1 - \cos heta) \ ext{Re}(\Phi) \qquad ext{Im}(\Phi)$$

Axion misalignment

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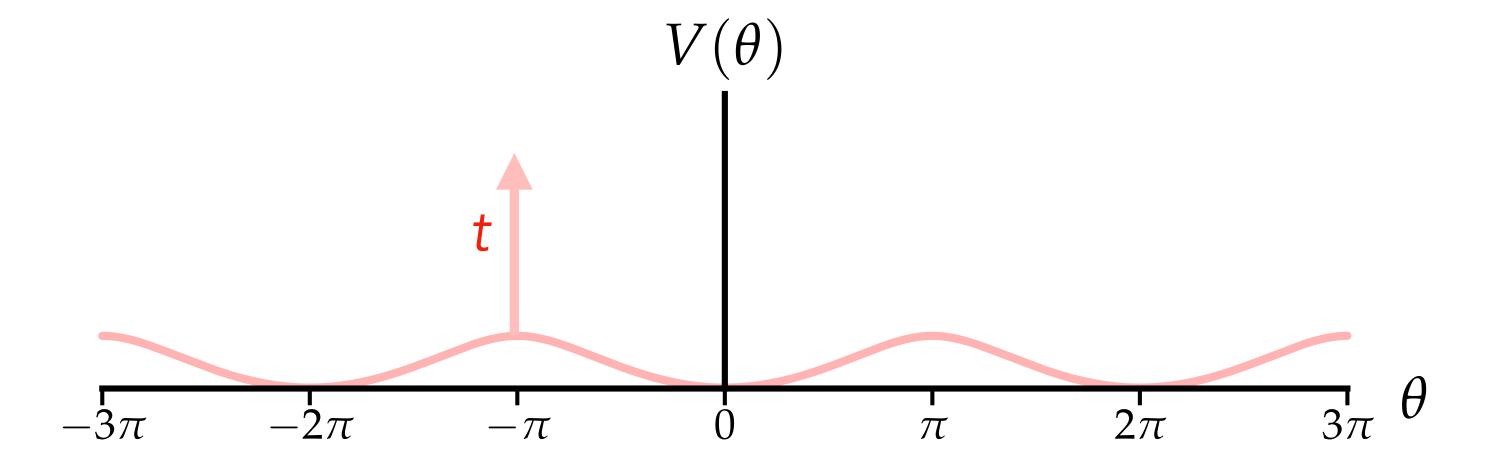
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Mass is generated by instantons whose effects are temperature-dependent In the literature this dependence is called the "topological susceptibility", $\chi(T)$

$$V(heta) pprox \chi(T)(1-\cos heta) = m_a^2(T)f_a^2(1-\cos heta)$$

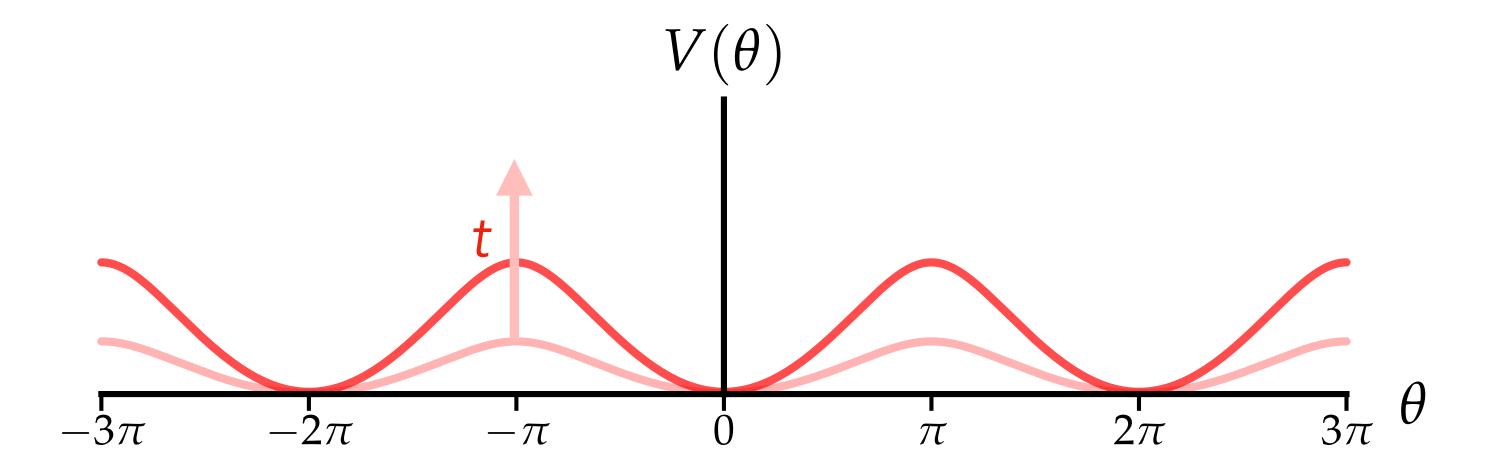
Axion mass grows as temperature drops, reaching a constant when $T < T_{\rm QCD}$



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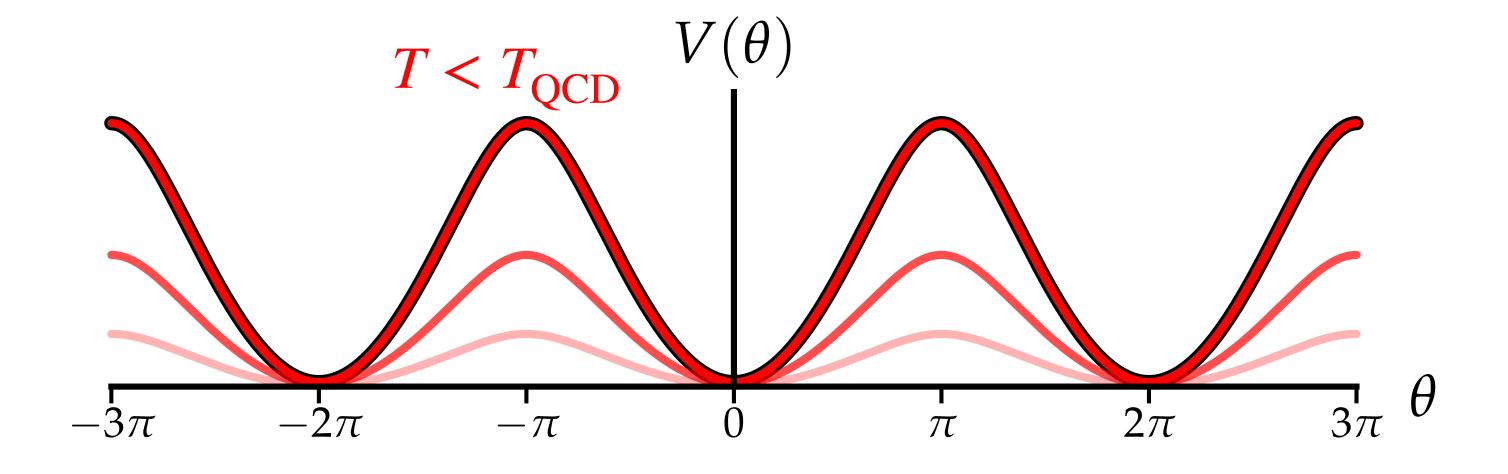
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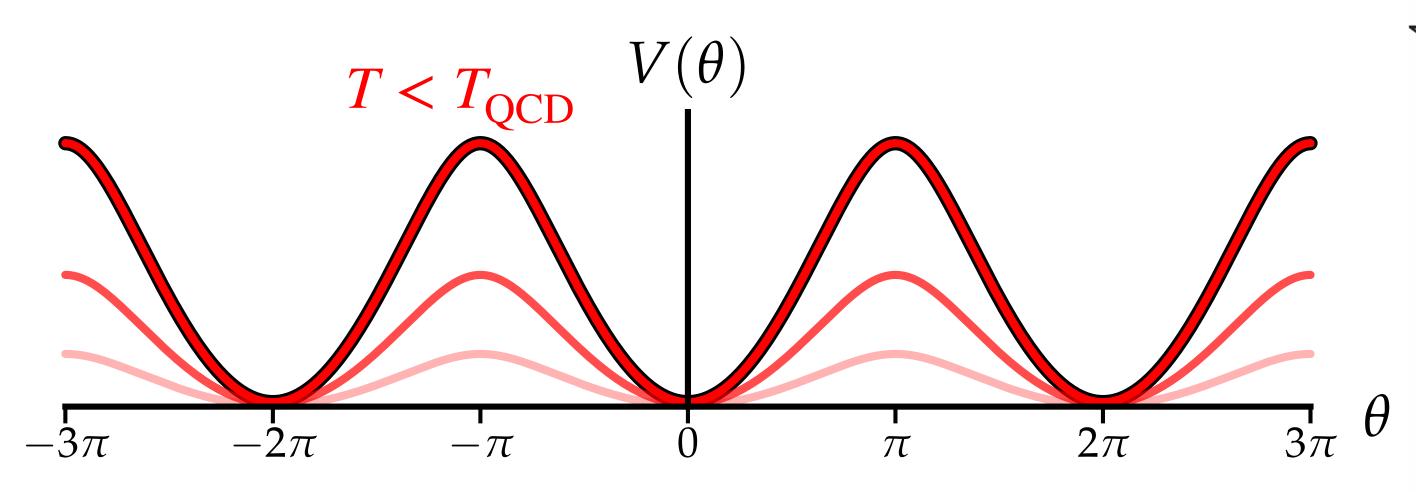
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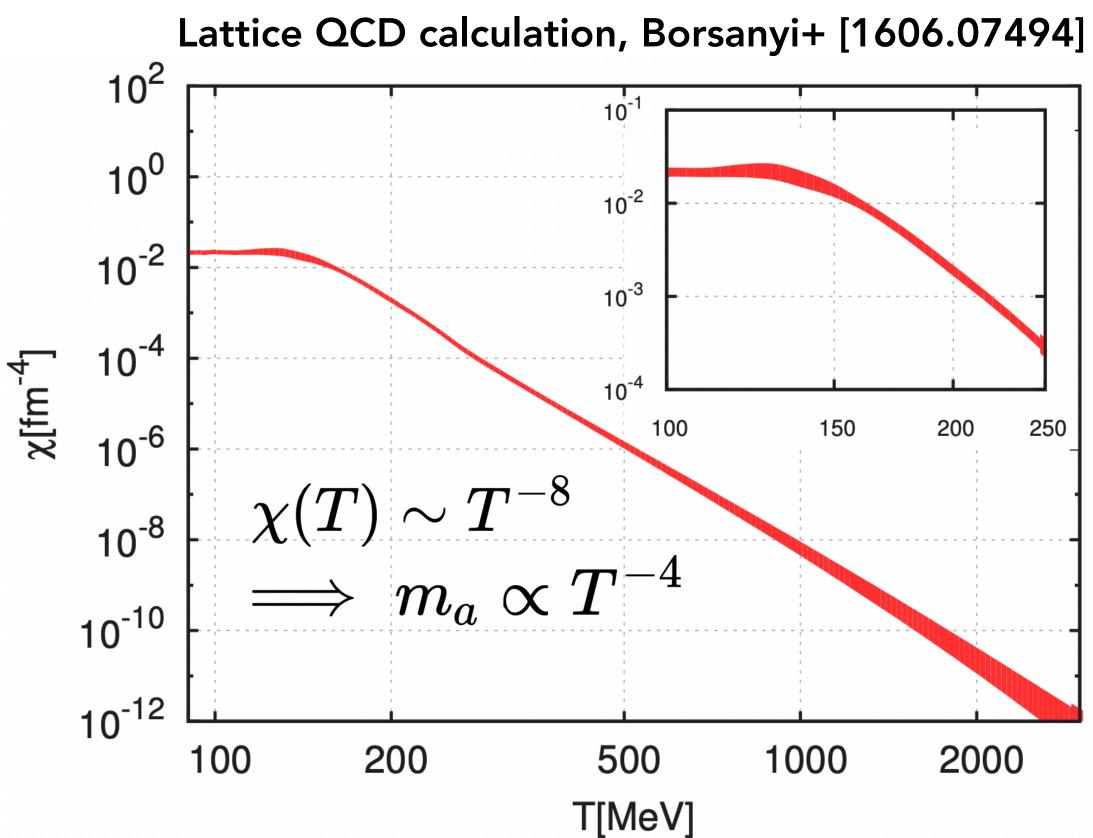


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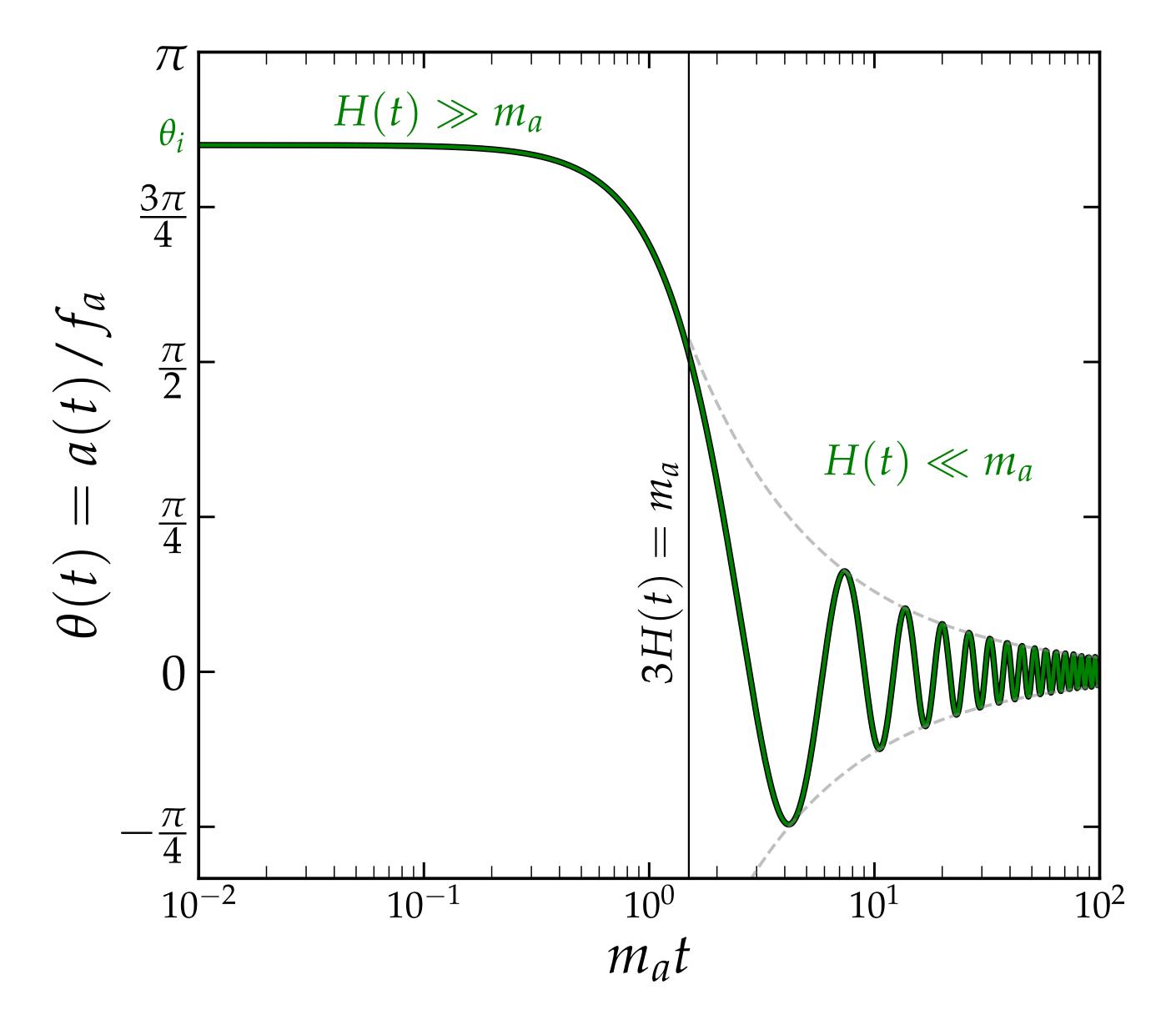
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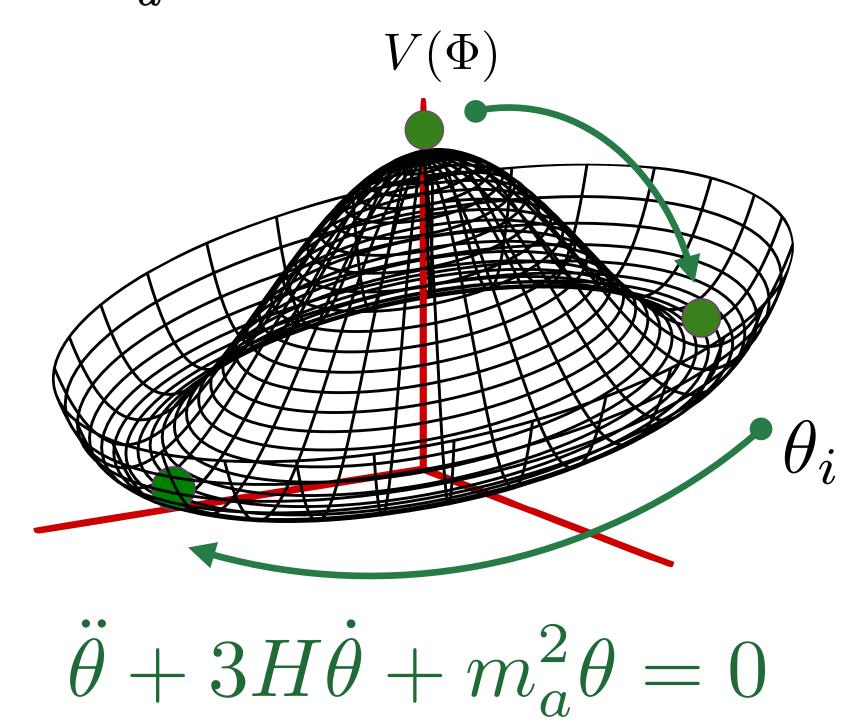


The QCD axion mass

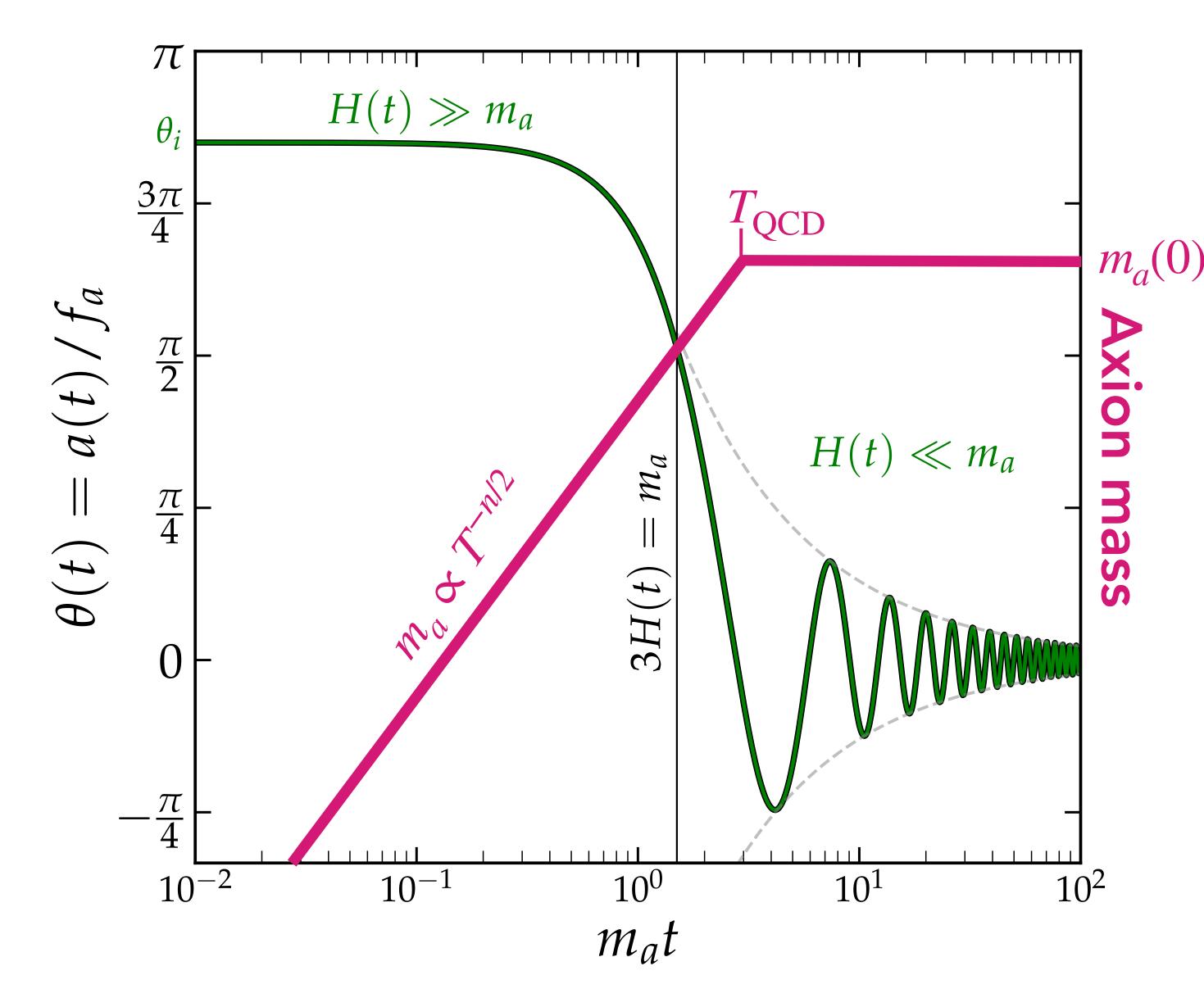


QCD topological susceptibility: The tilt comes on gradually as the temperature drops

$$\Longrightarrow m_a \propto T^{-n/2}$$

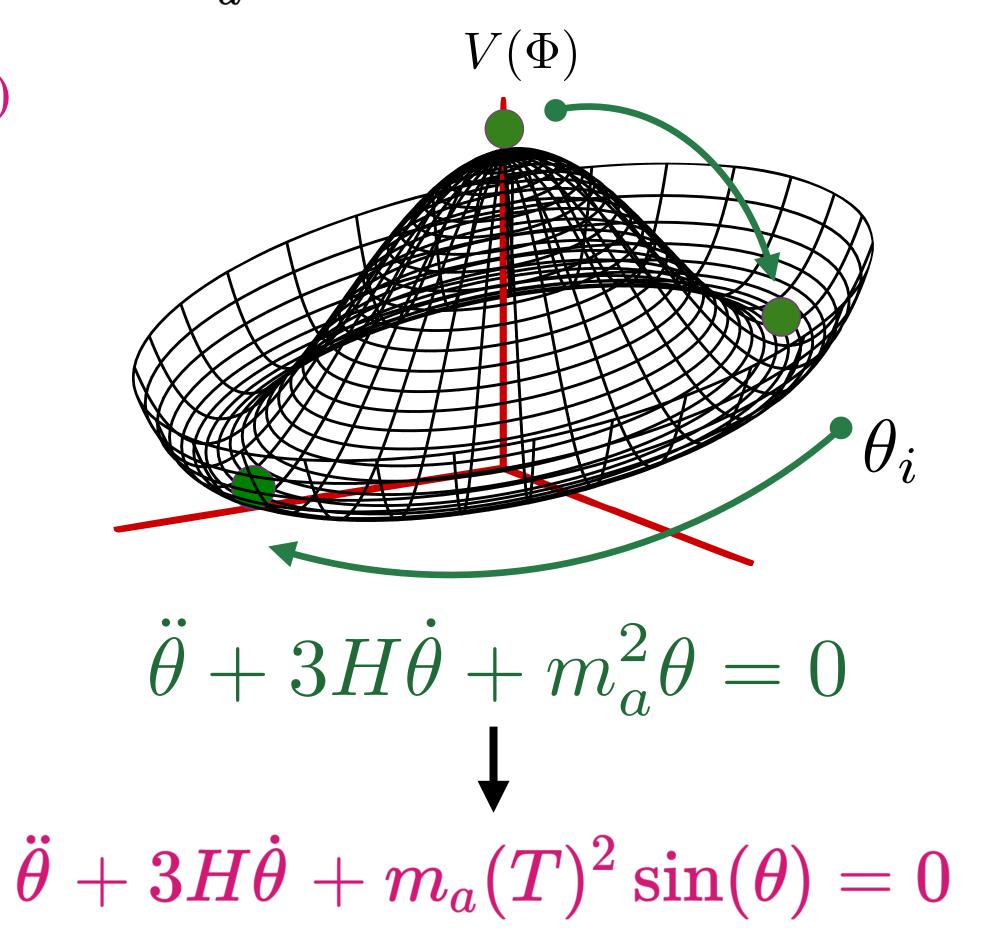


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QCD axion abundance

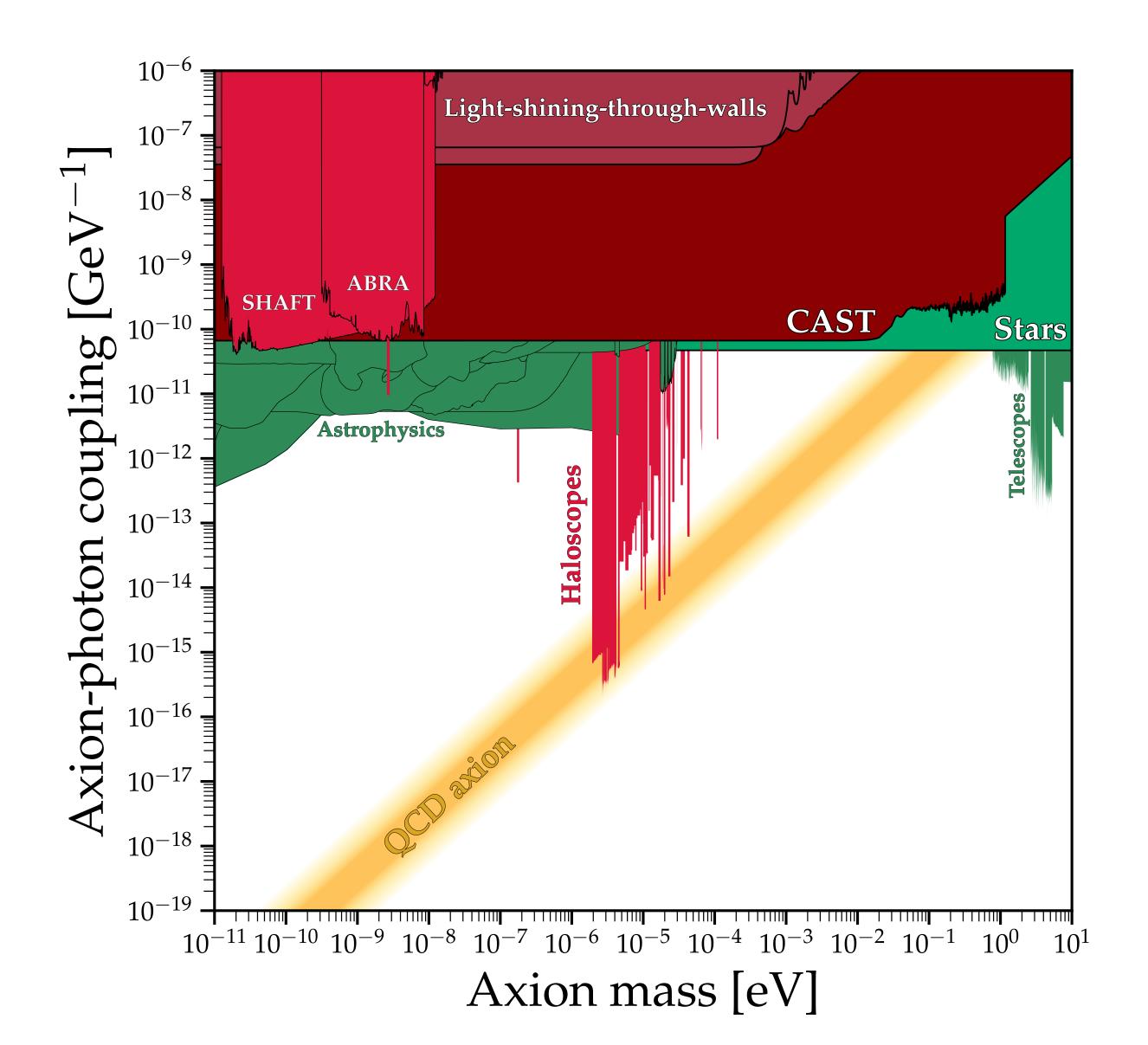
• Generic scalar misalignment:

$$\Omega_\phi h^2 \propto \phi_i^2 m^{1/2}$$

For QCD axion we get:

$$\Omega_a h^2 pprox 0.12\, heta_i^2 igg(rac{7.26\,\mu\mathrm{eV}}{m_a}igg)^{rac{n+6}{n+4}}$$

where $n \sim 8$ (from Lattice QCD, e.g. 1606.07494)



QCD axion abundance

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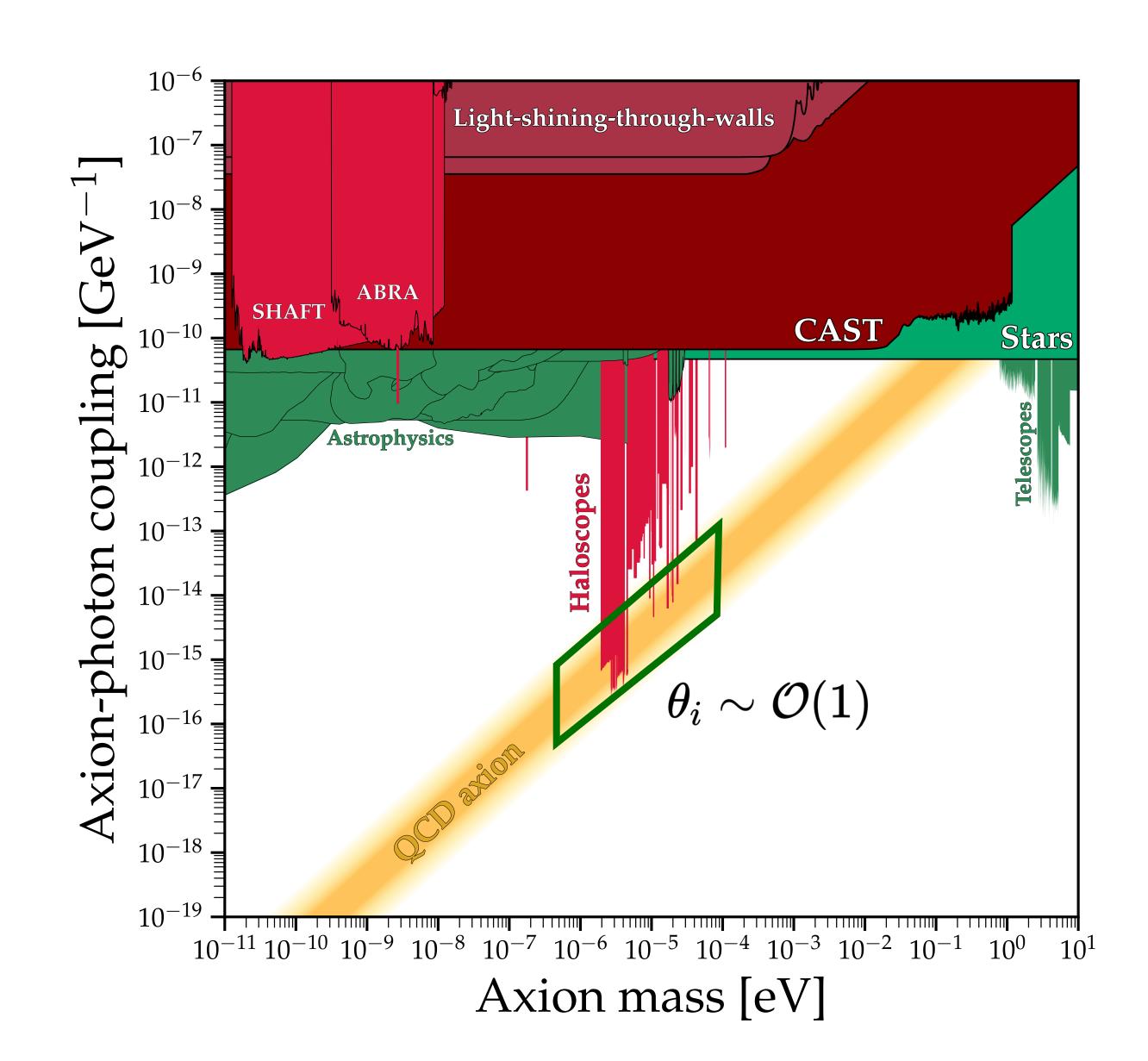
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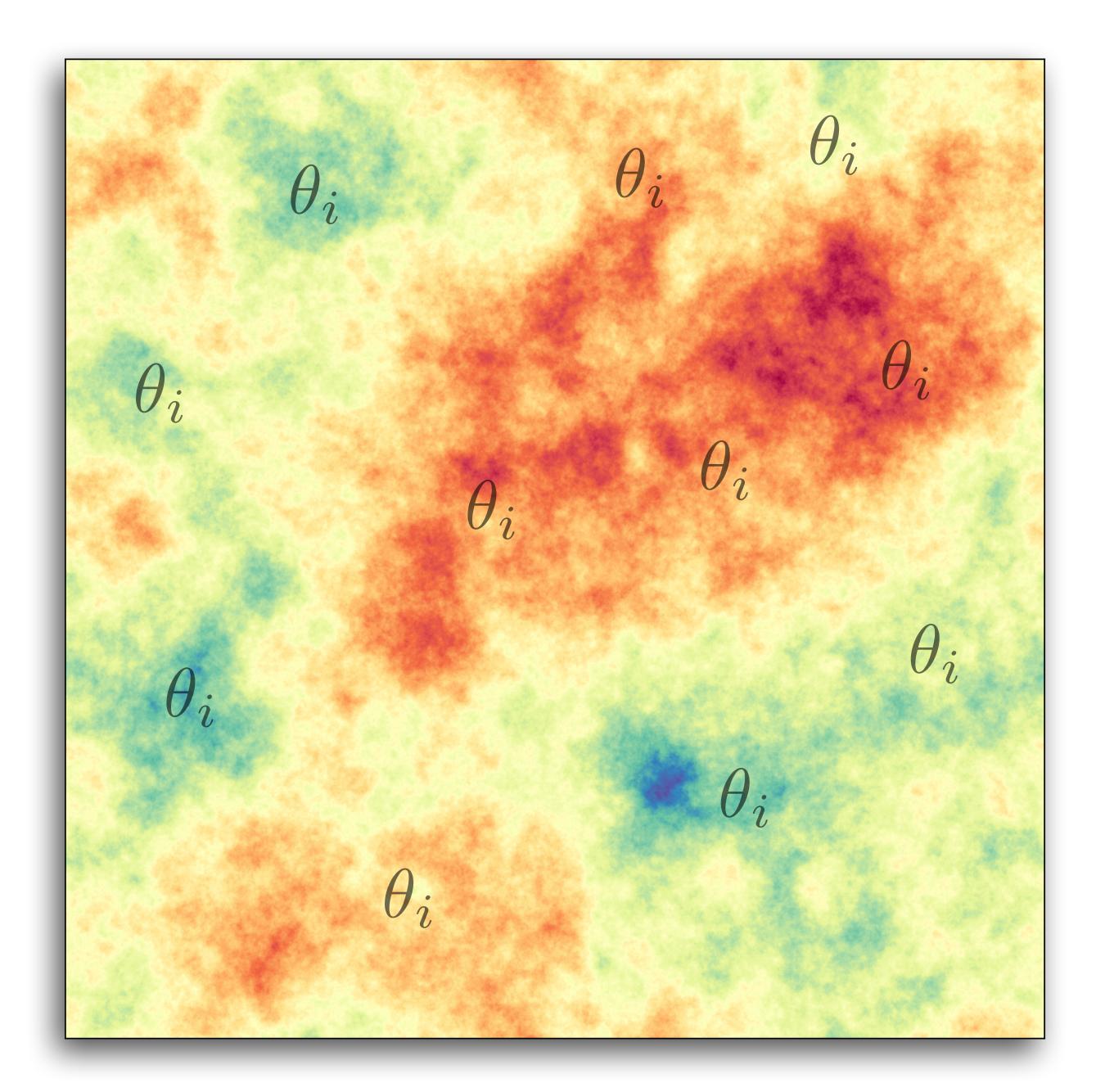
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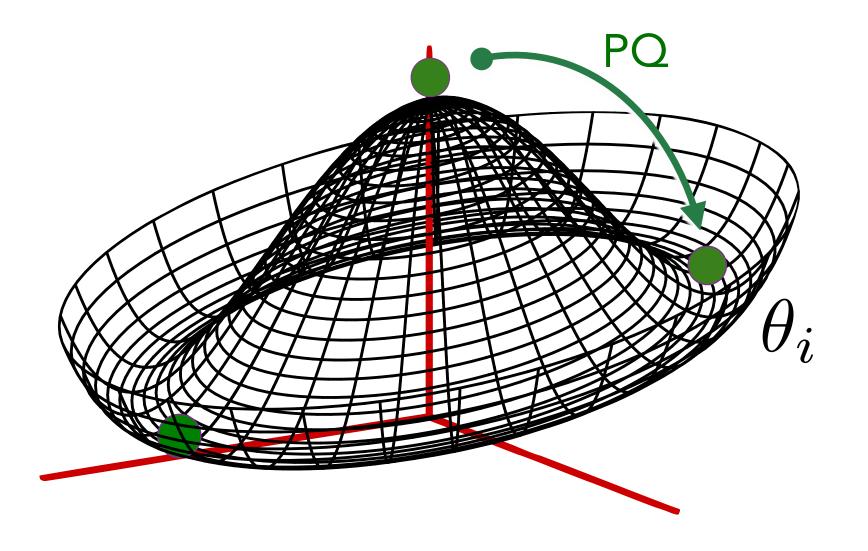
- Leads to "classic QCD axion window": $\mathcal{O}(1-10) \mu eV$
- \rightarrow but what should we pick for θ_i ?





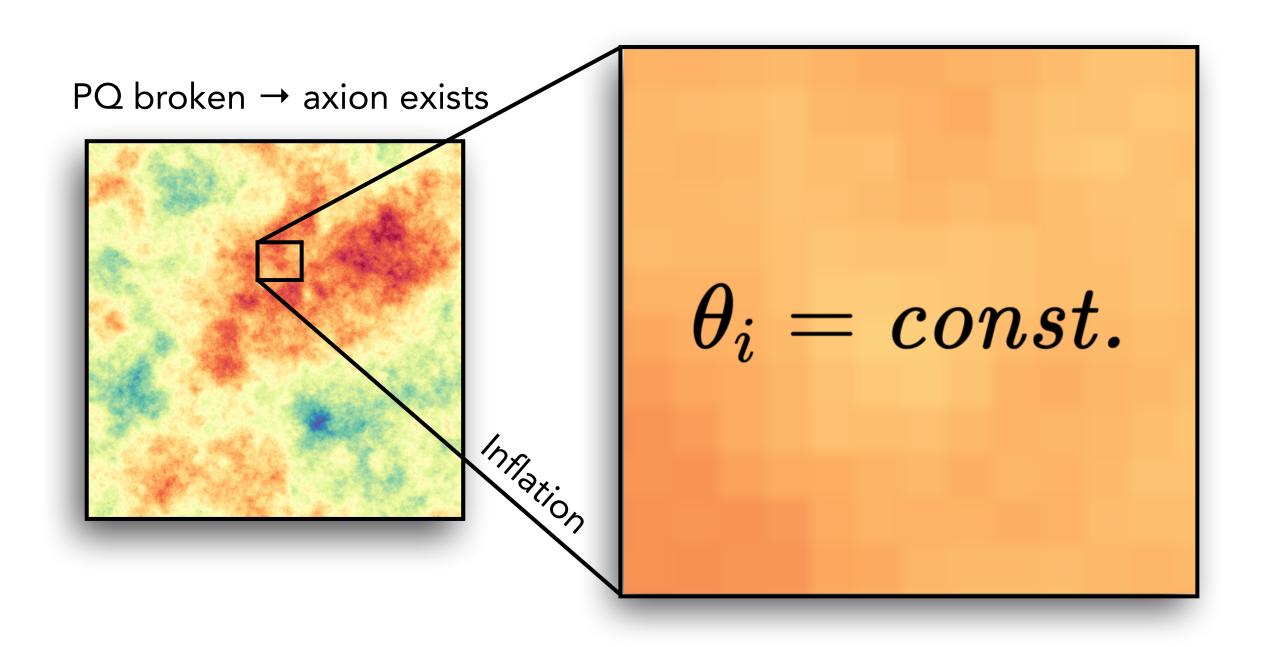
The issue is that $T_{\rm PQ}\gg T_{\rm QCD}$

The Universe should be filled with random θ_i everywhere since the axion was massless when it was born at PQ phase transition, i.e. it didn't know about the preferred angle



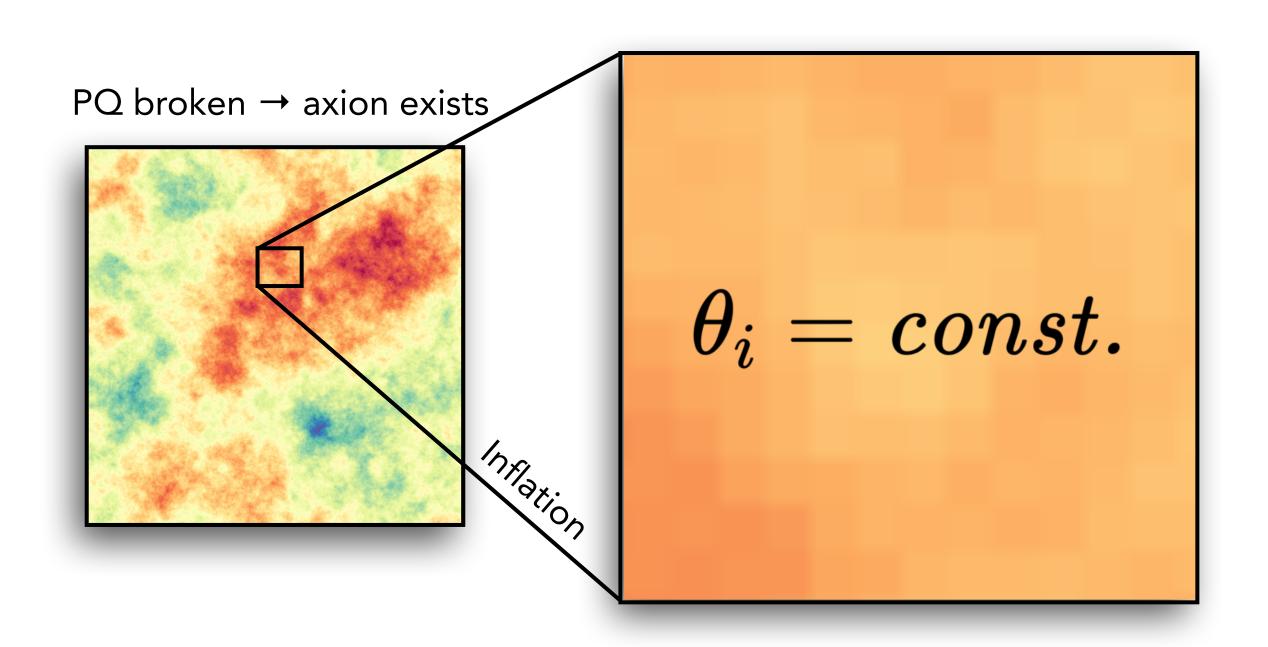
Option 1:

PQ is broken before and during inflation

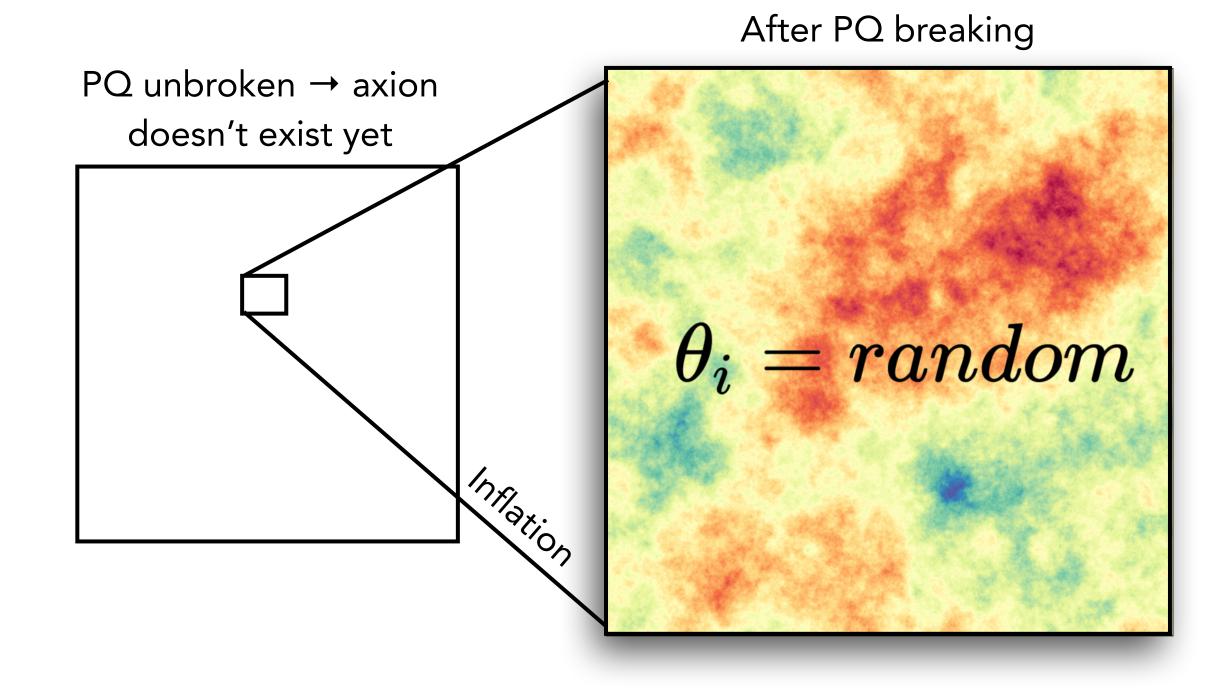


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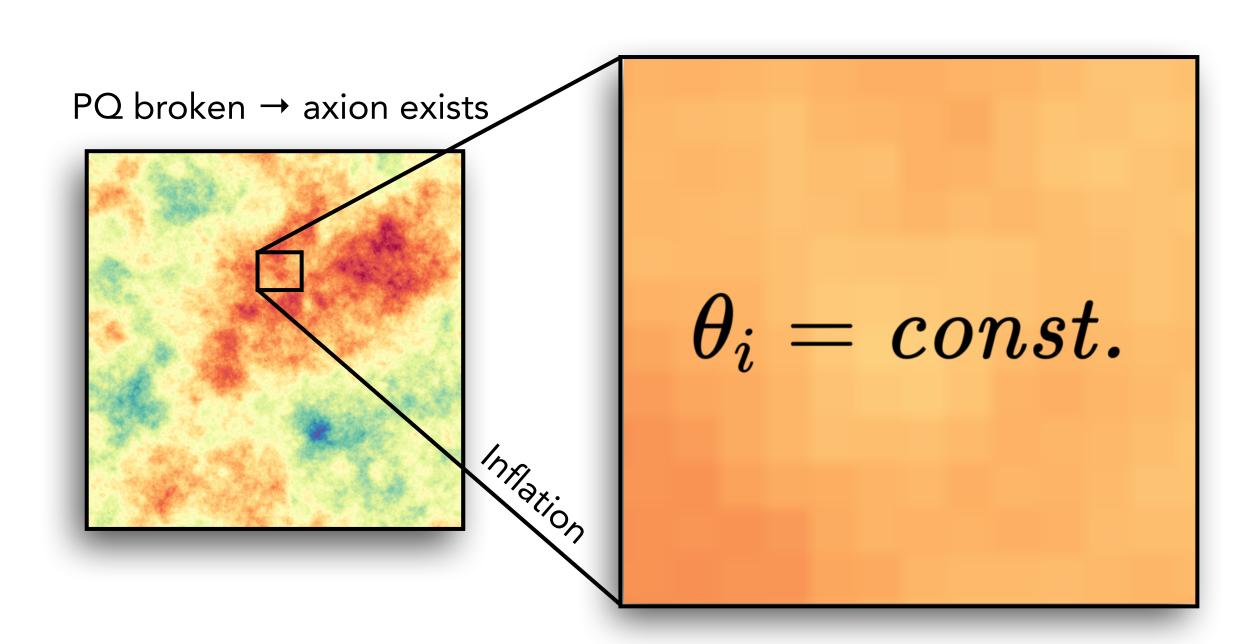


Option 2: PQ is broken *after* inflation



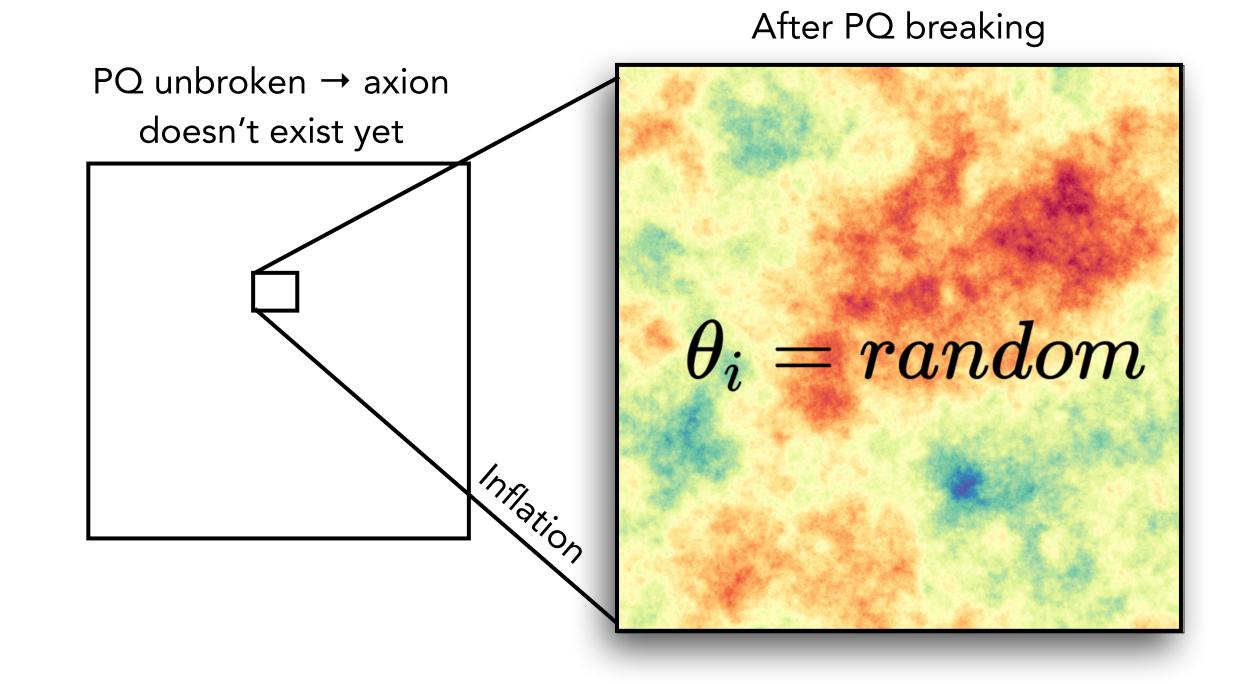
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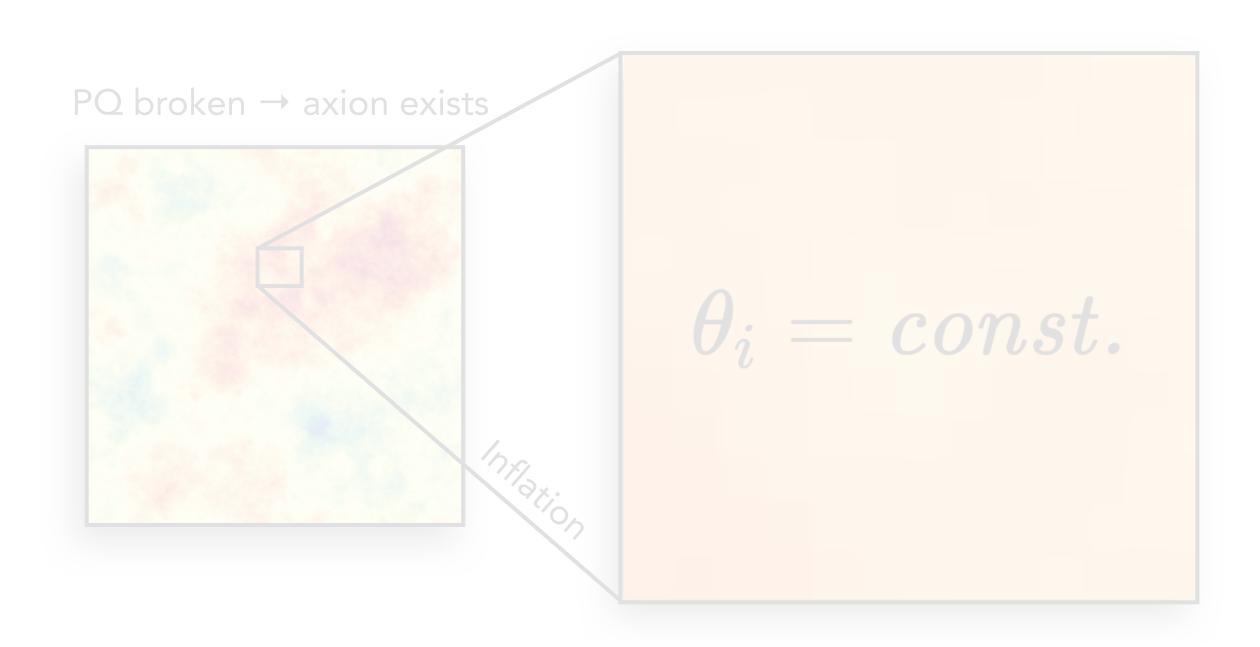


Pre-inflationary scenario

Post-inflationary scenario

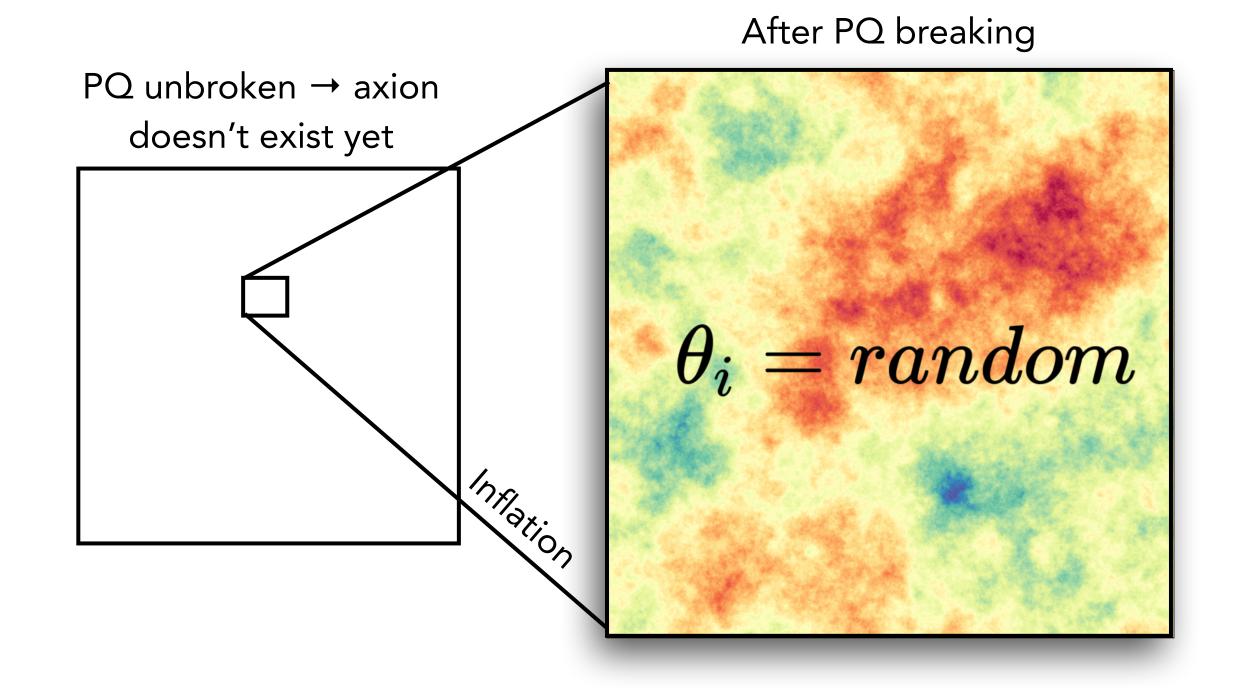
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Option 2:

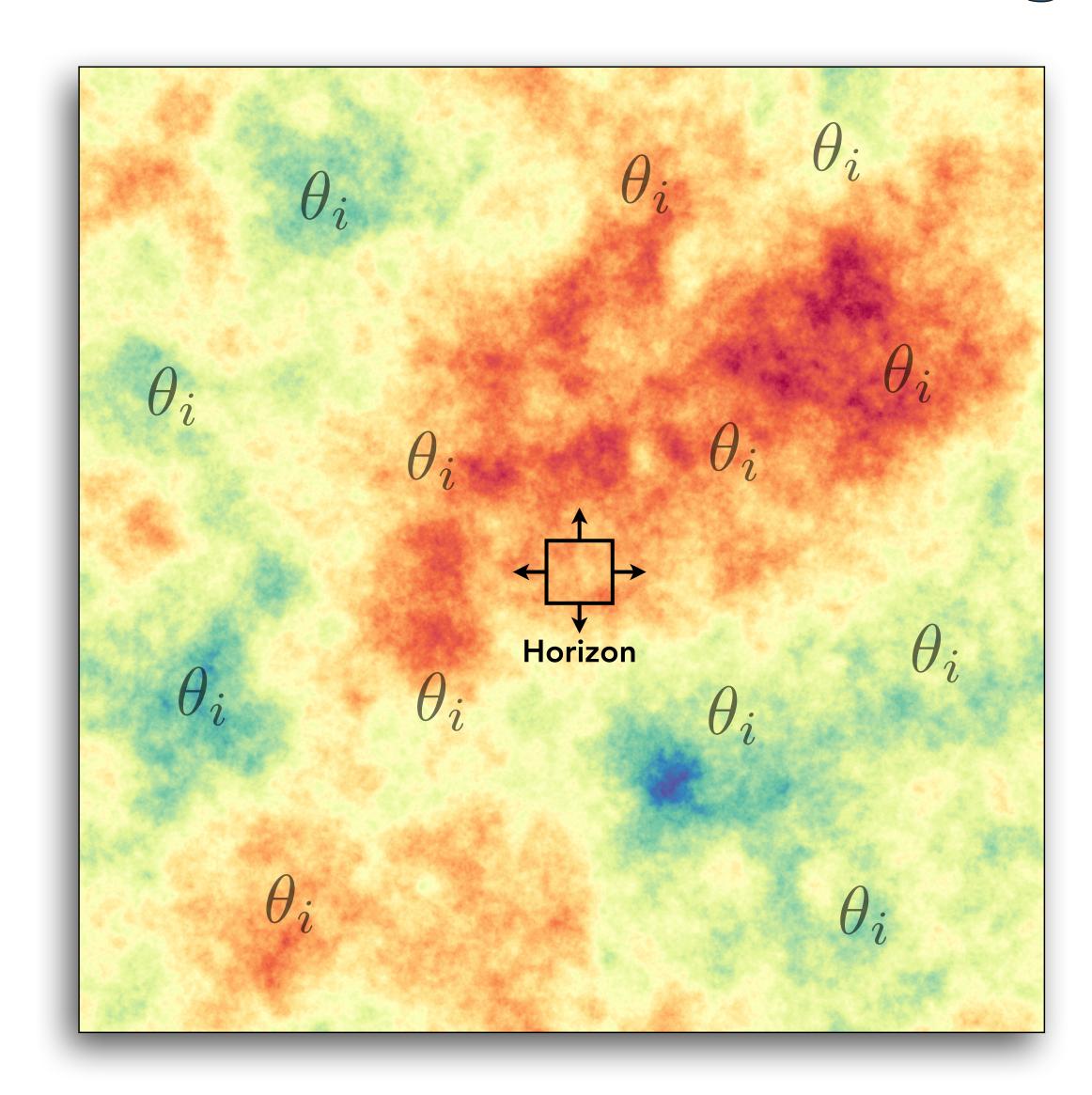
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Pre-inflationary scenario

Post-inflationary scenario

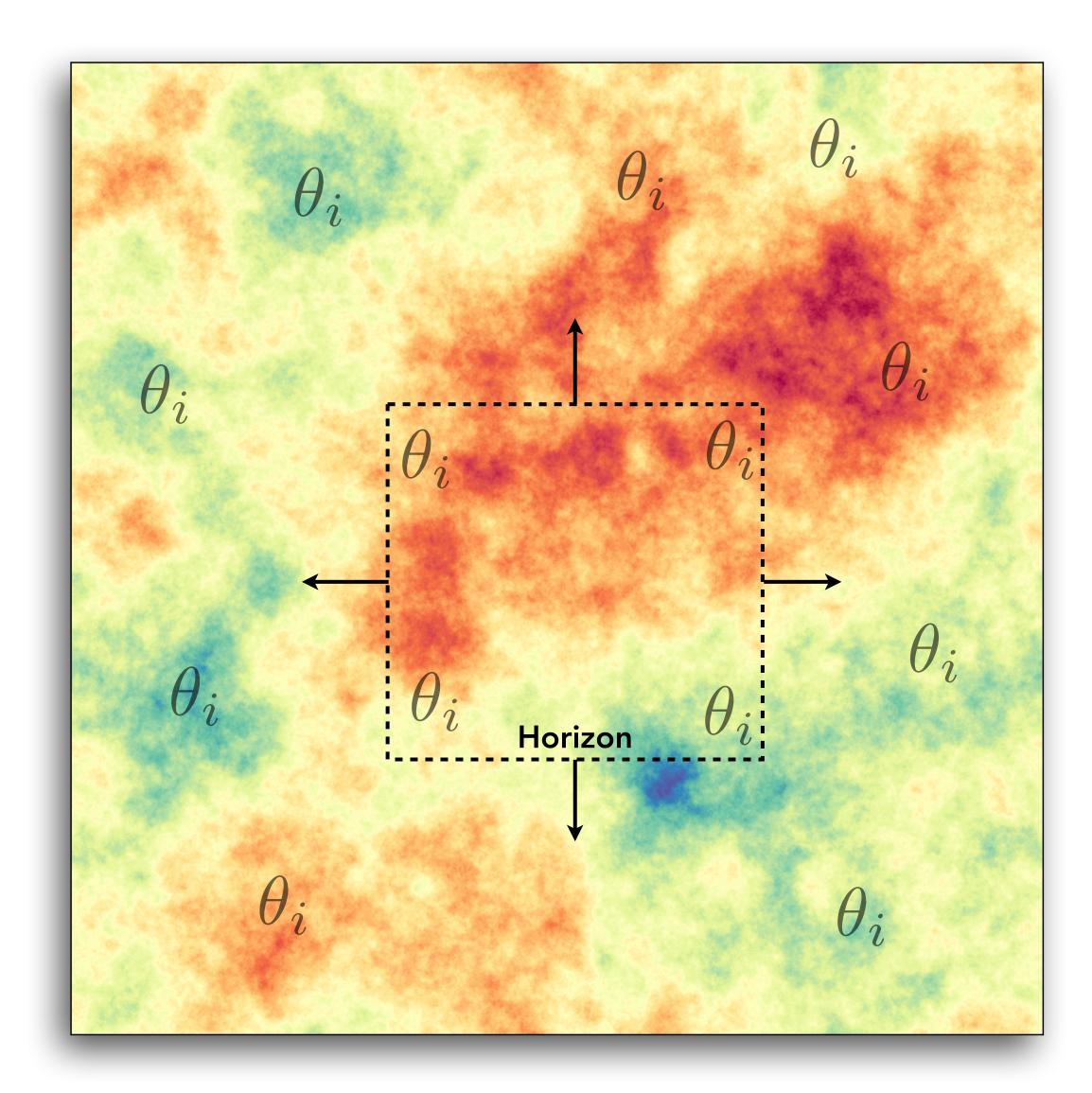
Post-inflationary scenario



Inflation has already happened *before* axion was born

- \rightarrow Universe filled with many values of θ_i
- → Different value in every causal patch

Post-inflationary scenario



Inflation has already happened *before* axion was born

- \rightarrow Universe filled with many values of θ_i
- → Different value in every causal patch
- → Patches come into contact as horizon grows.

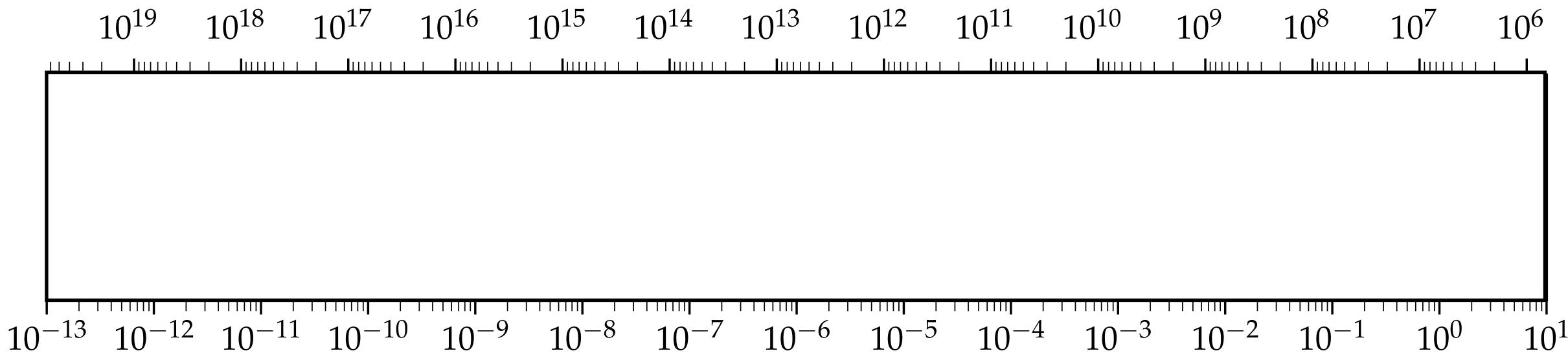
Post-inflationary scenario

- We have an ensemble of every possible θ_i sampled across our Universe.
- Stochastic average:

$$\langle heta_i^2
angle pprox \left(rac{\pi}{\sqrt{3}}
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$$\Omega_a h^2 pprox 0.12 rac{\left< heta_i^2
ight>}{(1.81)^2} \left(rac{20 \mu \mathrm{eV}}{m_a}
ight)^{rac{n+6}{n+4}}$$

Peccei-Quinn scale, fa [GeV]

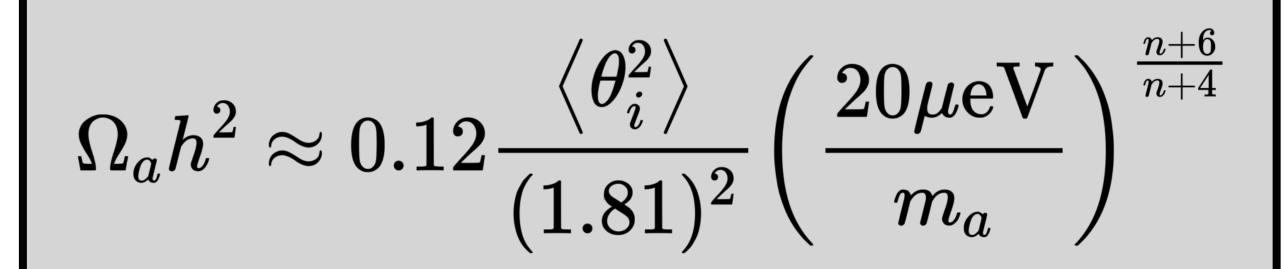


QCD axion mass, m_a [eV]

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Peccei-Quinn scale, f_a [GeV]

 $10^{19} \quad 10^{18} \quad 10^{17} \quad 10^{16} \quad 10^{15} \quad 10^{14} \quad 10^{13} \quad 10^{12} \quad 10^{11} \quad 10^{10} \quad 10^{9} \quad 10^{8} \quad 10^{7} \quad 10^{6}$

In the post-inflationary scenario only one mass

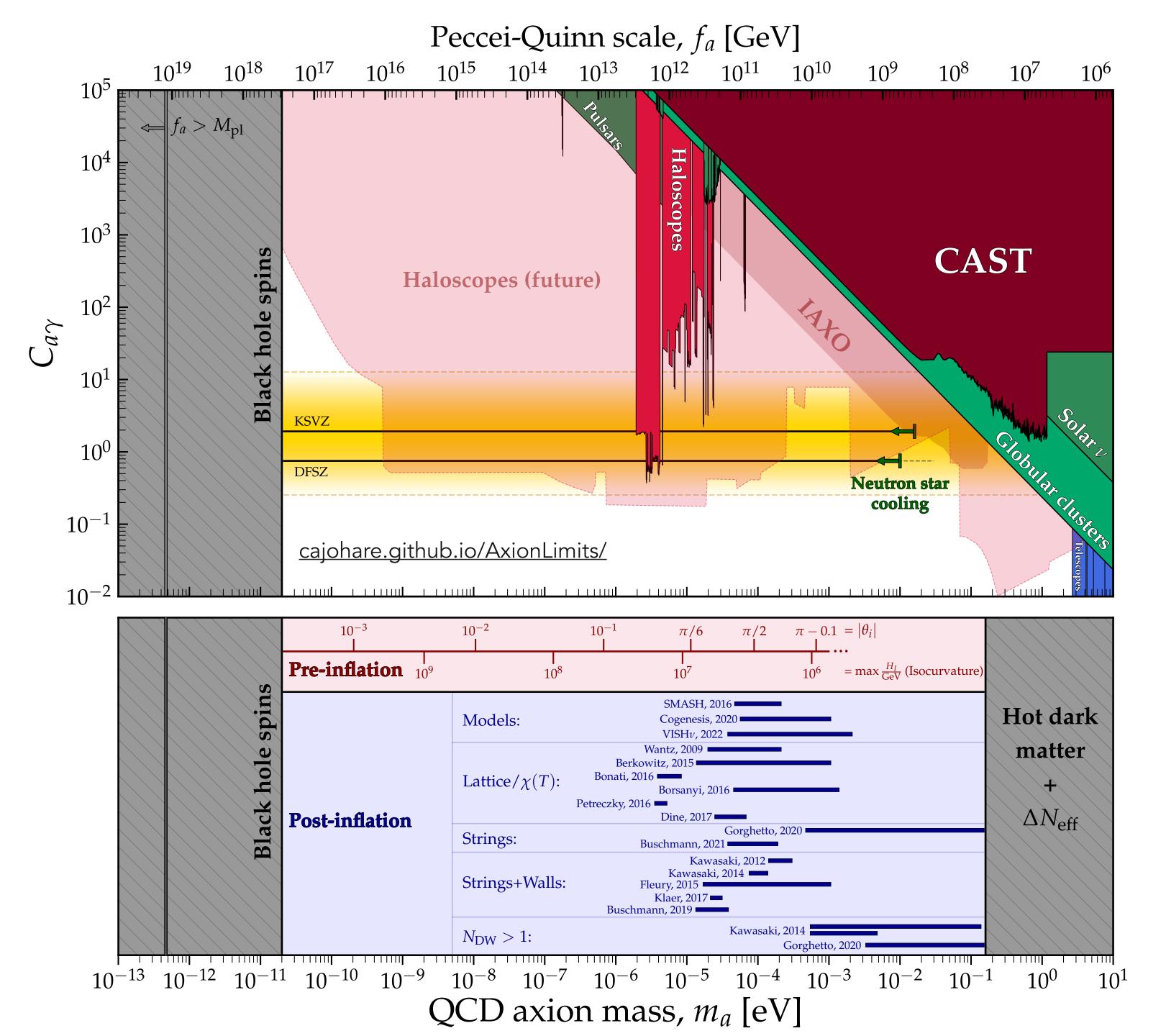
is consistent with observed DM abundance

(Up to theoretical uncertainties)

Overabundant - Underabundant

 $10^{-13} \ 10^{-12} \ 10^{-11} \ 10^{-10} \ 10^{-9} \ 10^{-8} \ 10^{-7} \ 10^{-6} \ 10^{-5} \ 10^{-4} \ 10^{-3} \ 10^{-2} \ 10^{-1} \ 10^{0} \ 10^{-1}$

QCD axion mass, m_a [eV]

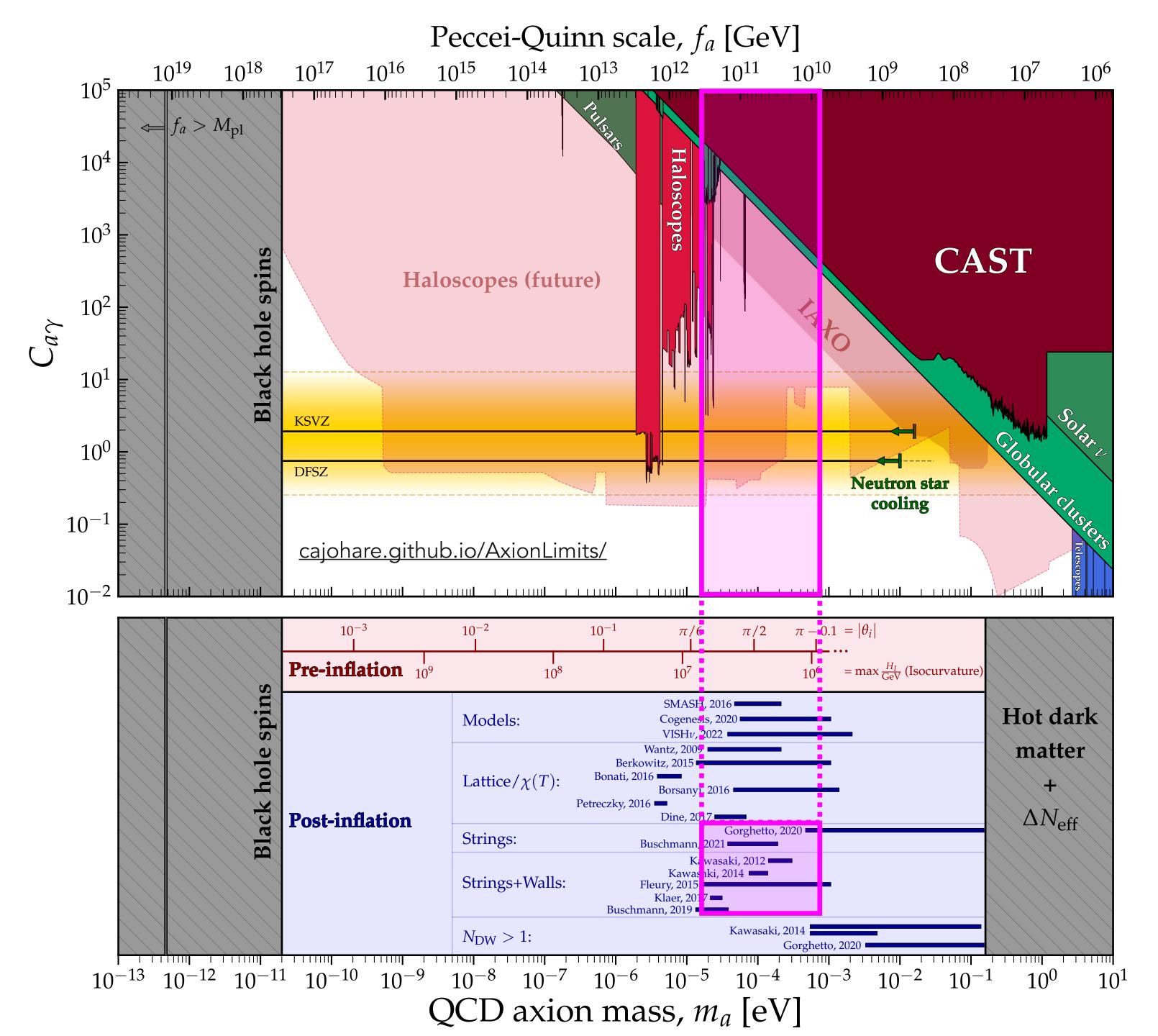


Post-inflationary axion mass range

 $\mathcal{O}(10 - 100 \,\mu\text{eV})$

Relevant for experiments like:

- \rightarrow QUAX
- → MADMAX
 - → ORGAN
 - → ALPHA
 - → DALI
 - → CADEx
 - → BRASS
 - → BREAD



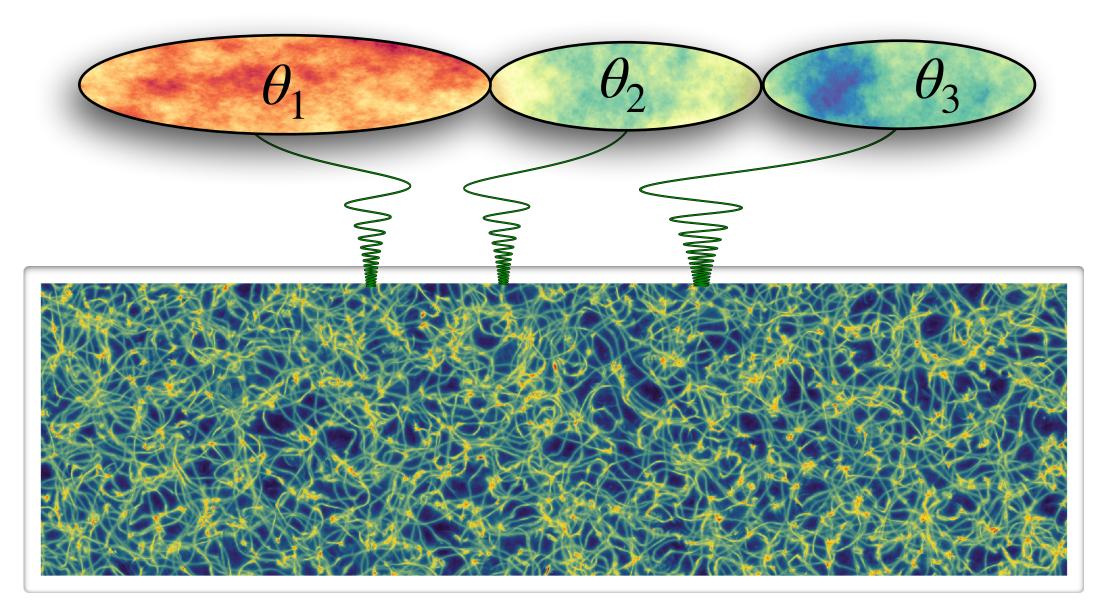
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But there's a complication: $\nabla \theta$

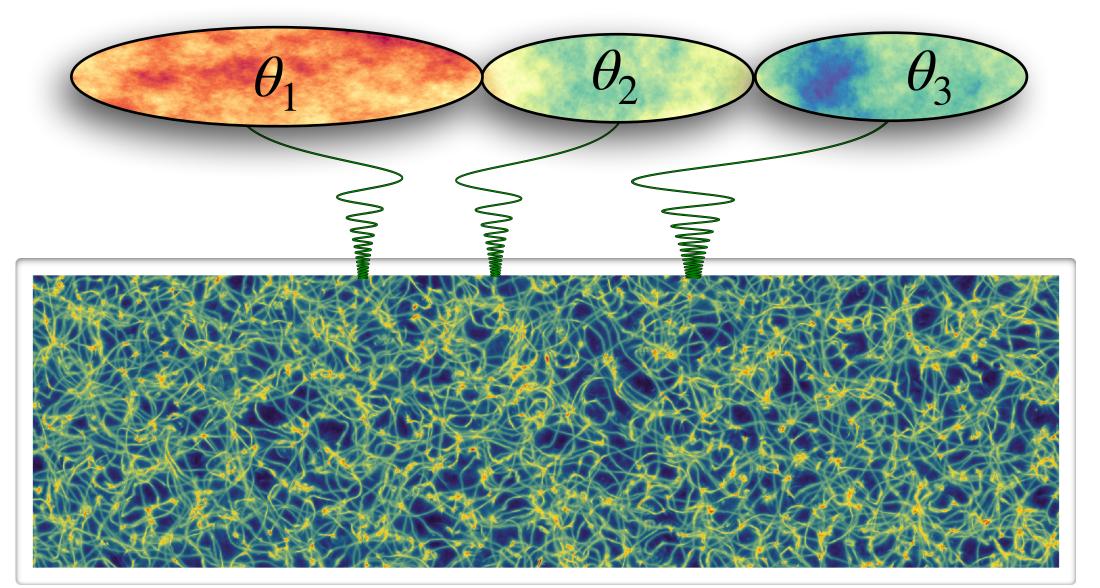


Different patches meet up

→ Field gradients!

$$\leftarrow \quad \ddot{\theta} + 3H\dot{\theta} - \frac{1}{a^2}\nabla^2\theta + m_a^2\theta = 0$$

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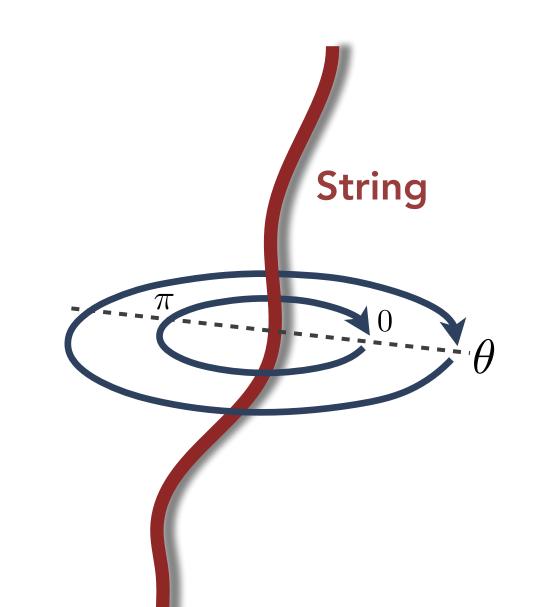


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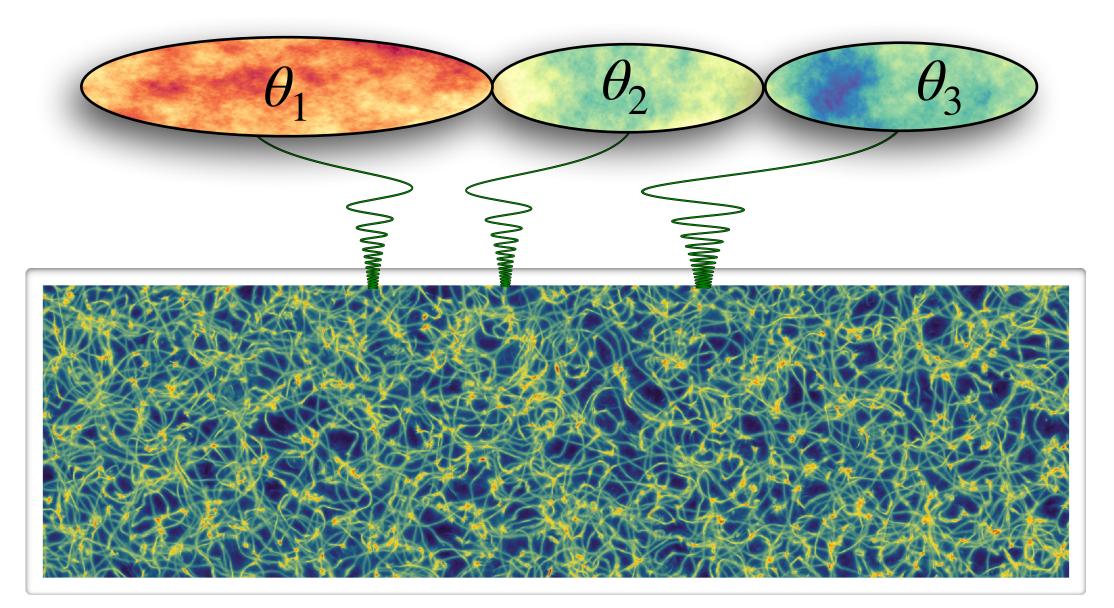
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 \Rightarrow Cosmic strings from axion field winding around 2π



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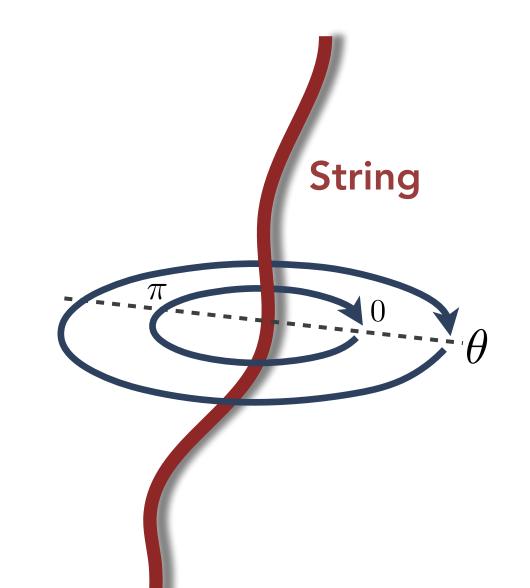


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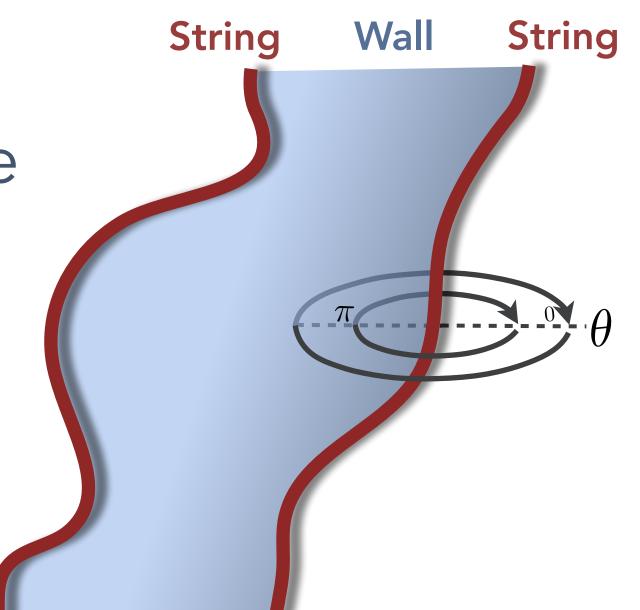
$$\leftarrow \quad \ddot{\theta} + 3H\dot{\theta} \left[-\frac{1}{a^2} \nabla^2 \theta + m_a^2 \theta = 0 \right]$$

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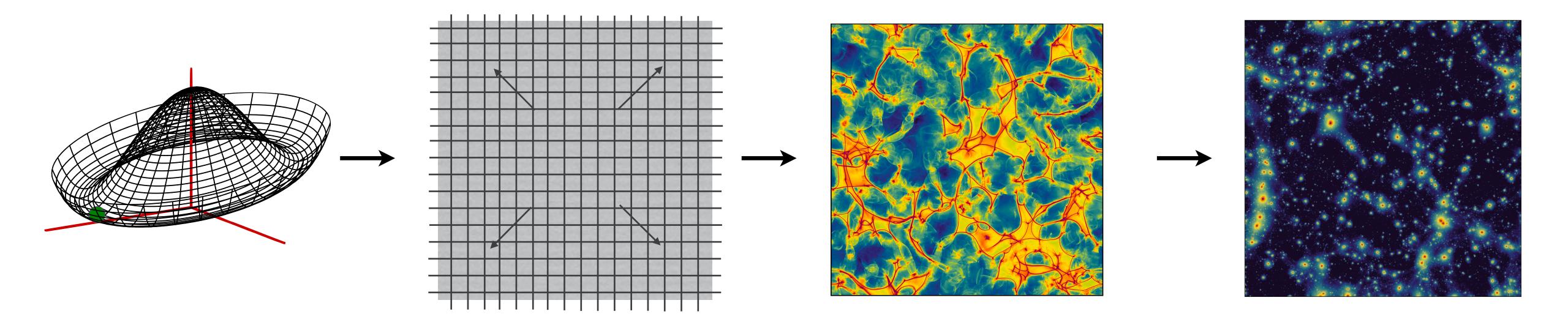


⇒ <u>Domain walls</u>

between true/false vacuum (0 and π)



Brute force solution: simulate



Evolve the axion field...

...on an expanding lattice...

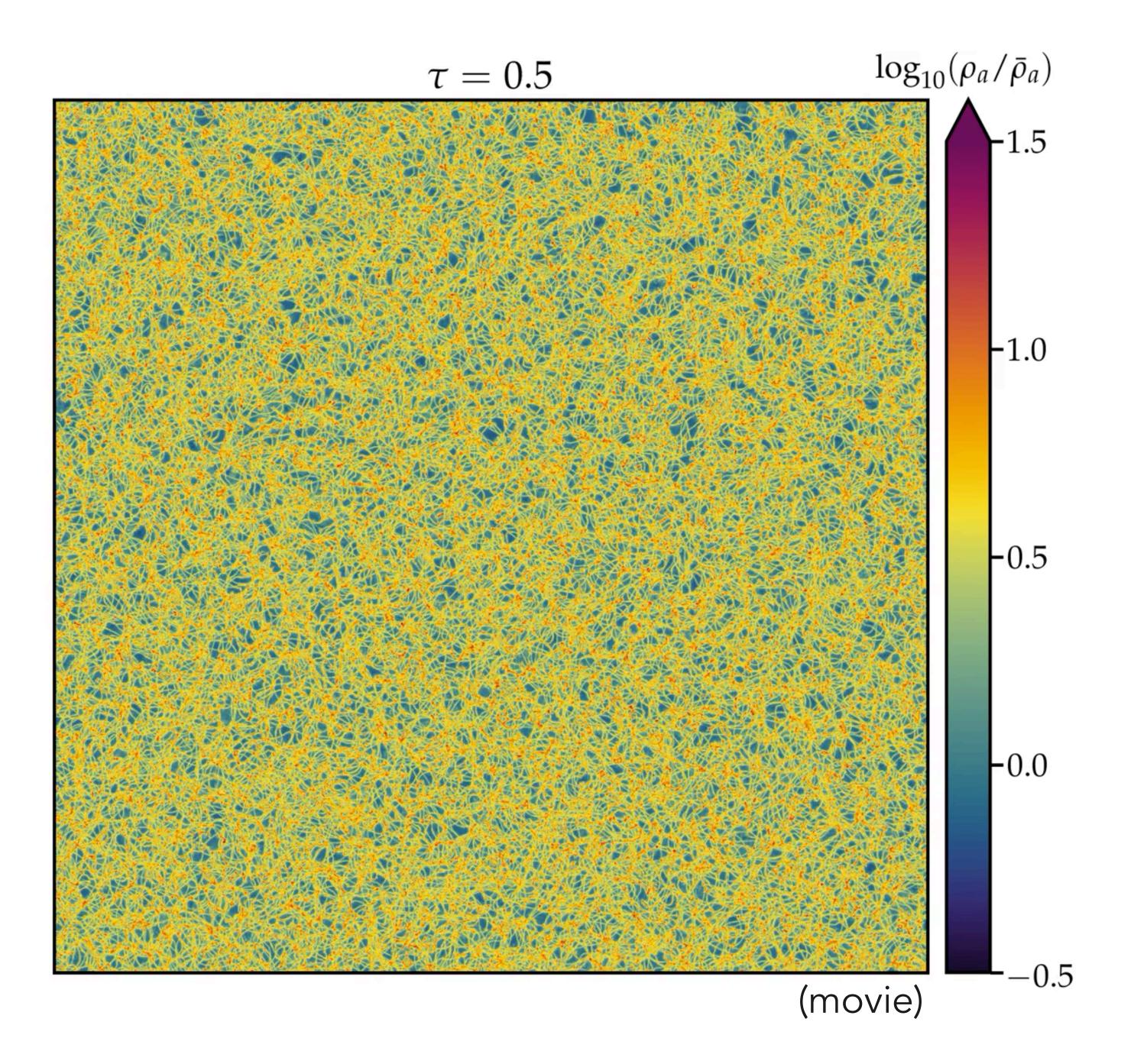
...to measure the relic abundance of axions...

...and predict its present day distribution

Evolution of the axion field in the post-inflationary scenario

Projection through 3D co-moving box, coloured by integrated axion energy density:

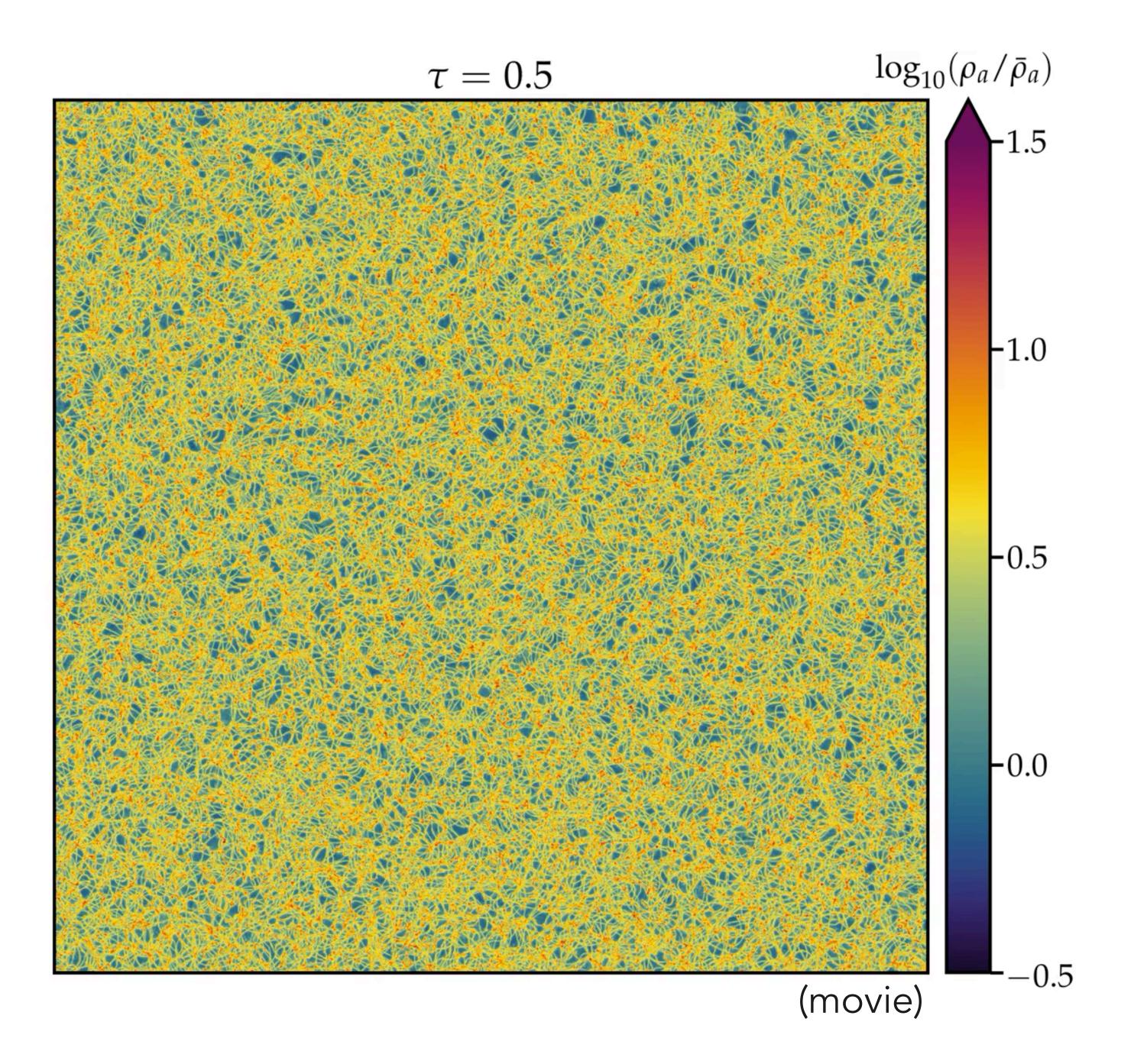
$$\rho_a = \frac{1}{2}\dot{a}^2 + \frac{1}{2R^2}(\nabla a)^2 + \chi(1 - \cos a/f_a)$$

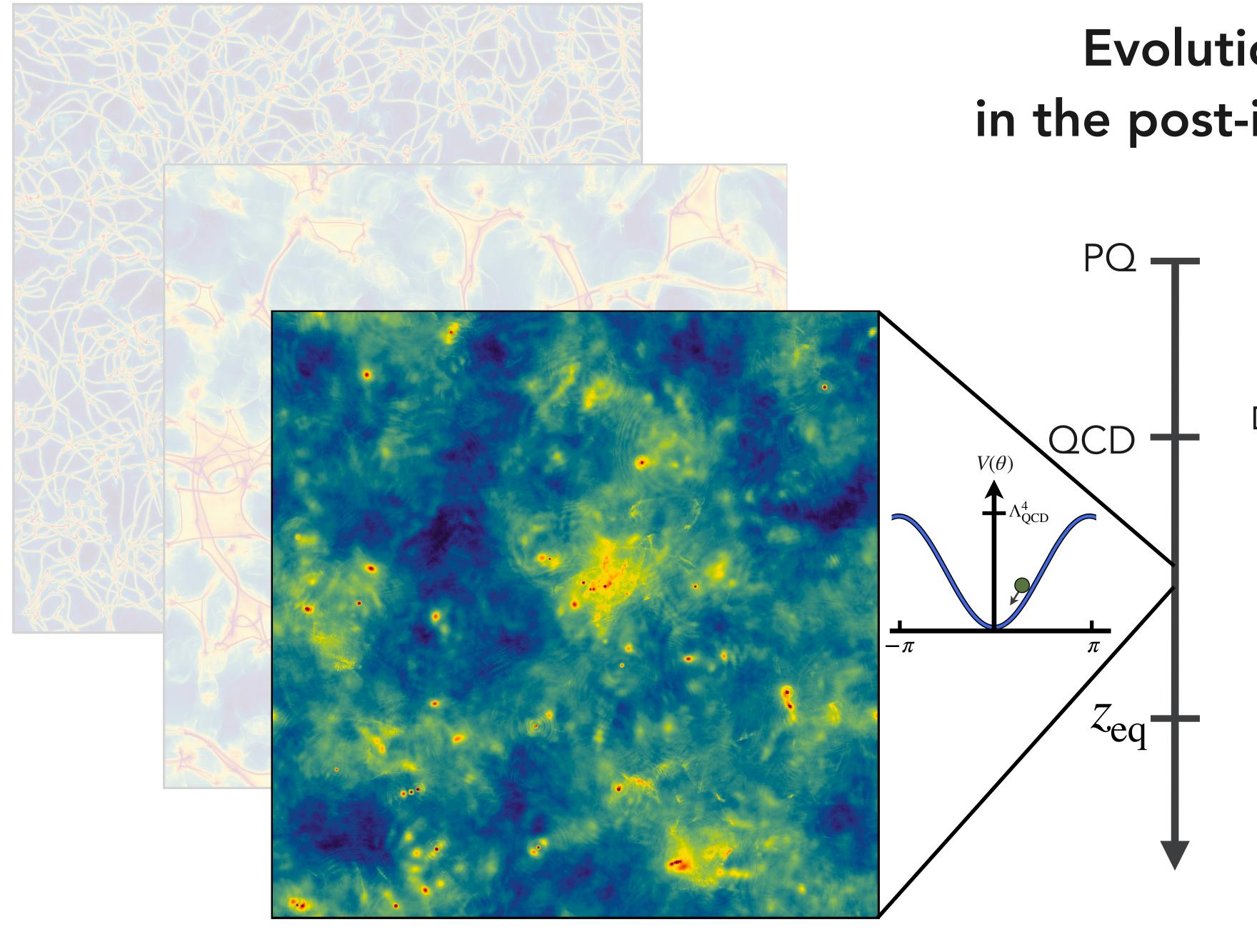


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Evolution of the axion field in the post-inflationary scenario

String network scaling

Domain walls attached to strings

→ network collapses

Inhomogeneous distribution of axions free streams until non-relativistic

Seeds of structure gravitationally collapse into miniclusters and halos

What is the ultimate distribution of axions in galaxies?

Will it be like vanilla Λ CDM halos?

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Pre-inflationary axion: probably, yes.

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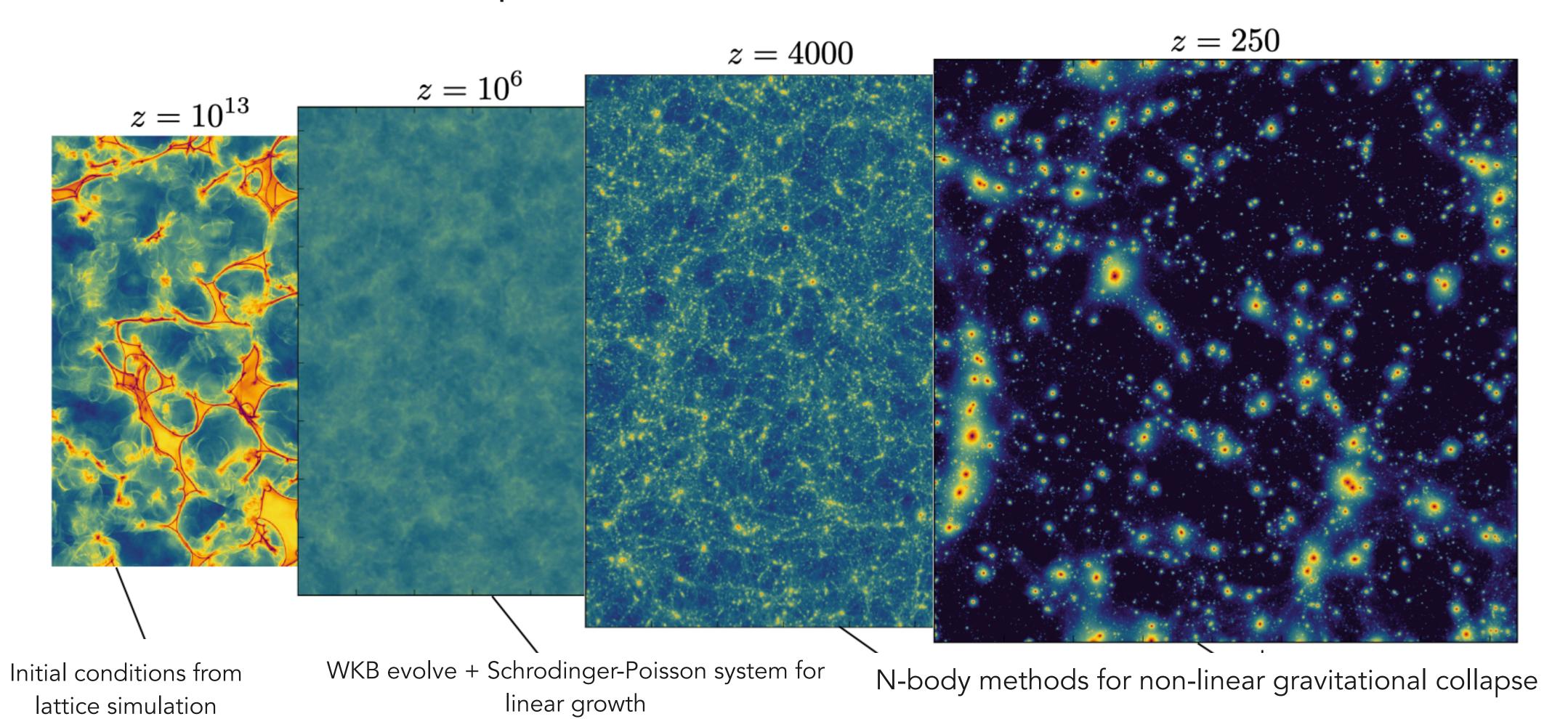
Will it be like vanilla Λ CDM halos?

Pre-inflationary axion: probably, yes.

Post-inflationary axion: NO

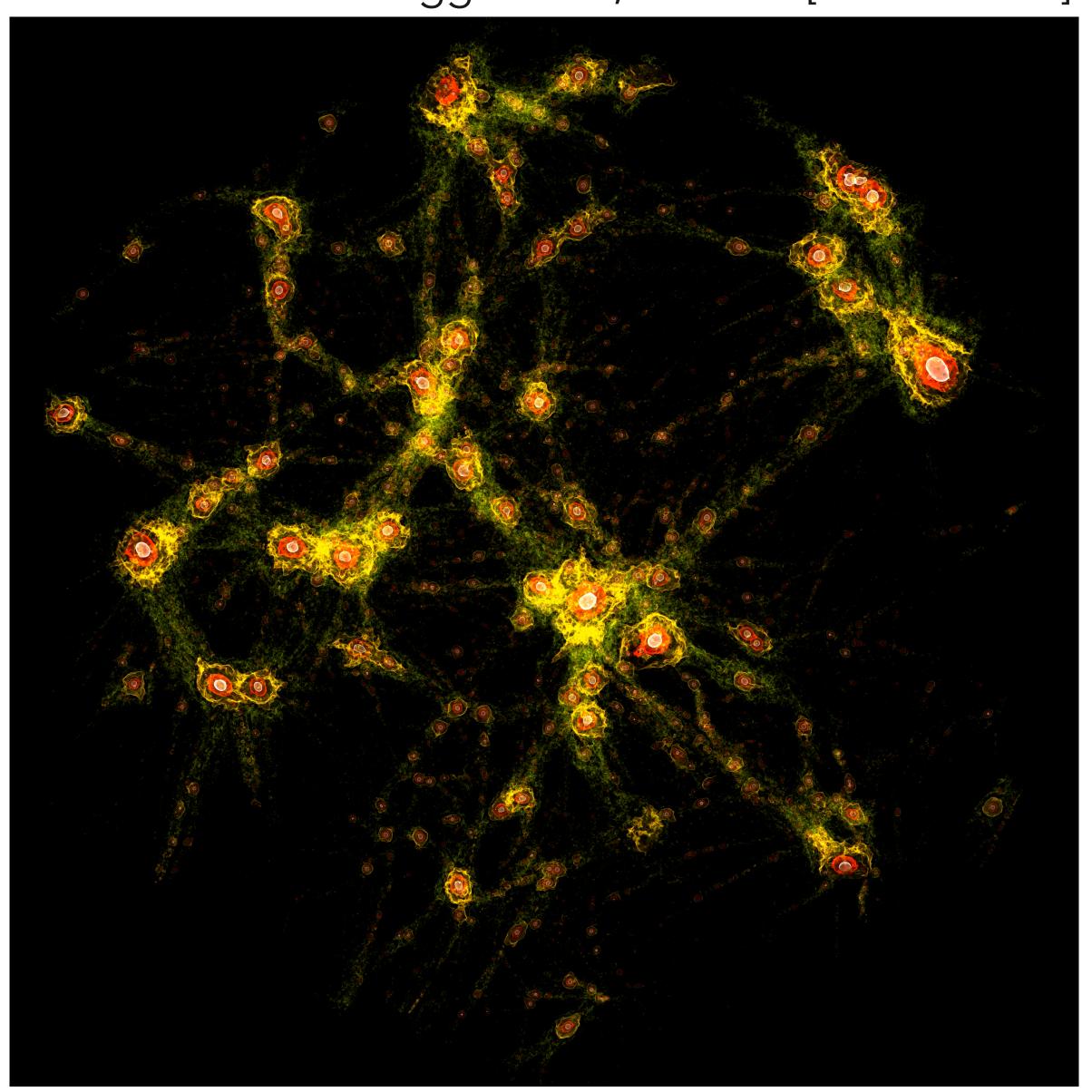
Gravitational collapse

Axion distribution is highly *inhomogeneous*. Large density fluctuations from QCD-horizon scale dynamics that can collapse prior to matter-radiation equality \rightarrow we need to keep simulating!



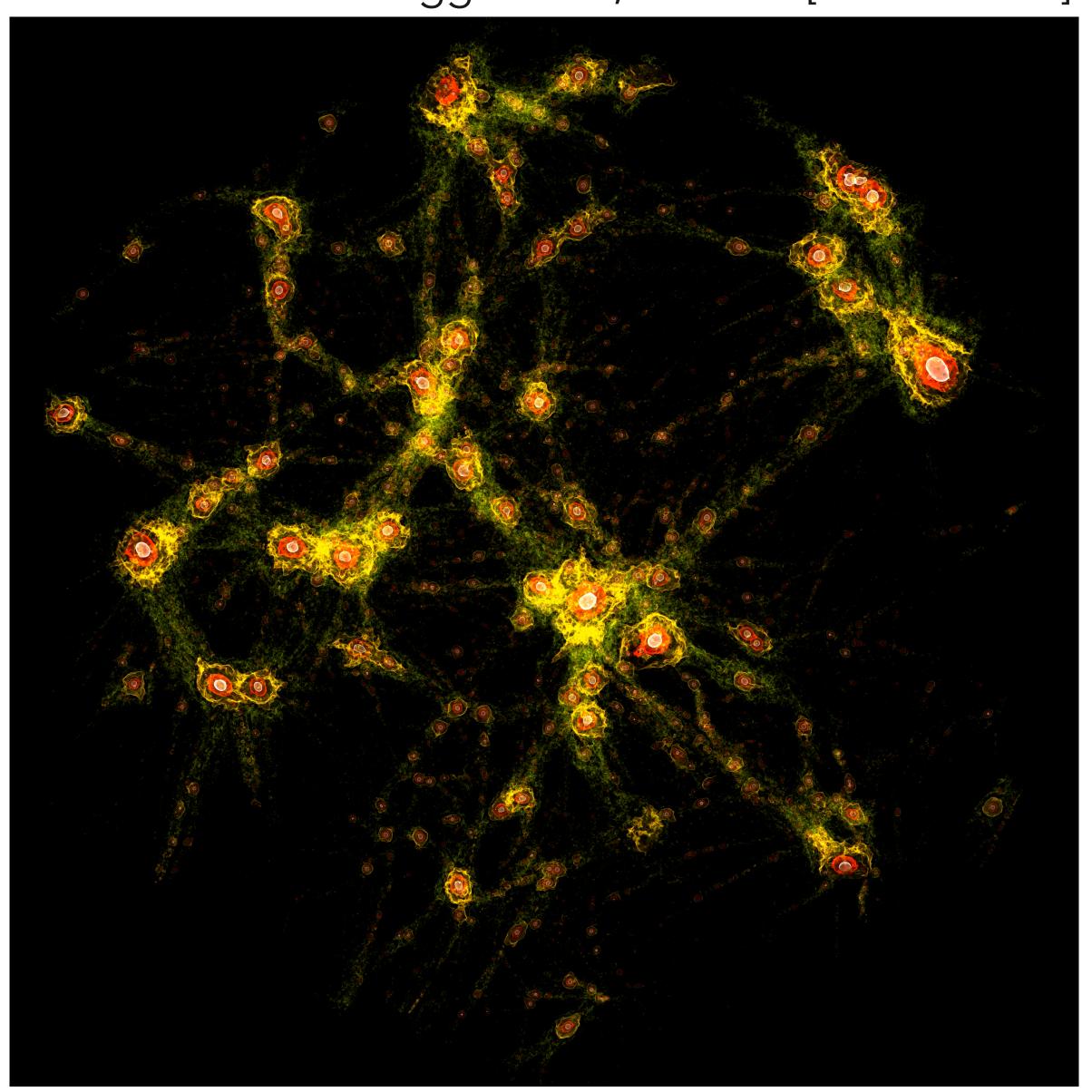
After $t_{\rm QCD}$ axion field forms quasi-stable solitons that lay down small-scale perturbations

These eventually seed AU—mpc gravitationally bound clumps of axions with masses $M \in [10^{-15}, 10^{-9}] M_{\odot}$

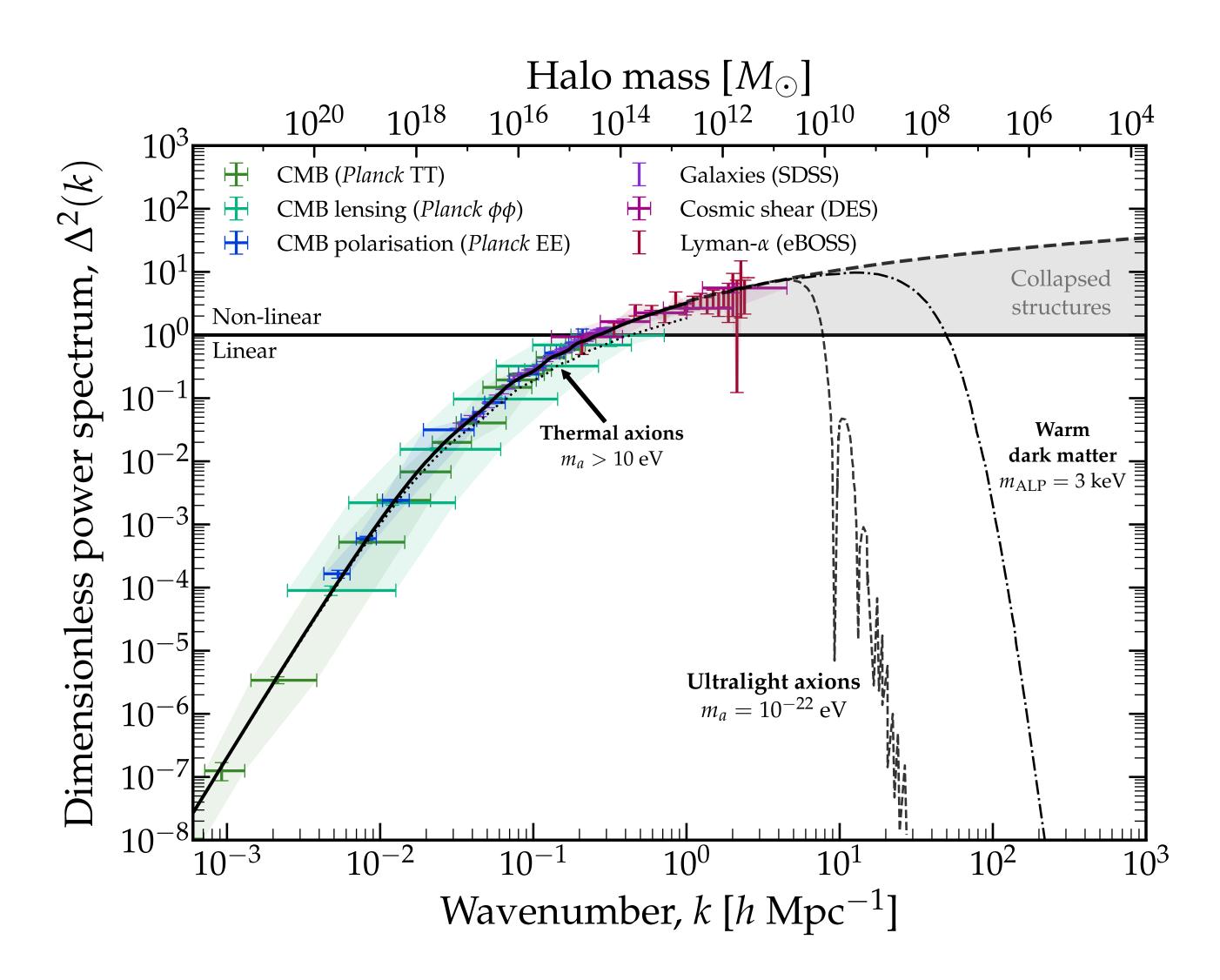


After $t_{\rm QCD}$ axion field forms quasi-stable solitons that lay down small-scale perturbations

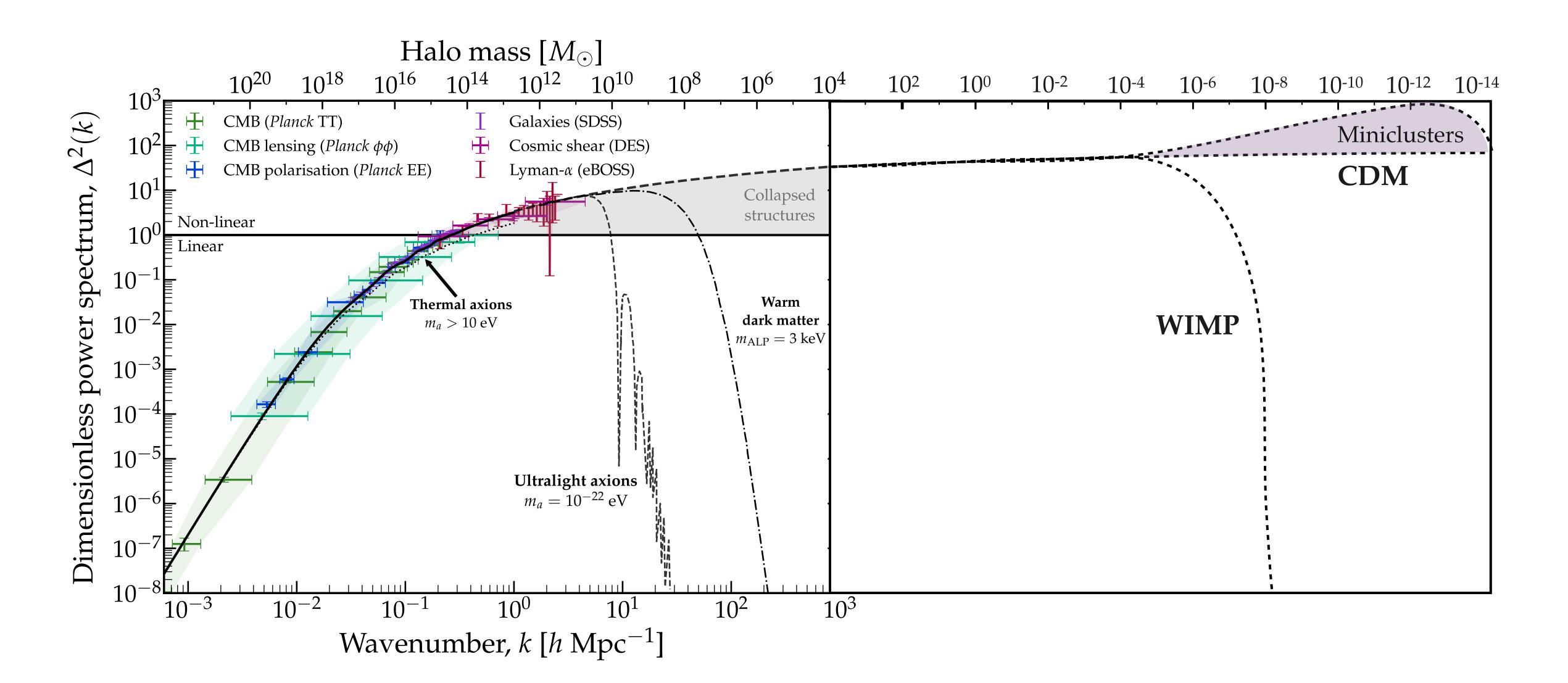
These eventually seed AU—mpc gravitationally bound clumps of axions with masses $M \in [10^{-15}, 10^{-9}] M_{\odot}$



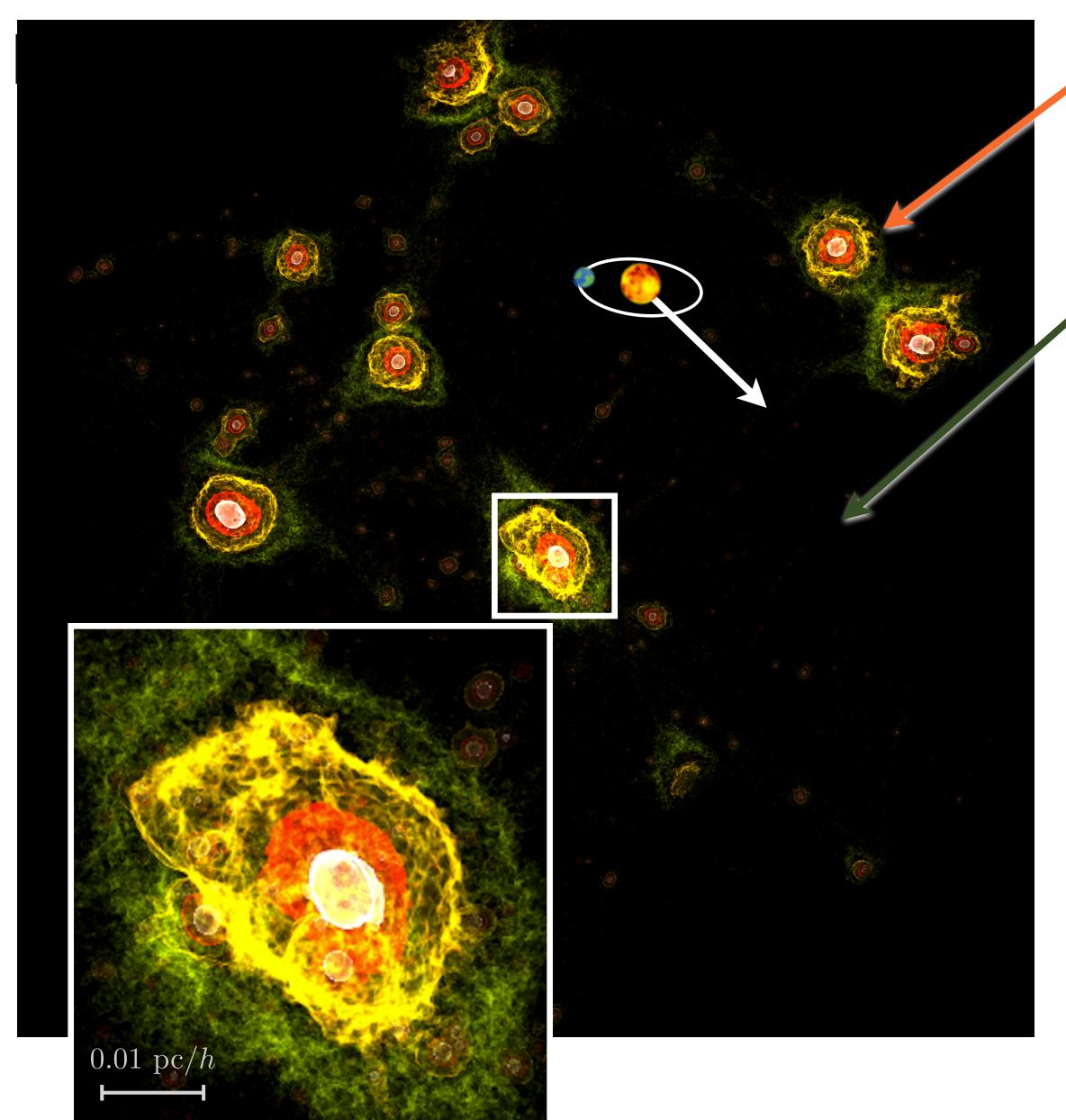
Axion miniclusters



Axion miniclusters



Eggemeier, CAJO+ [2212.00560]



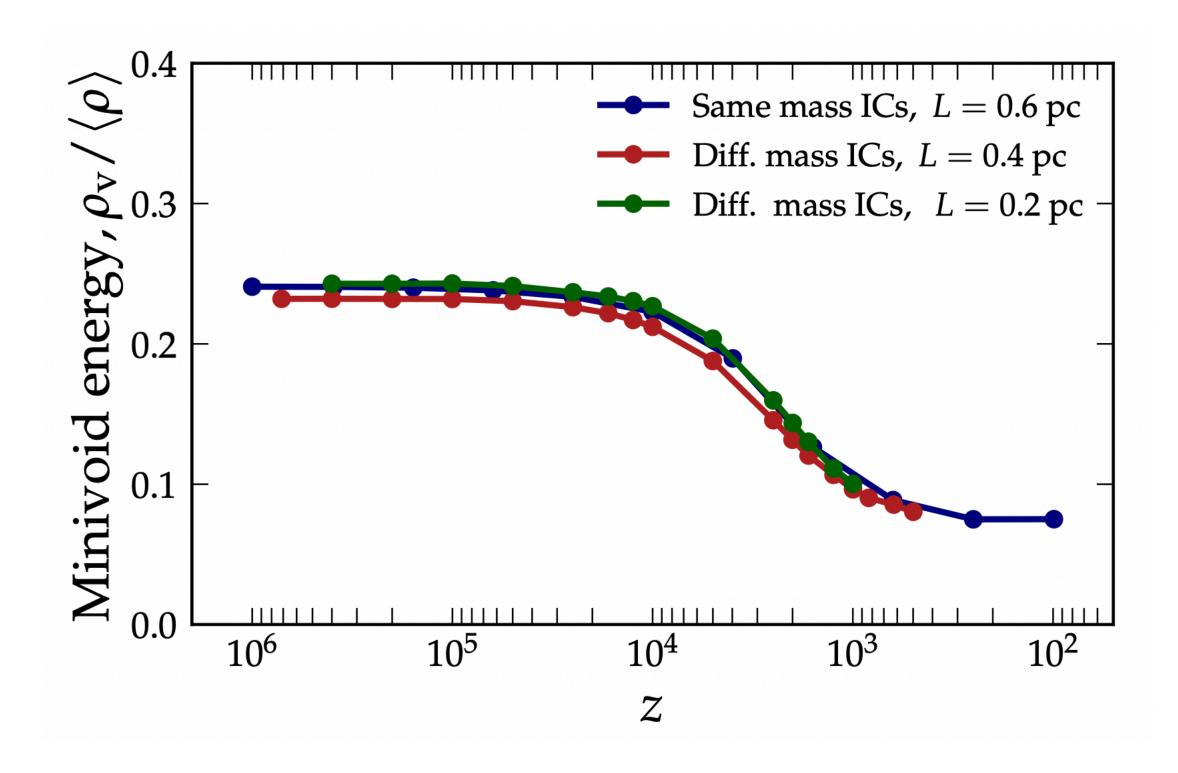
Miniclusters

Minivoids

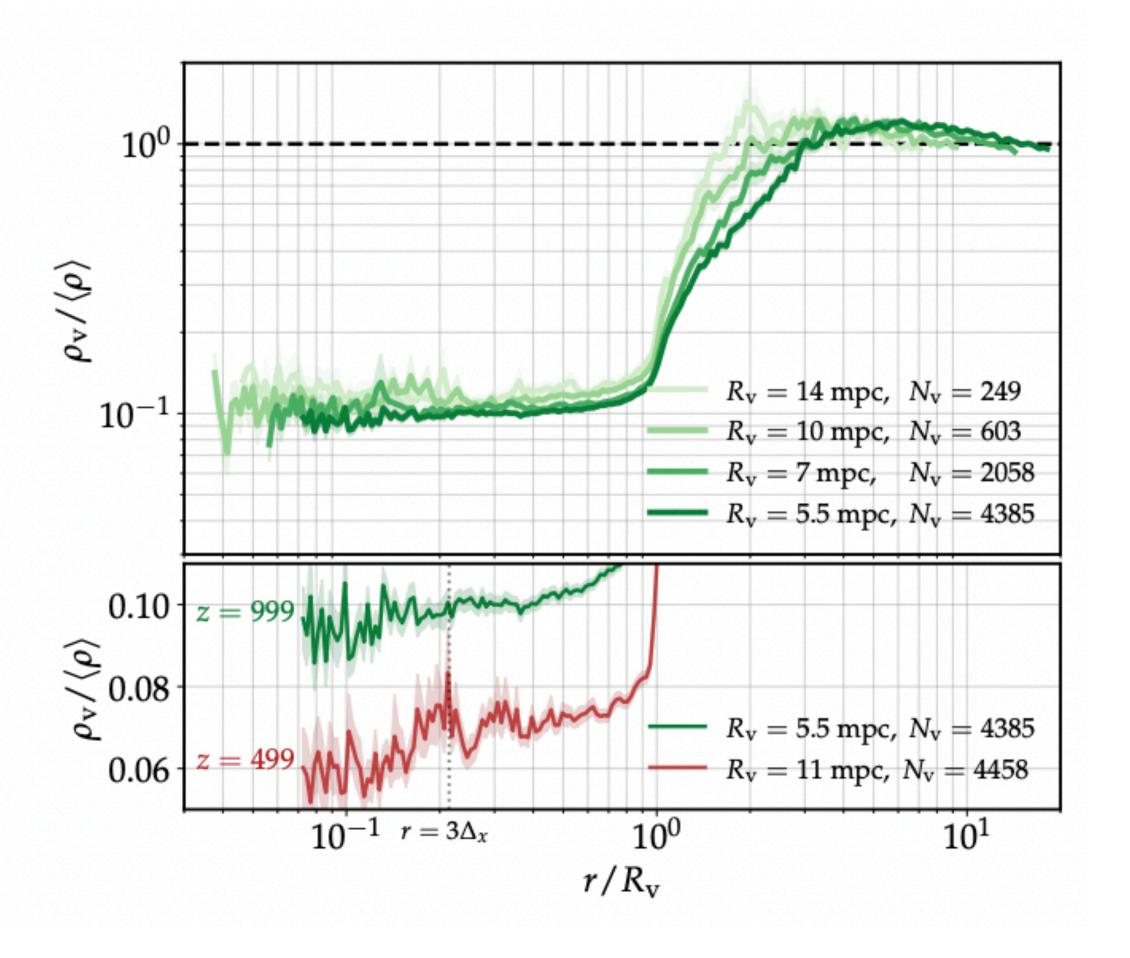
Miniclusters contain >80% of the axions but make up <1% of the volume

Earth travels through galaxy at about 0.2 mpc per year, so experiments are much more likely to sample the minivoids than the miniclusters

Minivoids are mostly stable by final simulation time (z~100)

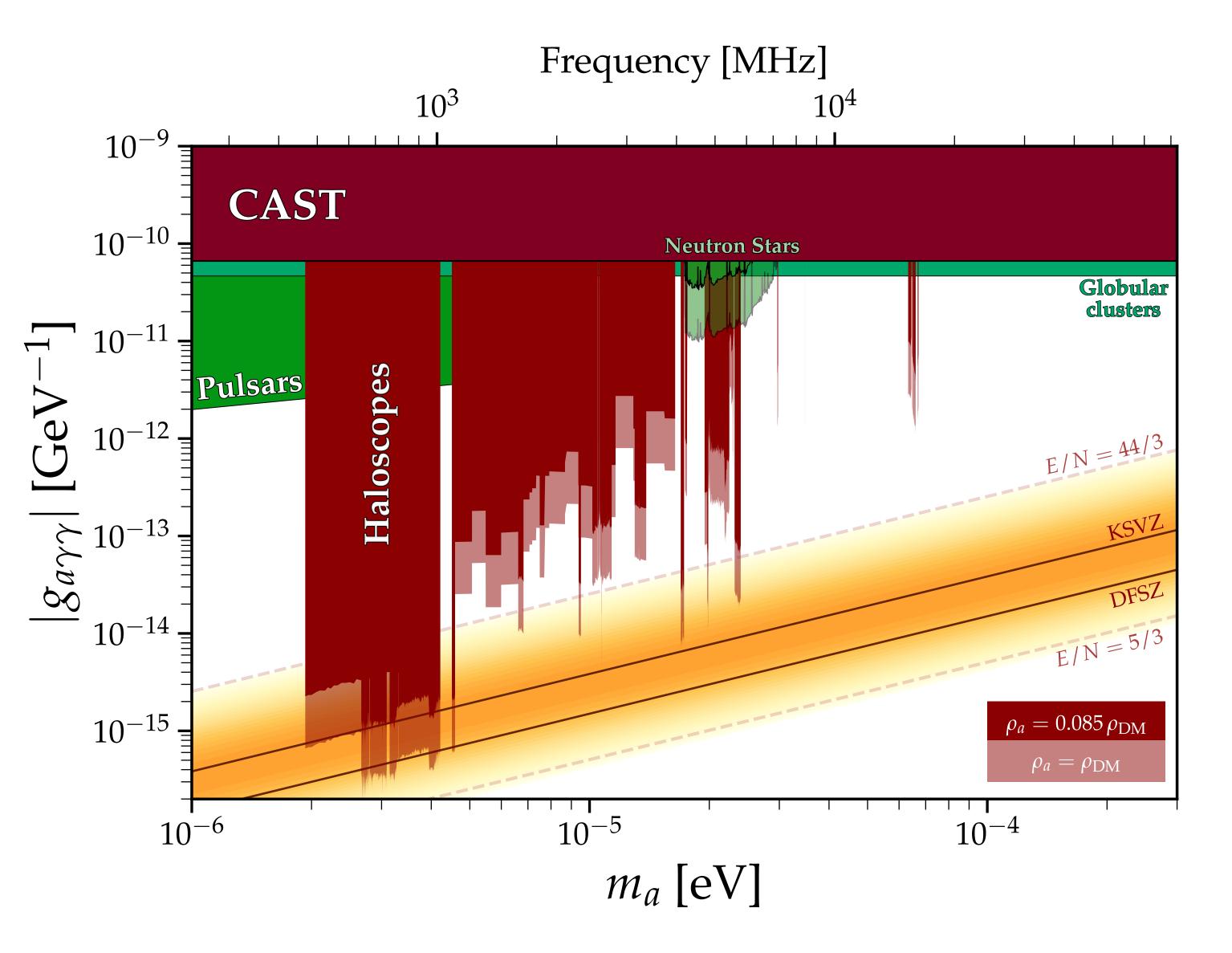


Typical "worst case scenario" density would be inside the minivoids ~10% of large-scale average density



Eggemeier, CAJO+ [2212.00560]

Why is the dark matter density a problem?



Haloscope sensitivity scales slowly.

$$\sqrt{\rho_{\rm DM}}g_{a\gamma} \propto \frac{1}{\sqrt{T}}$$

Usually assume $\rho_{\rm DM}=0.45~{\rm GeV/cm^3}$ inspired by from inferences using Milky Way stars on >100 pc scales

If true (local) value was only ~10% of large-scale average then this is equivalent to a haloscope thinking they've excluded DFSZ when they've only excluded KSVZ

Is this the end of the story?

Not the end of the story...

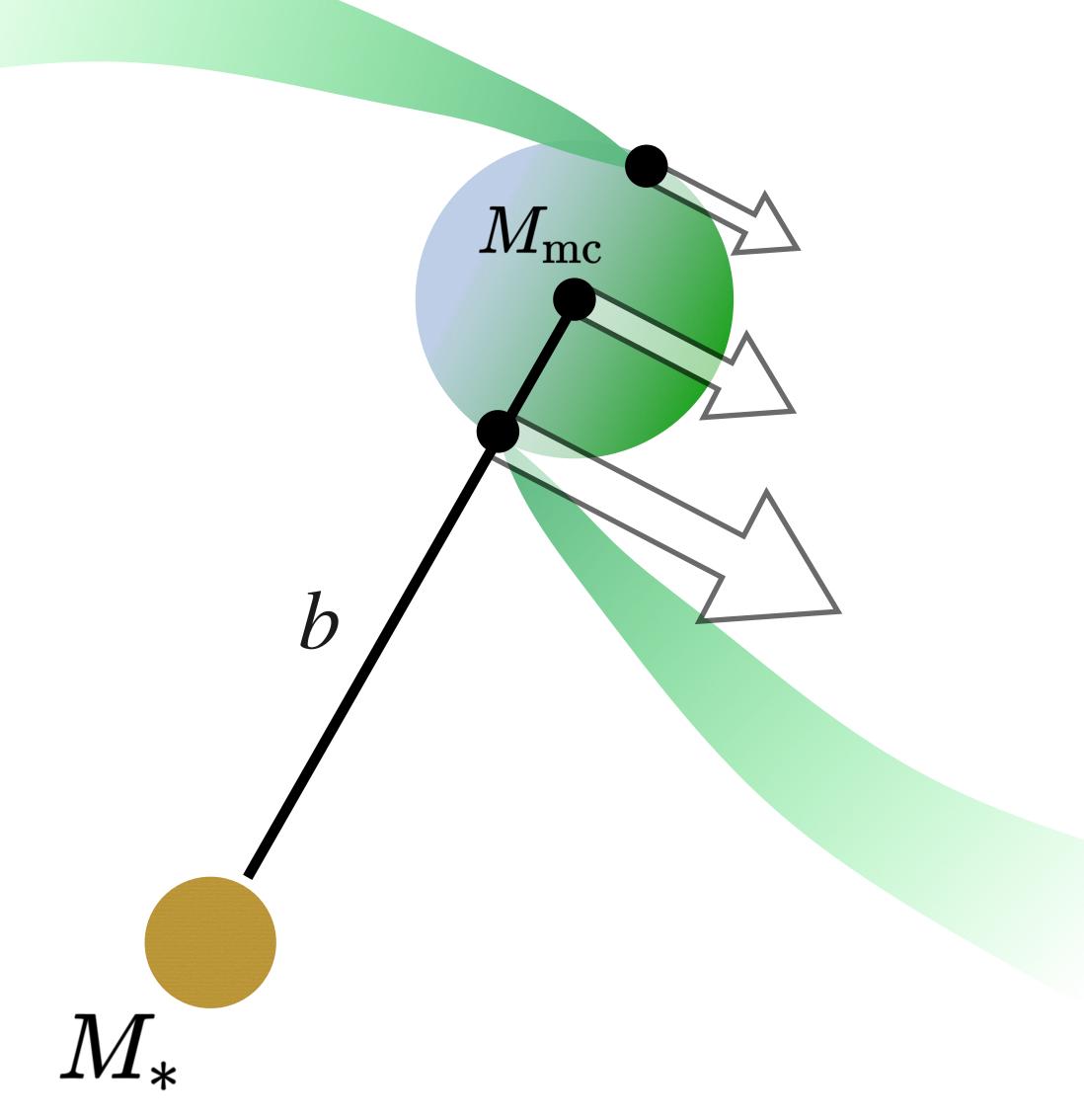
Miniclusters are susceptible to tidal disruption by stars

$$\Delta E \simeq \left(rac{2GM_*}{bv_{
m rel}}
ight)^2 rac{M_{
m mc}R_{
m mc}^2}{3}$$

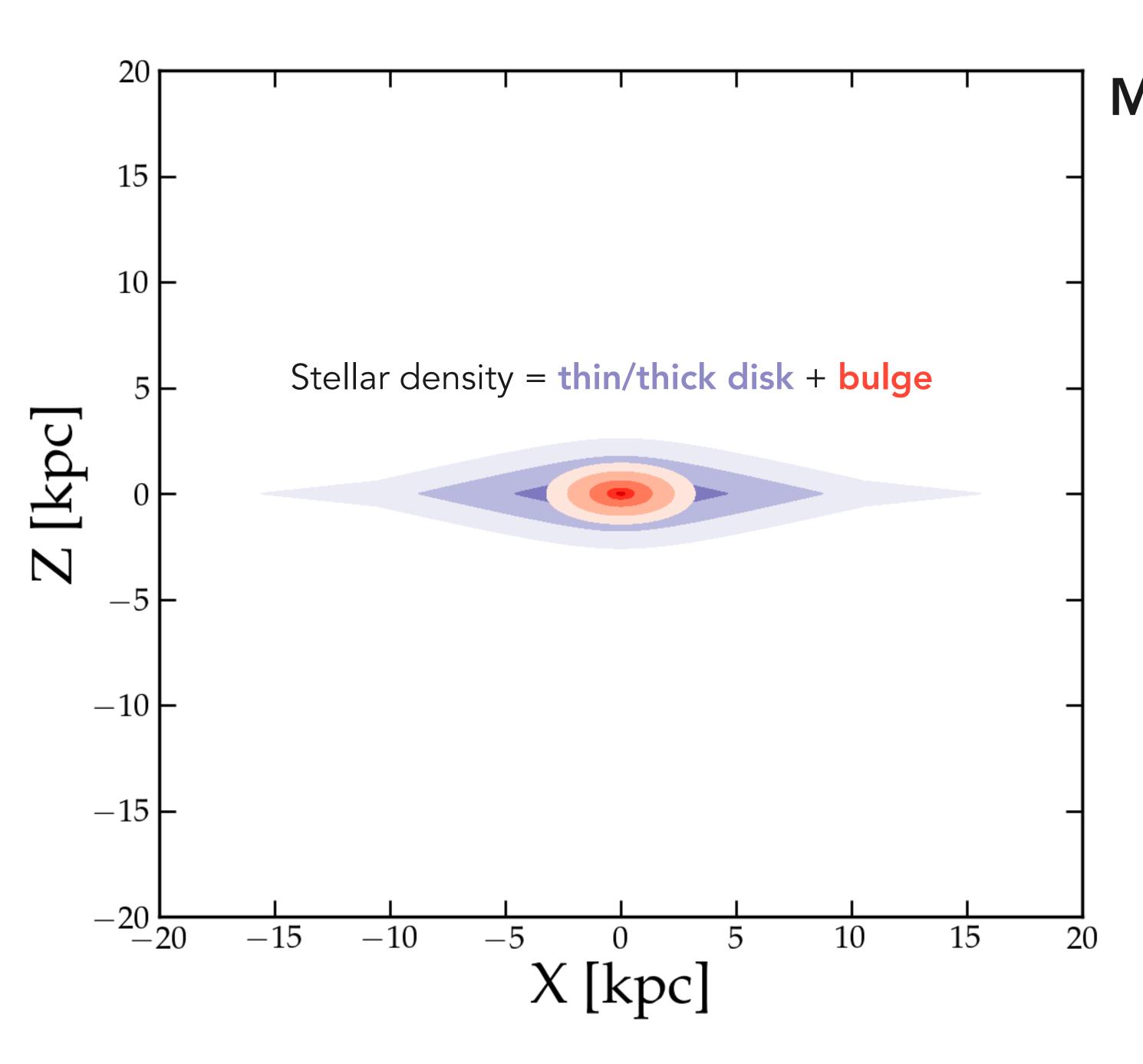


Energy injected into minicluster

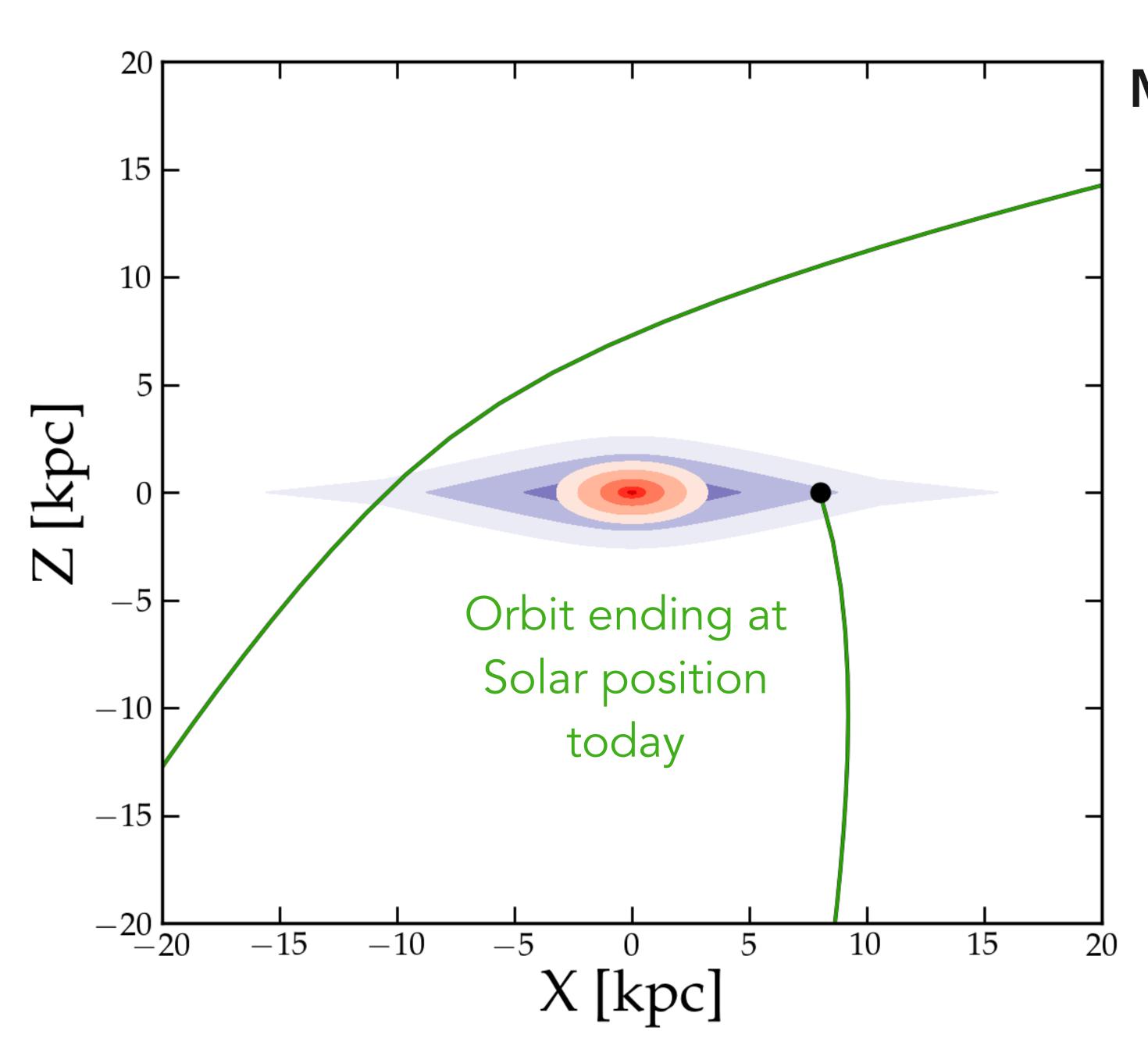
Axions with E>Binding energy will evaporate away \rightarrow form **tidal stream**



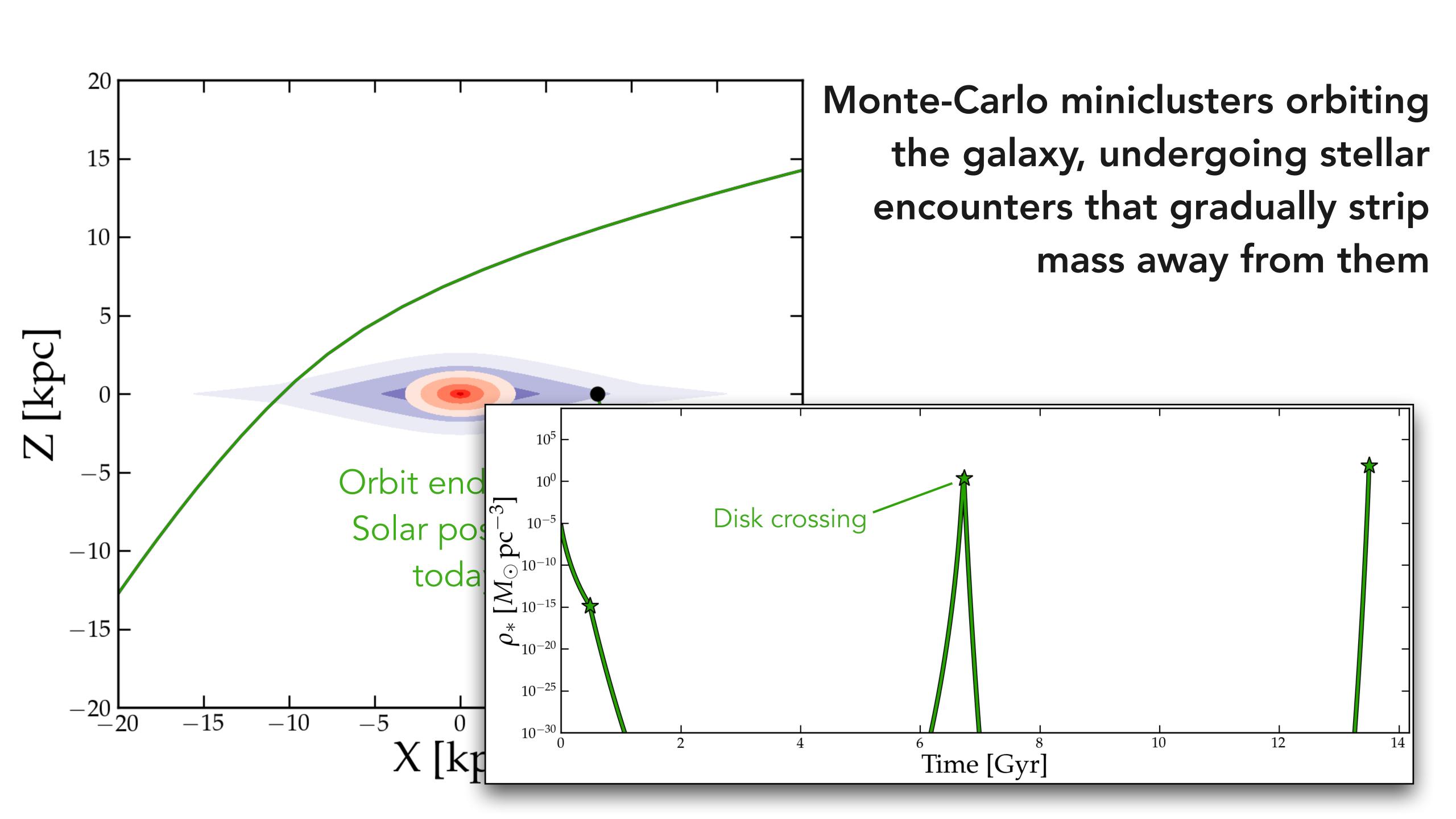
See e.g., Tinyakov+ [1512.02884], Kavanagh+ [2011.05377]

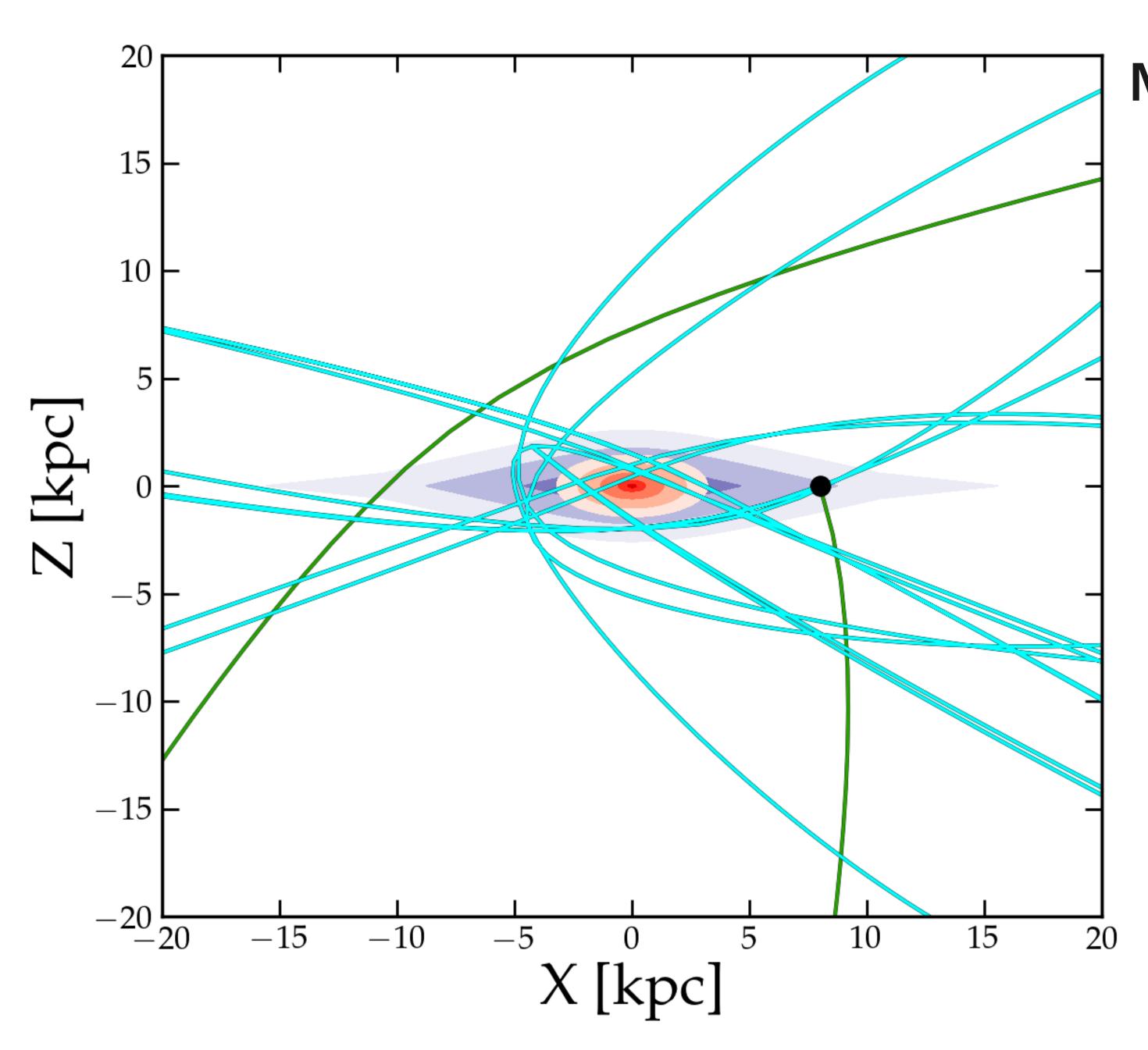


Monte-Carlo miniclusters orbiting the galaxy, undergoing stellar encounters that gradually strip mass away from them

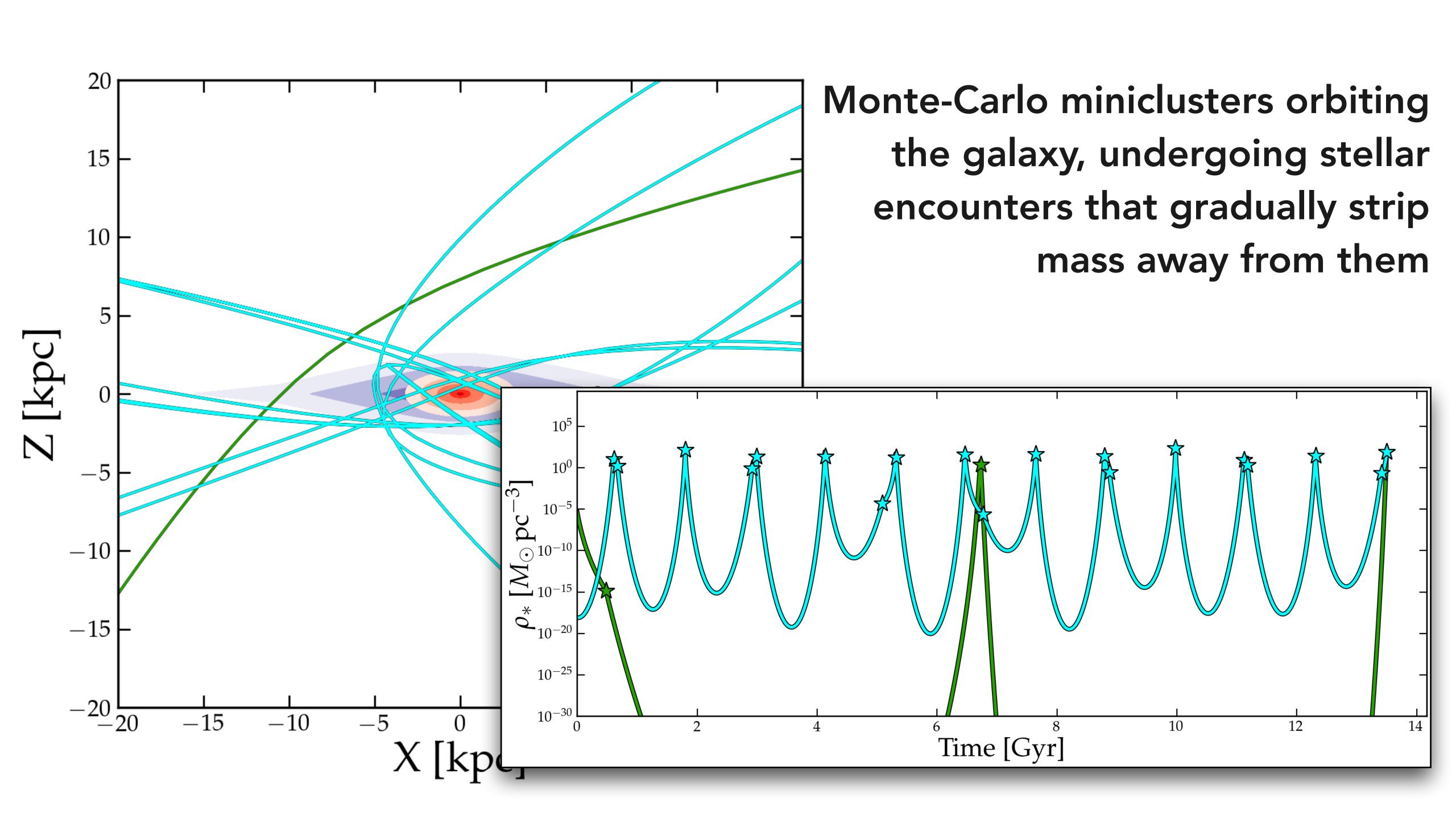


Monte-Carlo miniclusters orbiting the galaxy, undergoing stellar encounters that gradually strip mass away from them

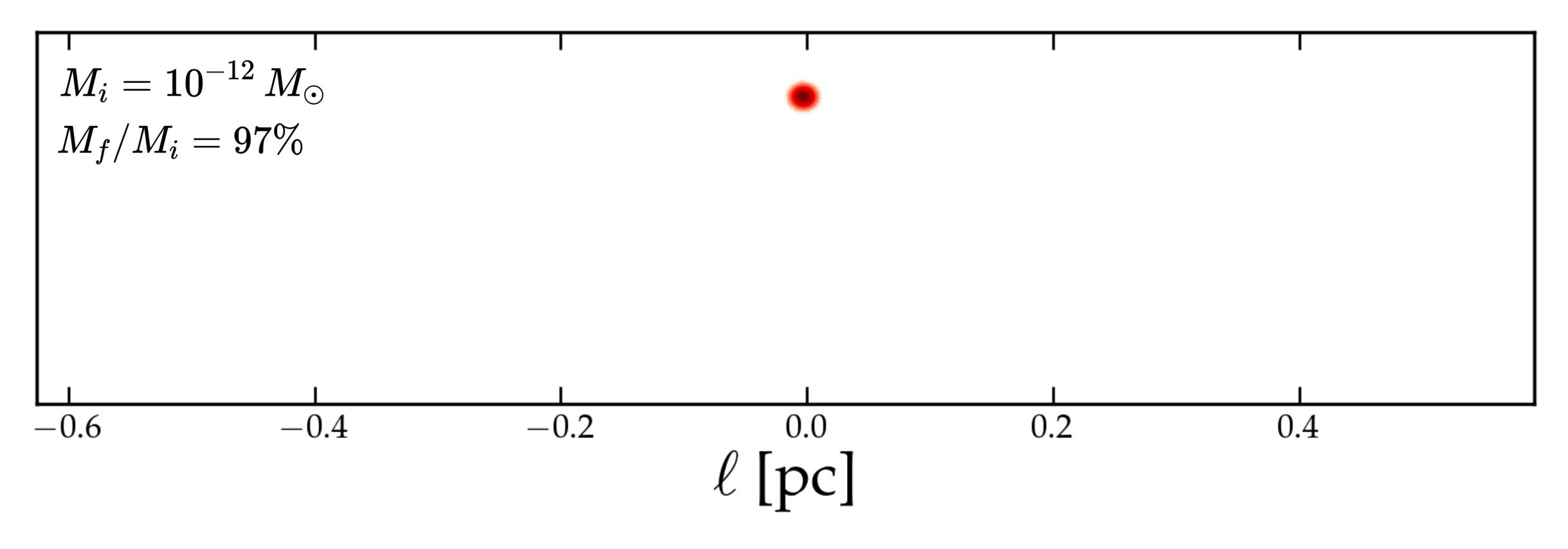




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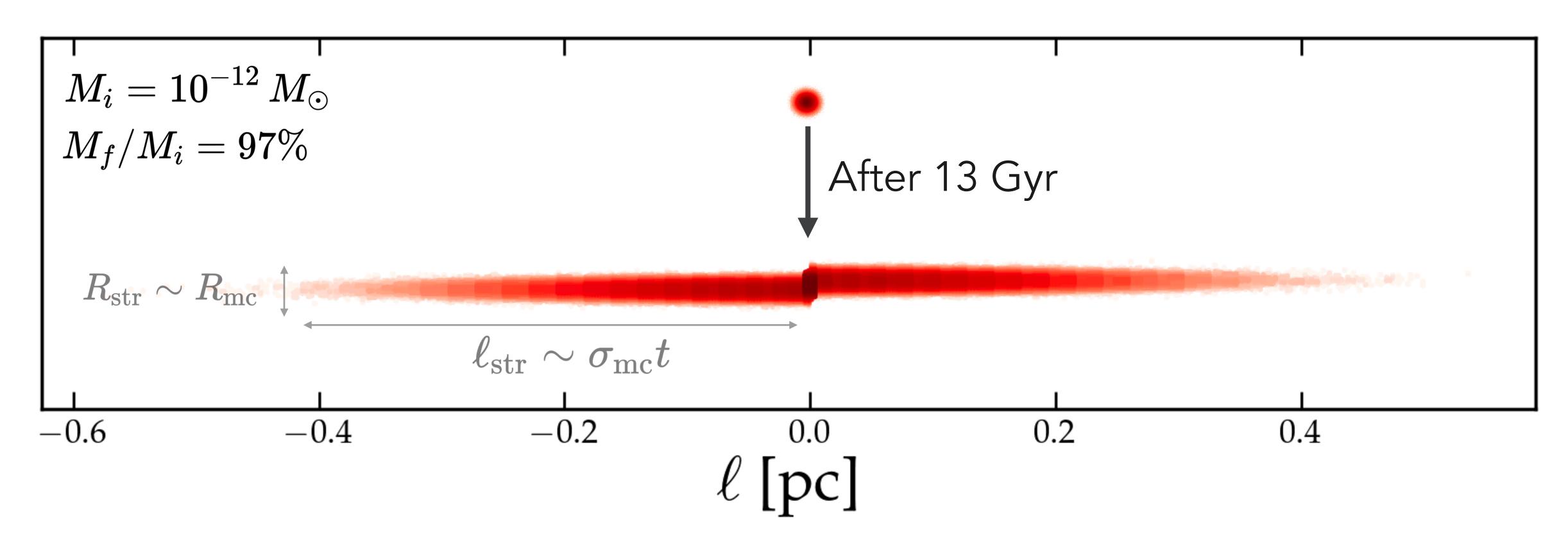


Tidal stream formation



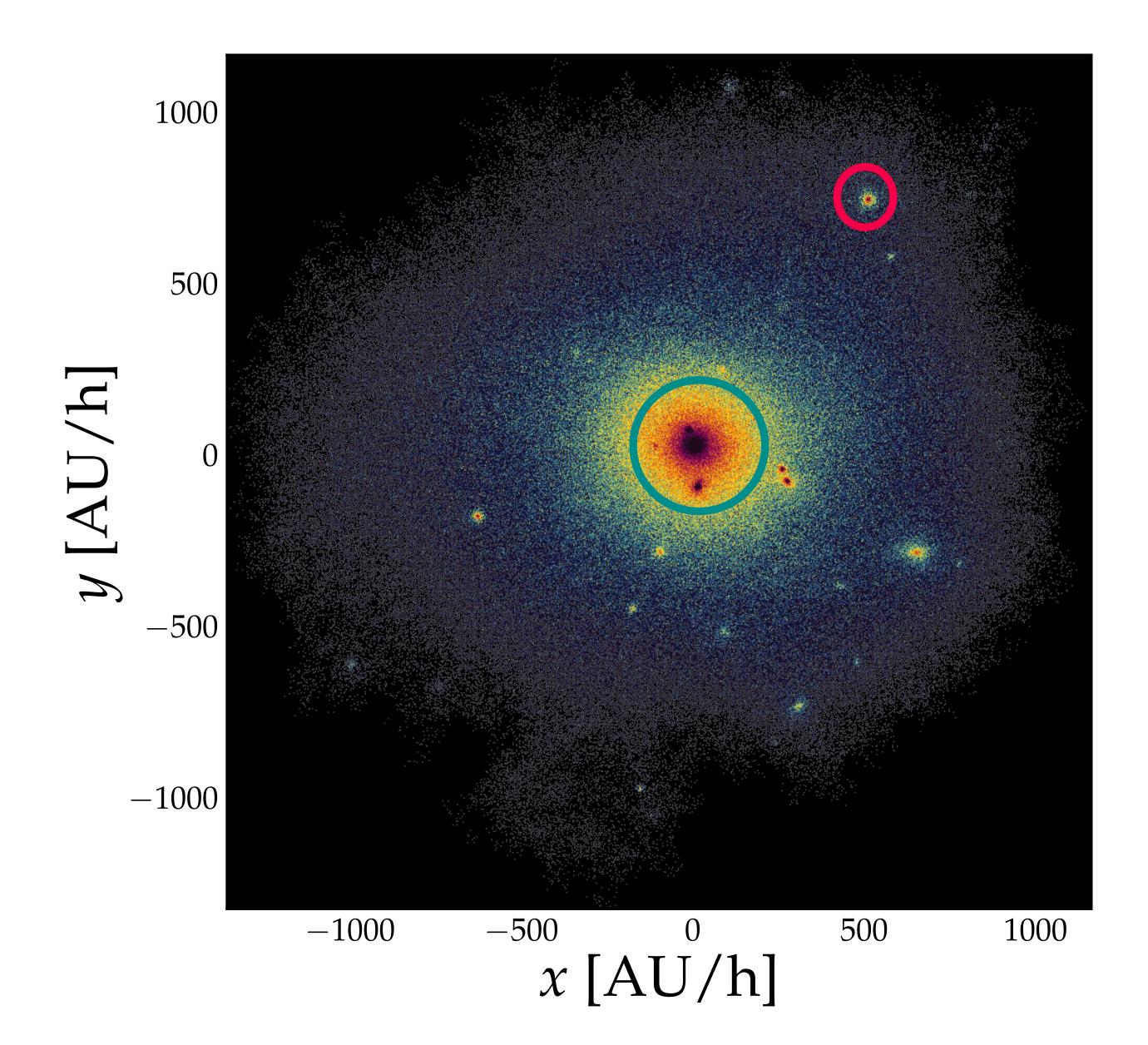
At solar position, most miniclusters are not 100% disrupted. However, a sizeable amount will form ~pc-long tidal streams

Tidal stream formation



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Different populations of miniclusters



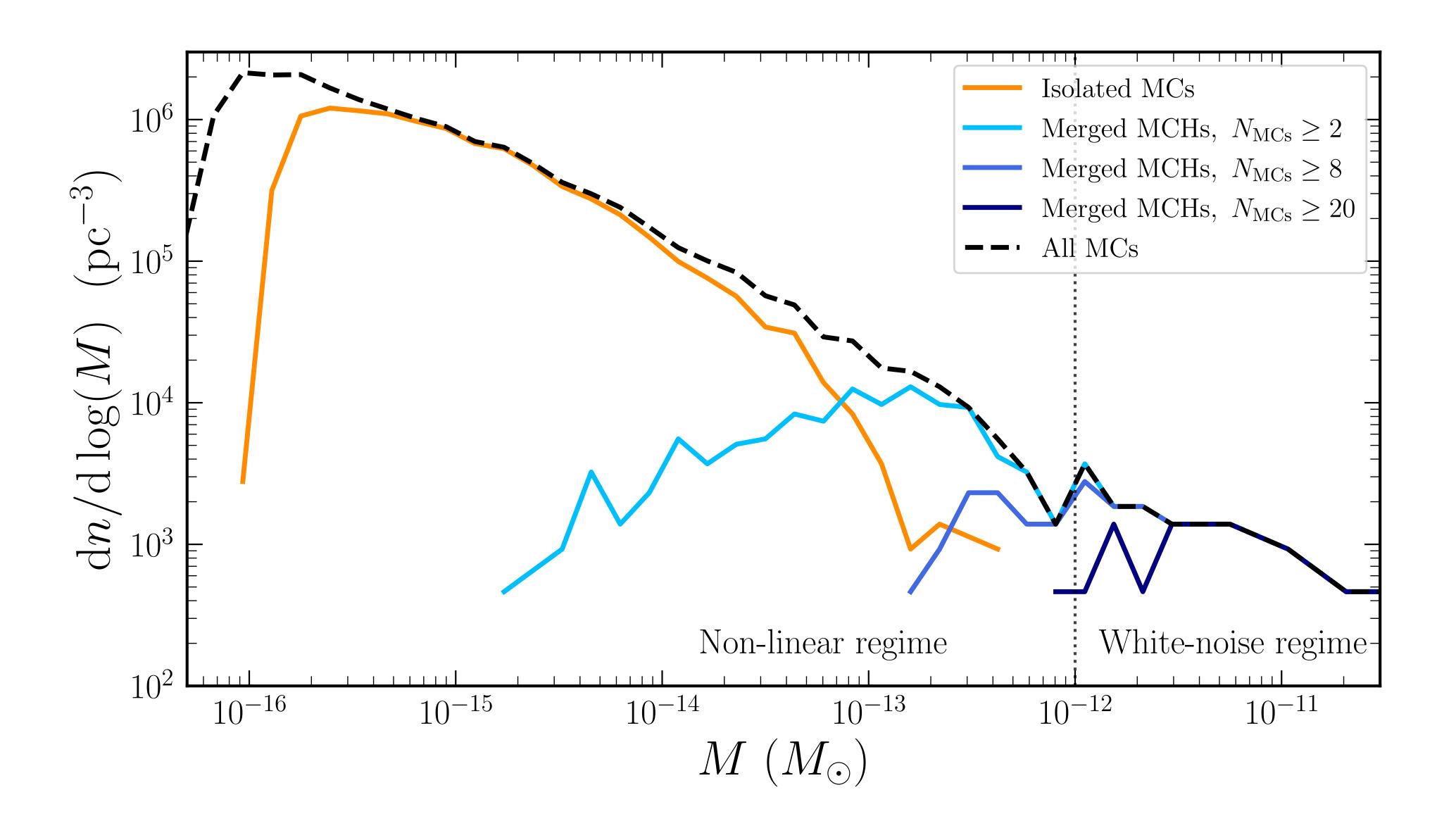
Isolated

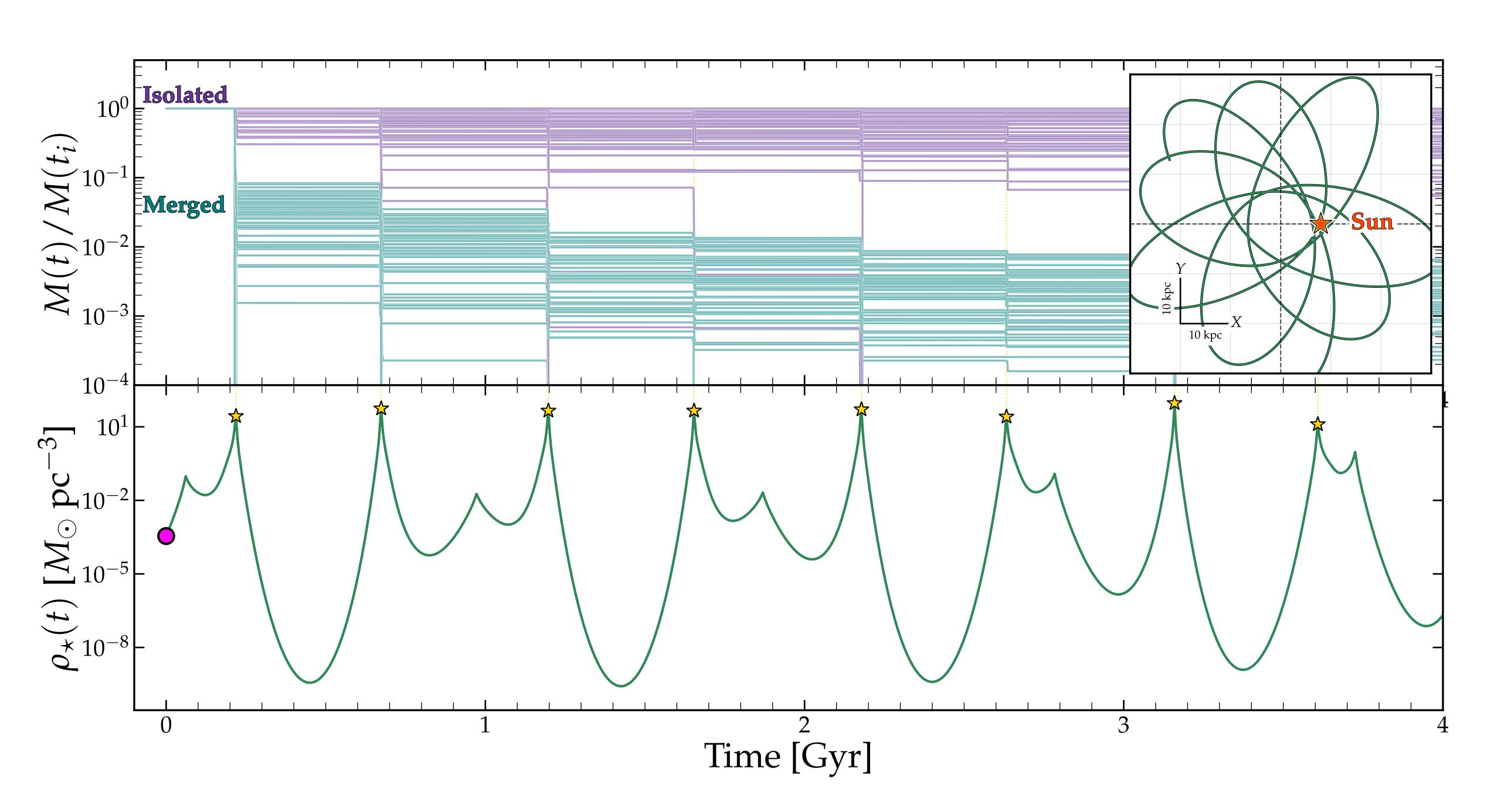
- → About 70% of MCs by number
- → Masses $M \in [10^{-16}, 10^{-12}] M_{\odot}$
- → Form from prompt collapse
- \rightarrow Power law density profiles $\rho \sim r^{-2.71}$
- → ~0% are fully disrupted

Merged

- → About 30% of MCs by number
- → Masses $M \in [10^{-12}, 10^{-7}] M_{\odot}$
- → Form from mergers of MCs
- → NFW density profile
- → 45% are fully disrupted

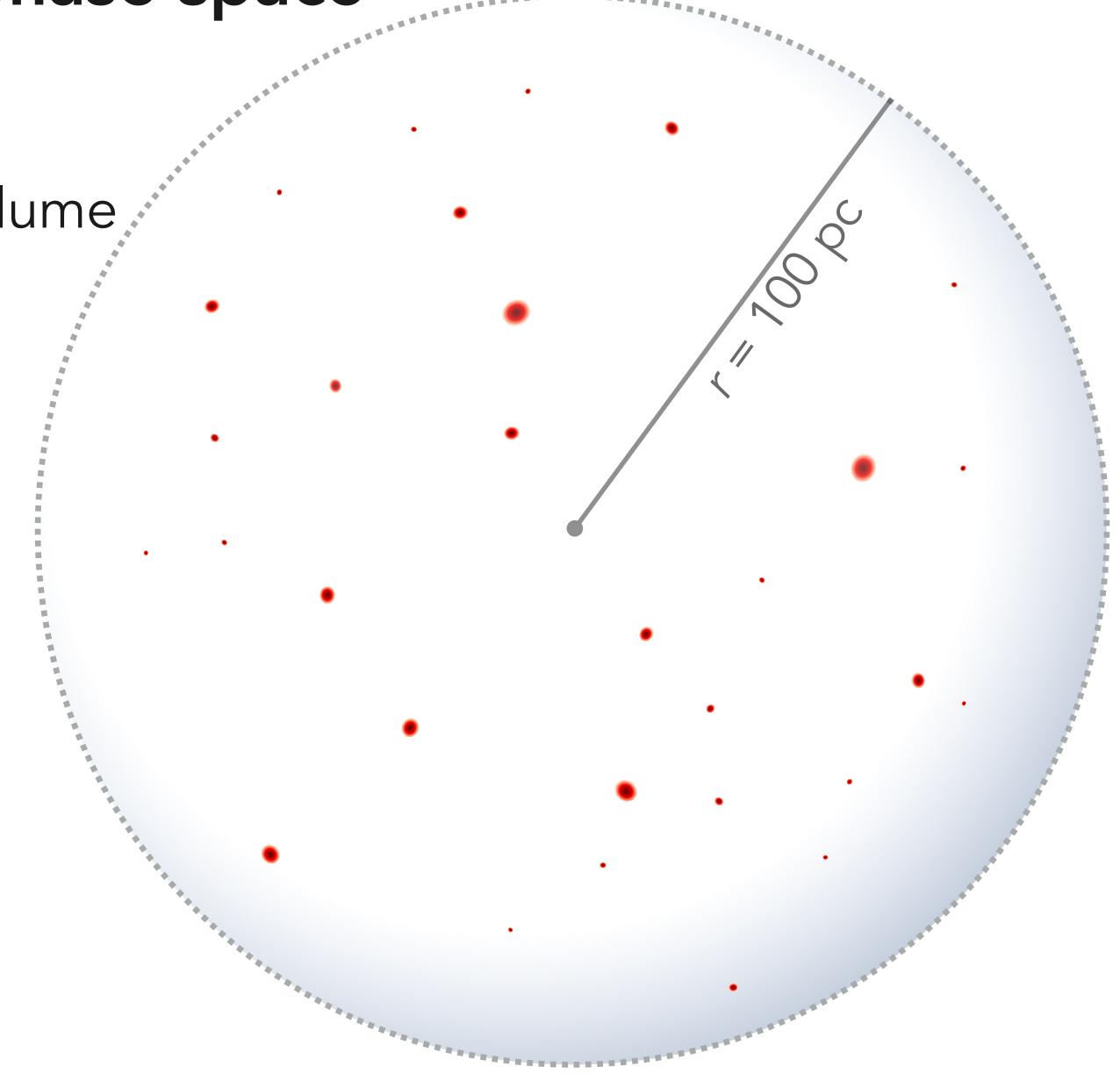
Minicluster mass function





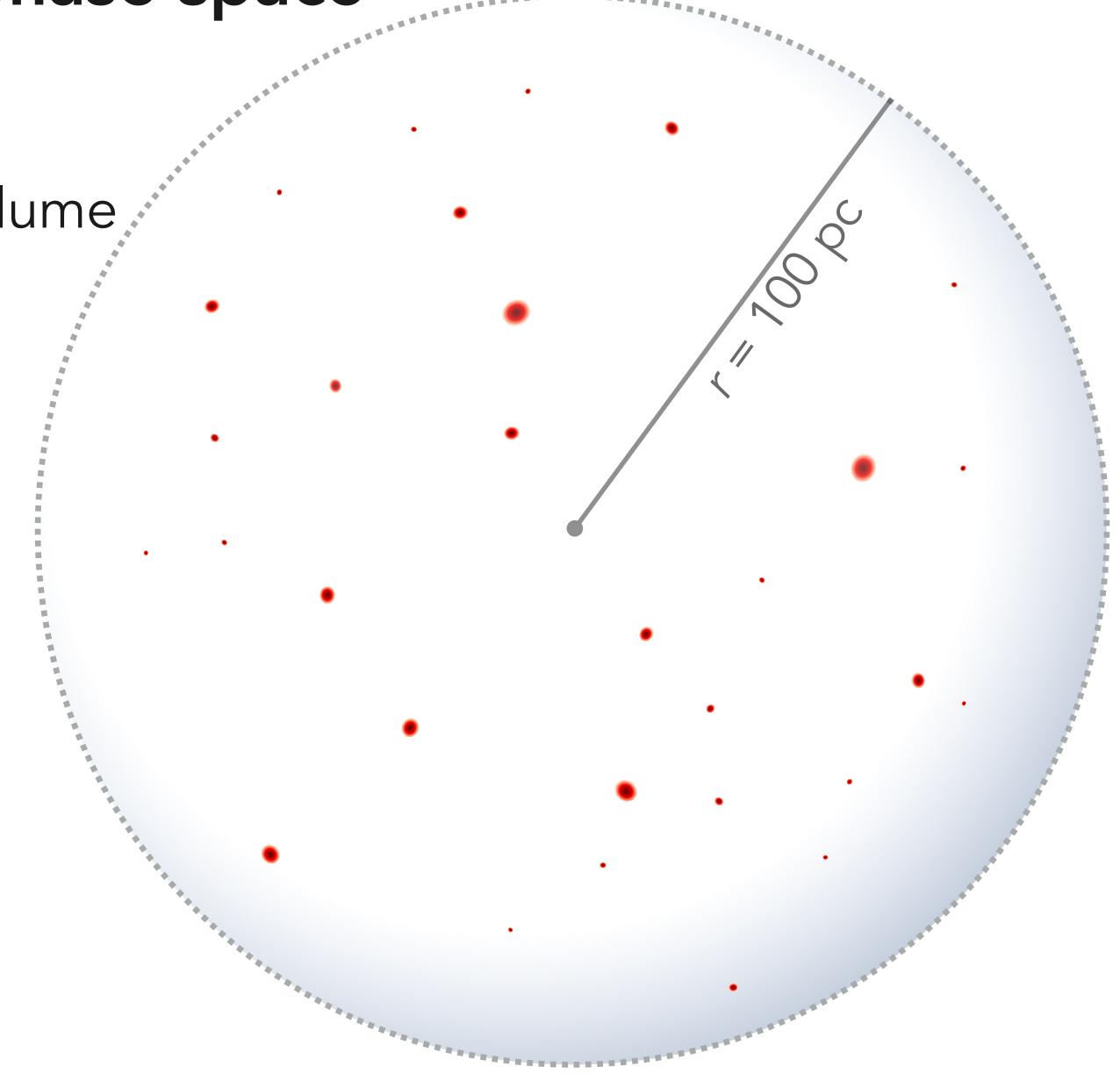
We measure ho_{DM} on scales ~100 pc

 \rightarrow Must be ~ 10^{14} miniclusters in that volume.



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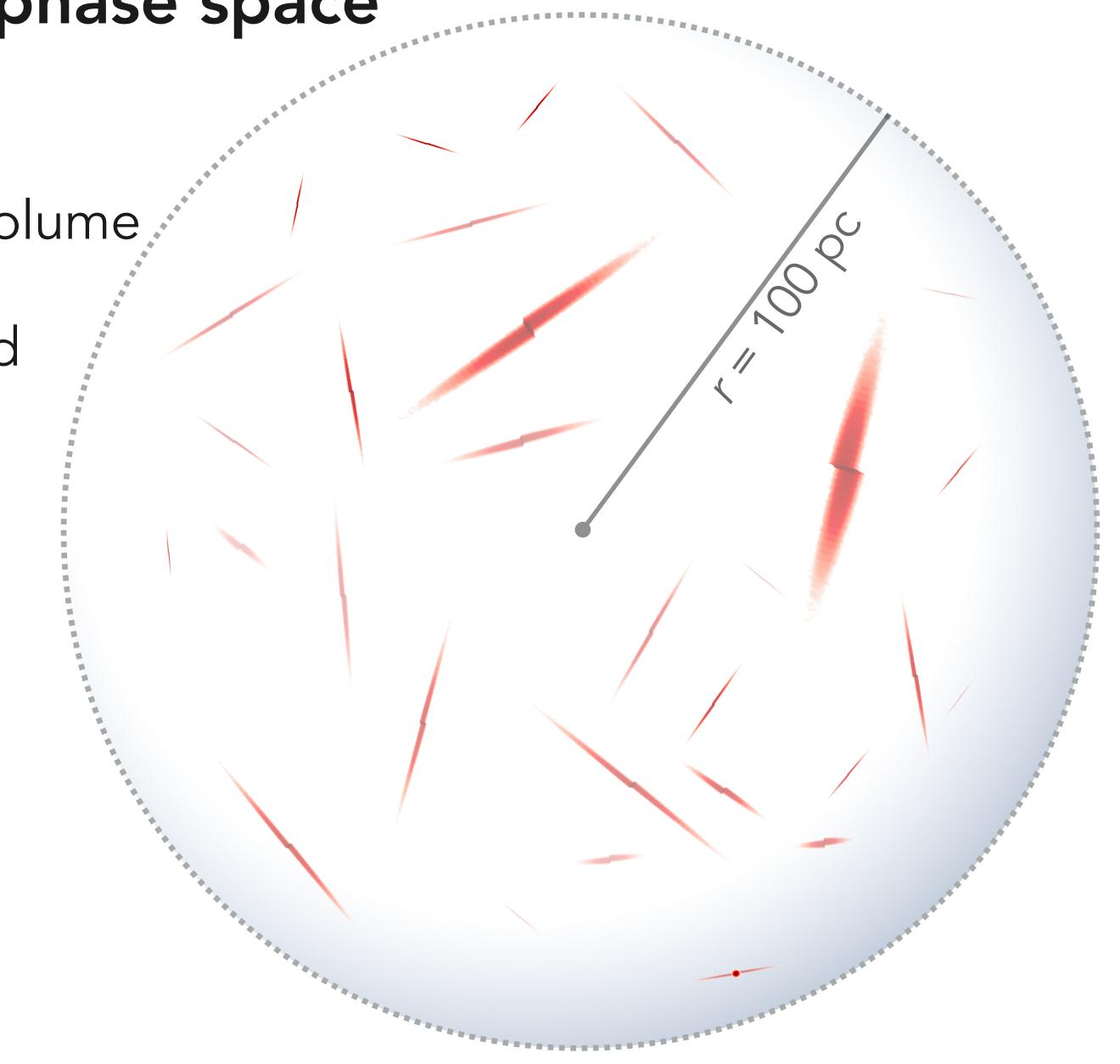
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We measure $ho_{
m DM}$ on scales ~100 pc

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After disruption, MCs turn into extended \sim pc-long streams. Volume filled with axions is enhanced by a factor of $\sim 10^4$



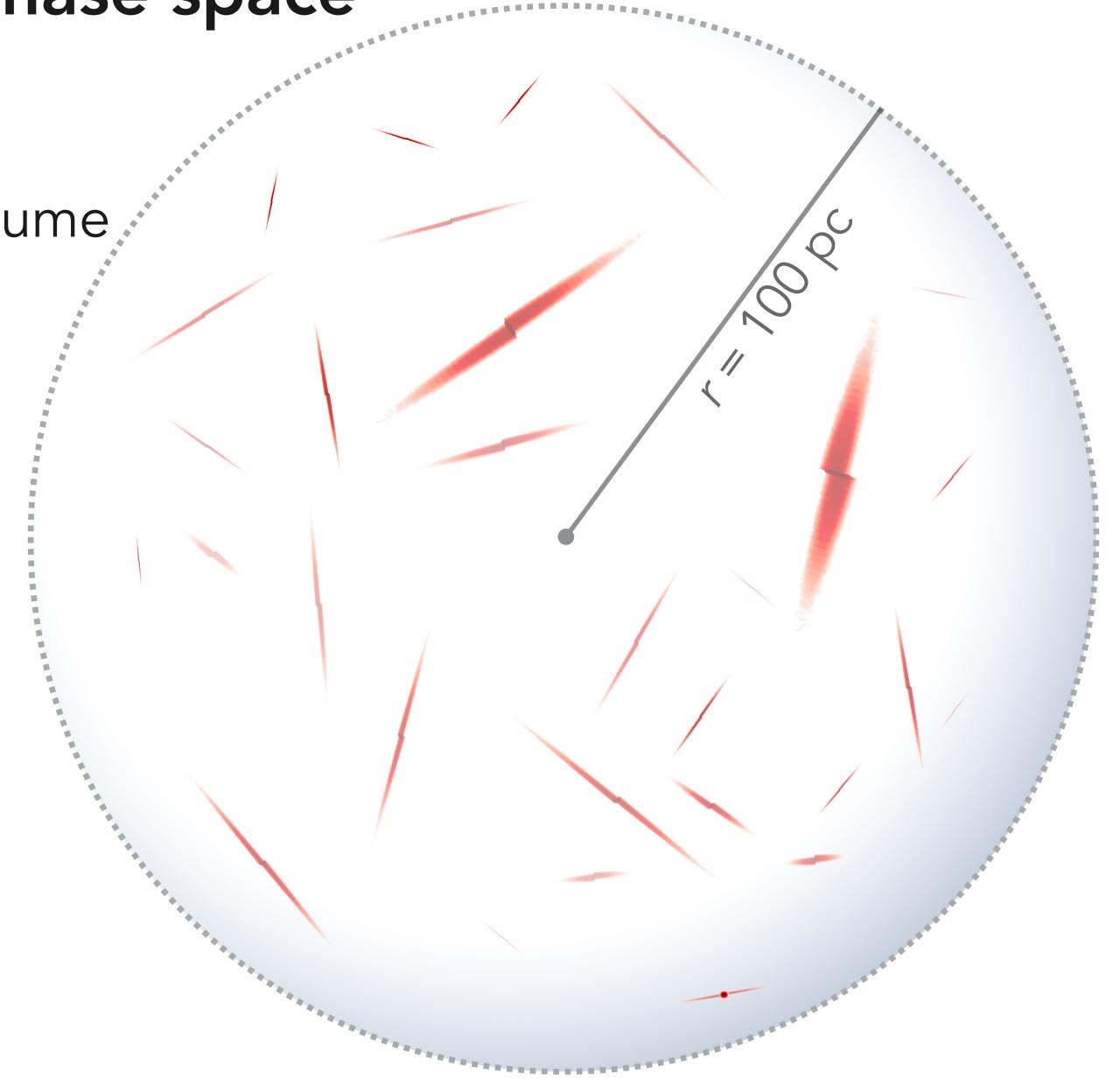
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Q: How many streams overlap at a given position in the box?

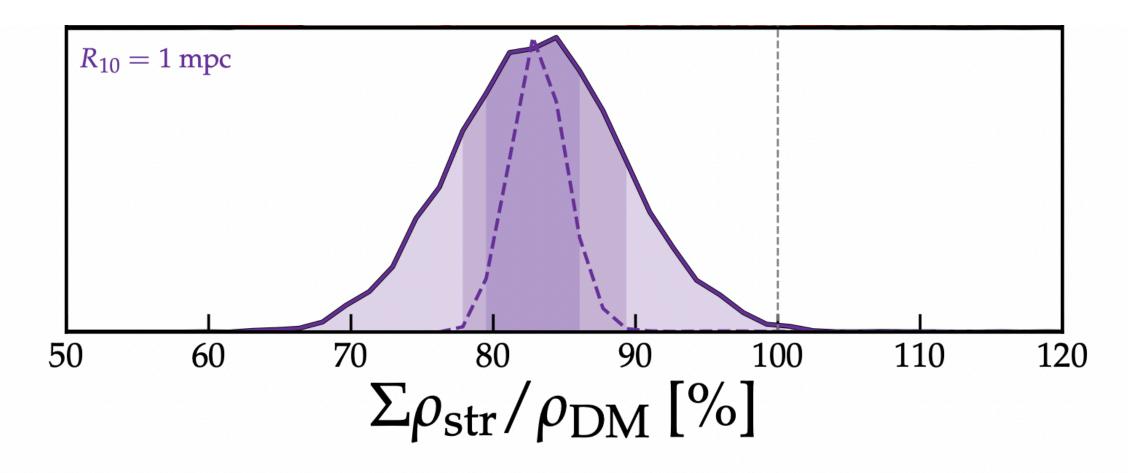
Q: How much is the density enhanced due to the re-filling of phase space

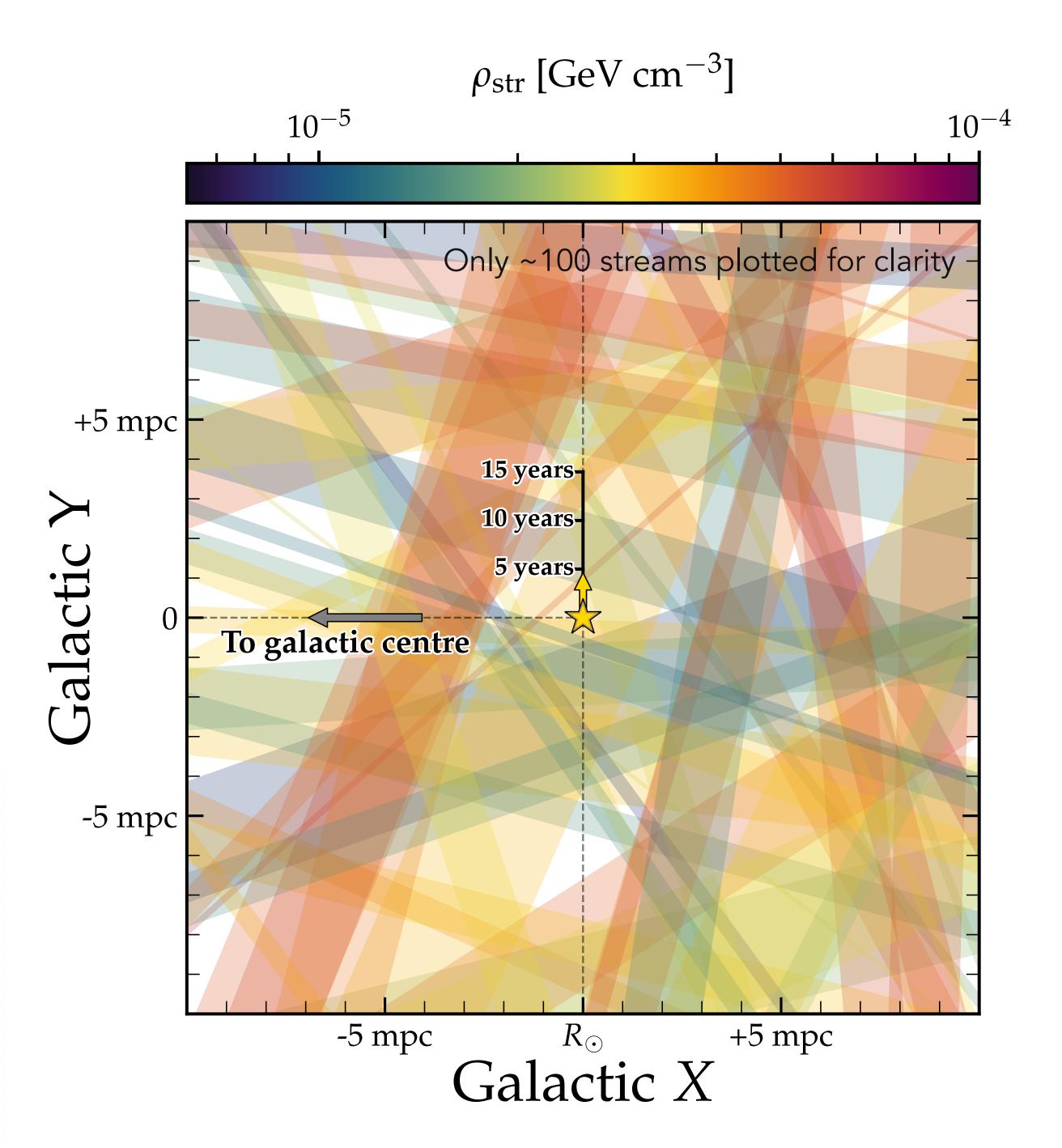


Axion streams at the Solar position

Answer: typically there are O(100-1000) tidal streams overlapping a given position. Vast majority do not contribute substantially to the density

Together they add up to ~70-90% of large-scale measured value of $\rho_{\rm DM}$

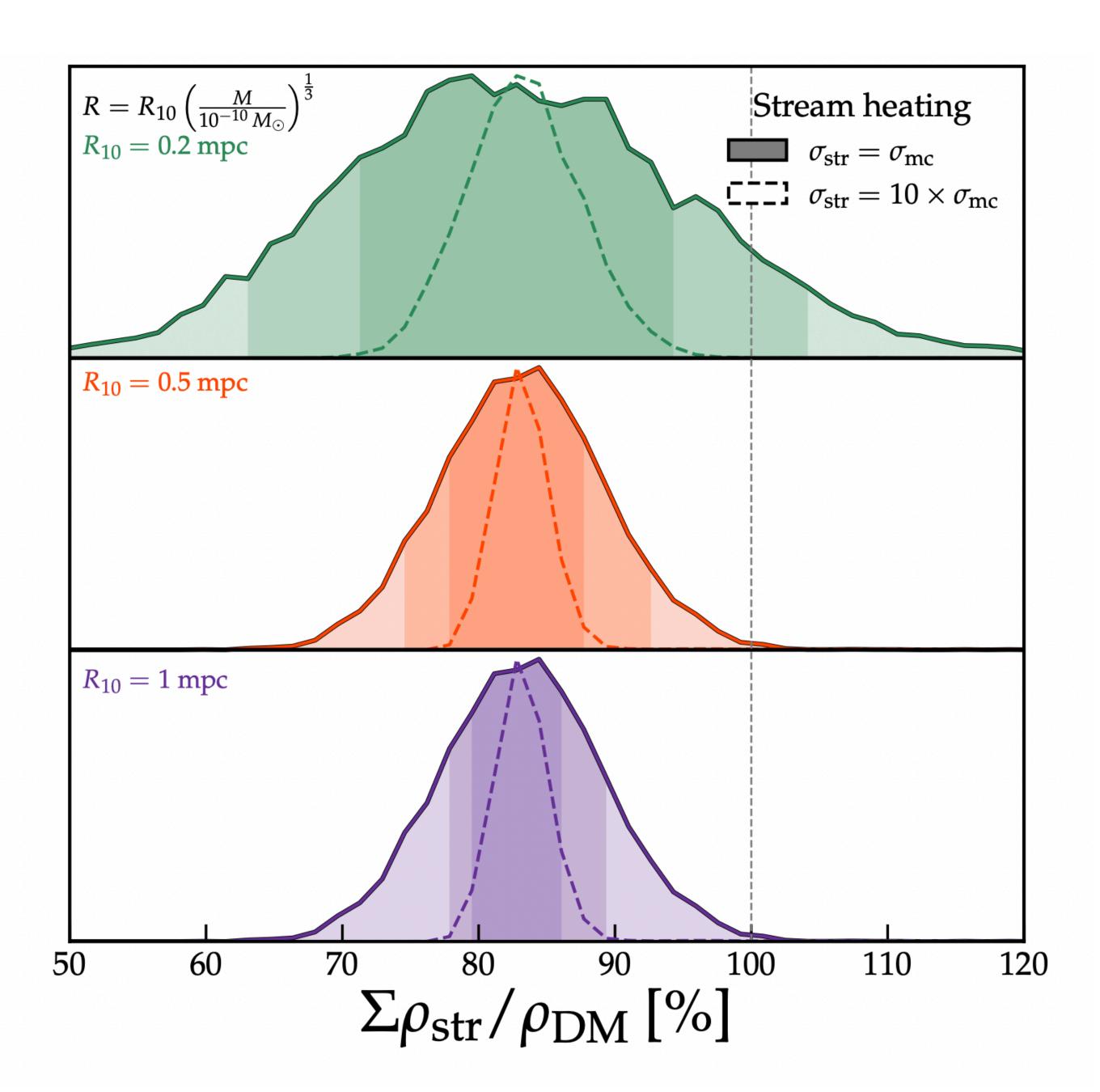




Uncertainties

We find very little dependence on the details of the mass function or the orbit models, which can be supported up with a back-of-the-envelope calculation. The only things that matter are:

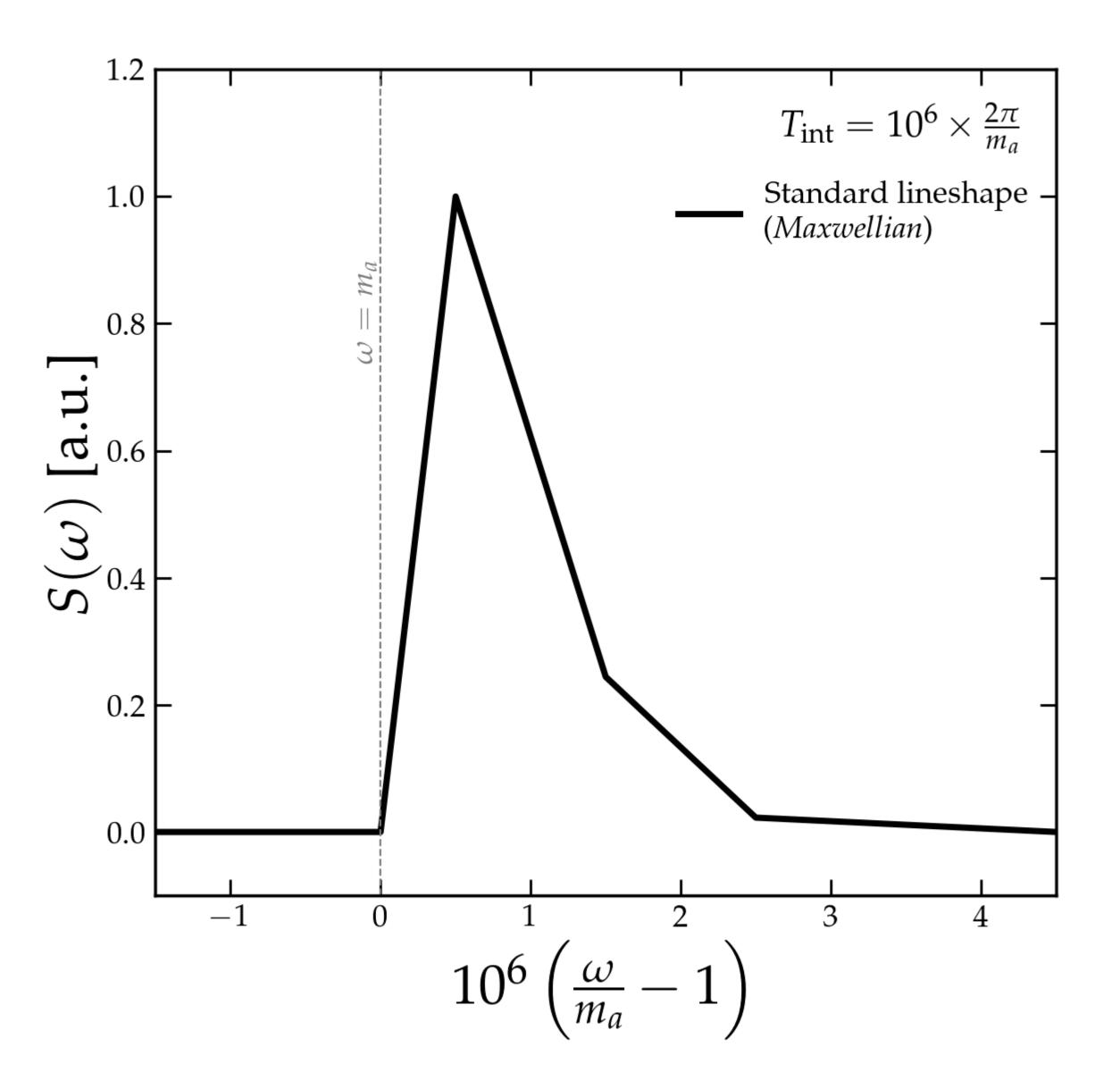
- → That the most massive miniclusters are described by smooth NFW halos. If they are "clusters of miniclusters" they are probably more resiliant.
- → The NFW concentration parameter (or Mass-radius relation), which affects the variance in our answer.



The power spectrum of the oscillating axion signal in a haloscope have a distinct Maxwellian **lineshape**.

Frequency resolution depends on the duration of the timestream samples that are put through a discrete Fourier transform in order to calculate that power spectrum

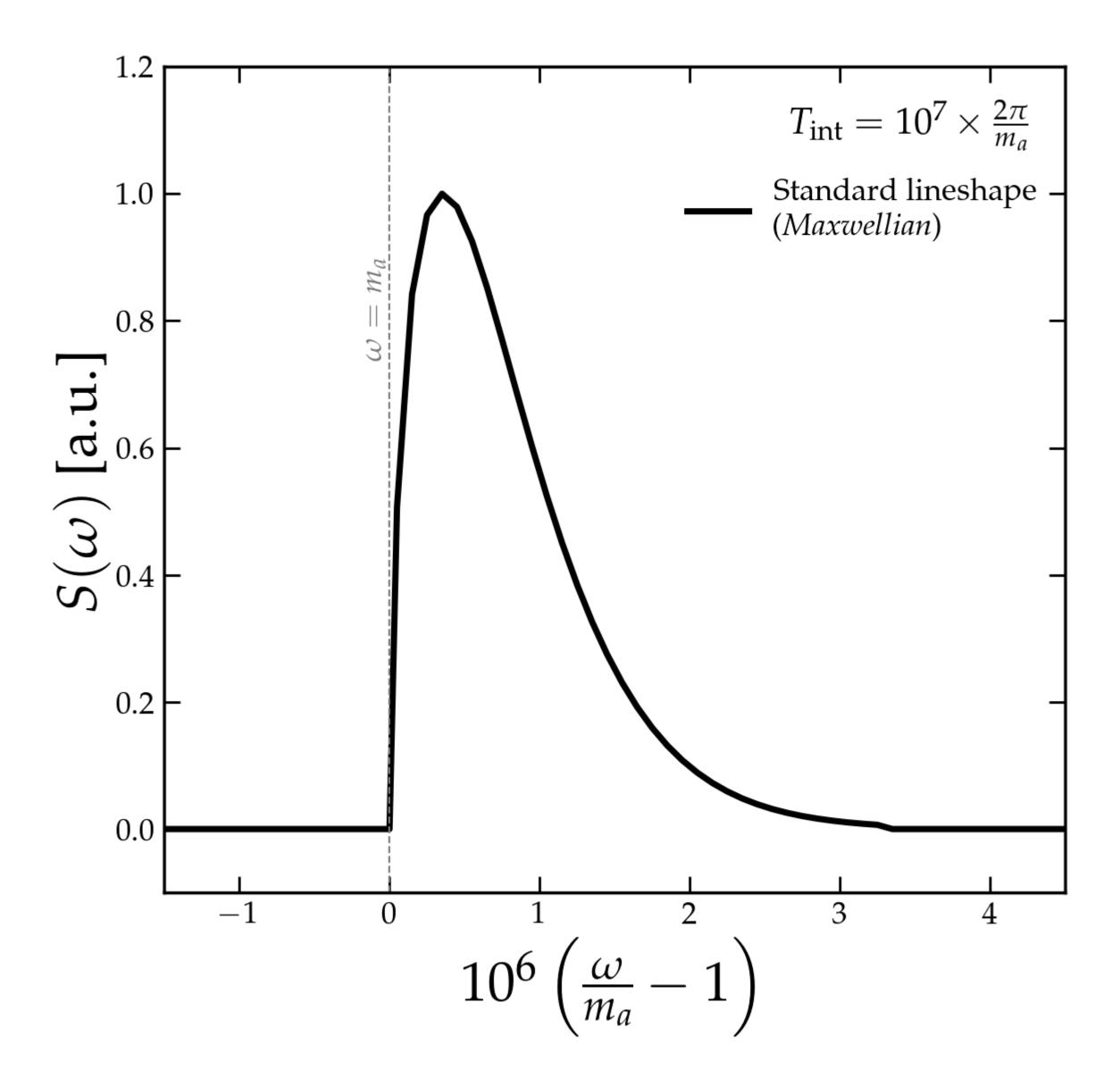
$$S(\omega) \propto rac{
ho_{
m DM}}{m_a^2} g_{a\gamma}^2 f(\omega)$$



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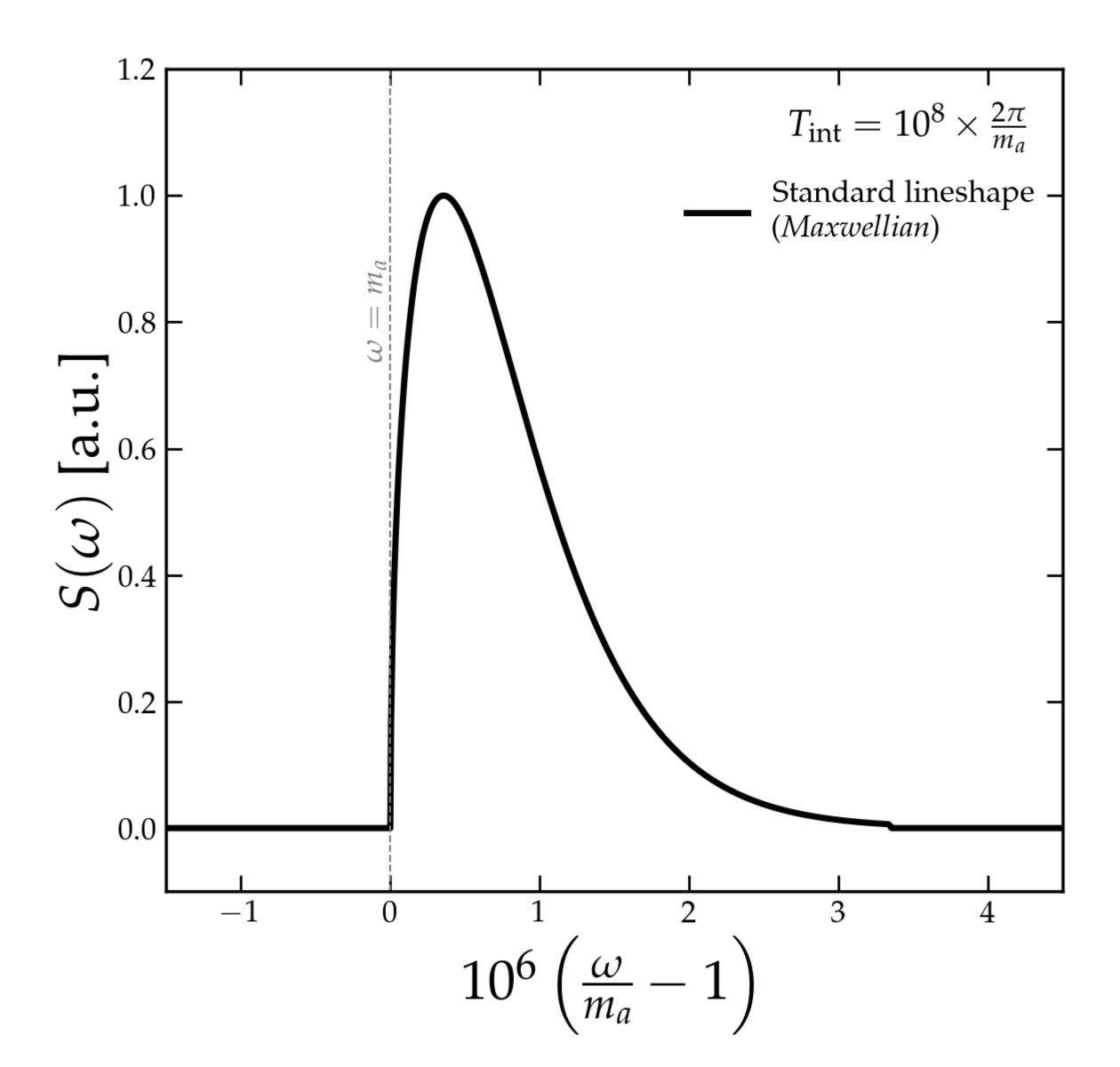
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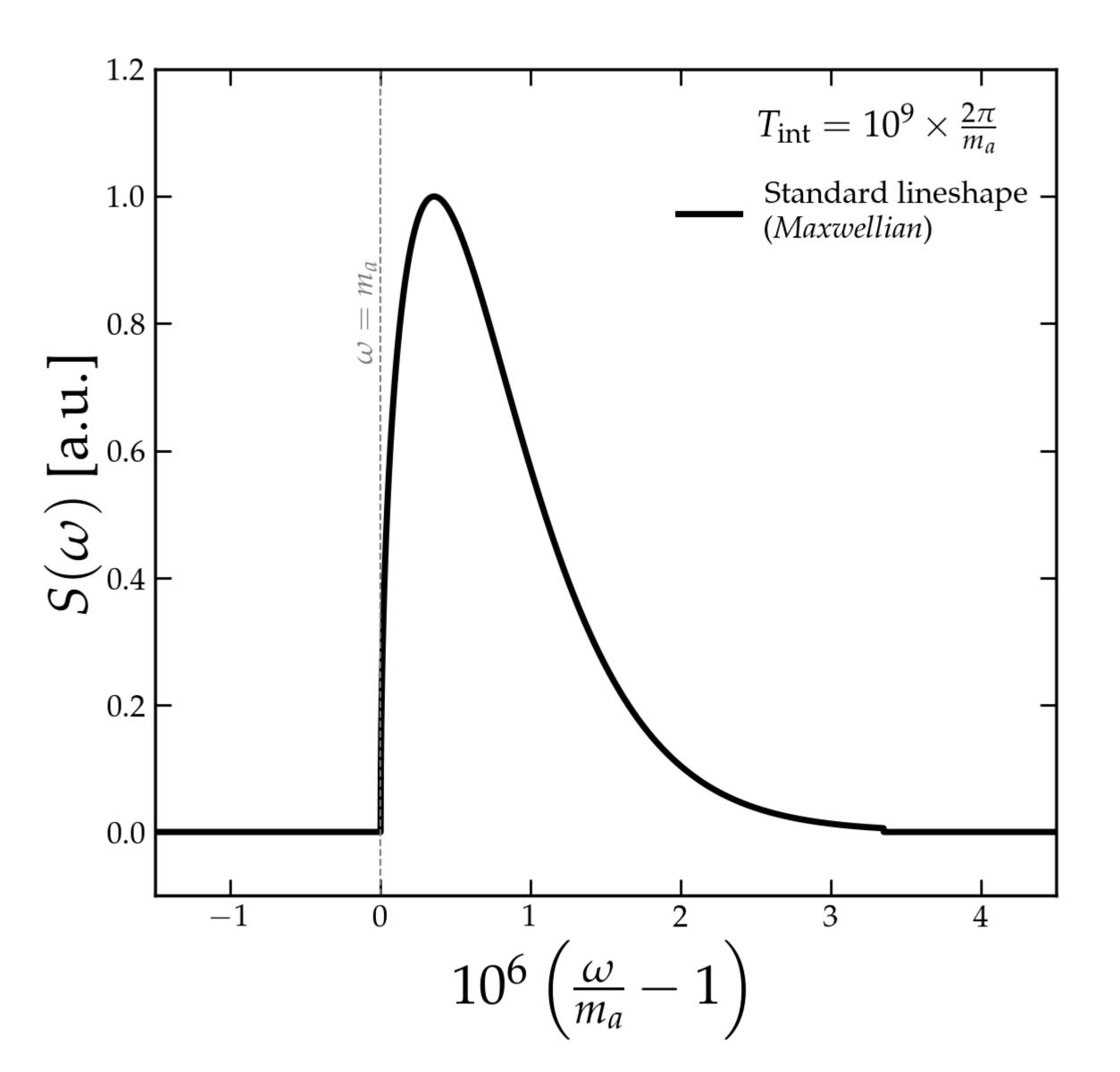
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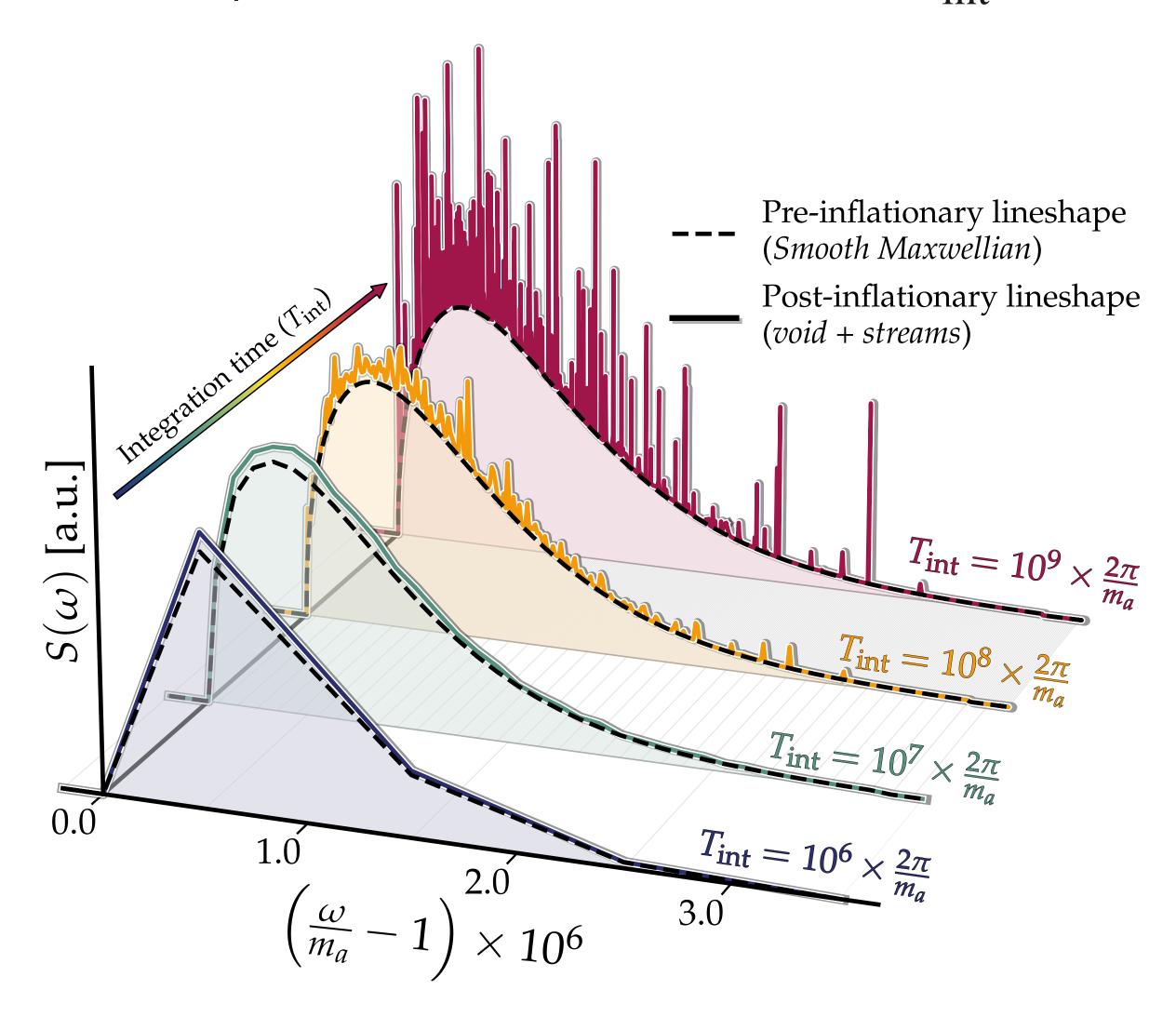
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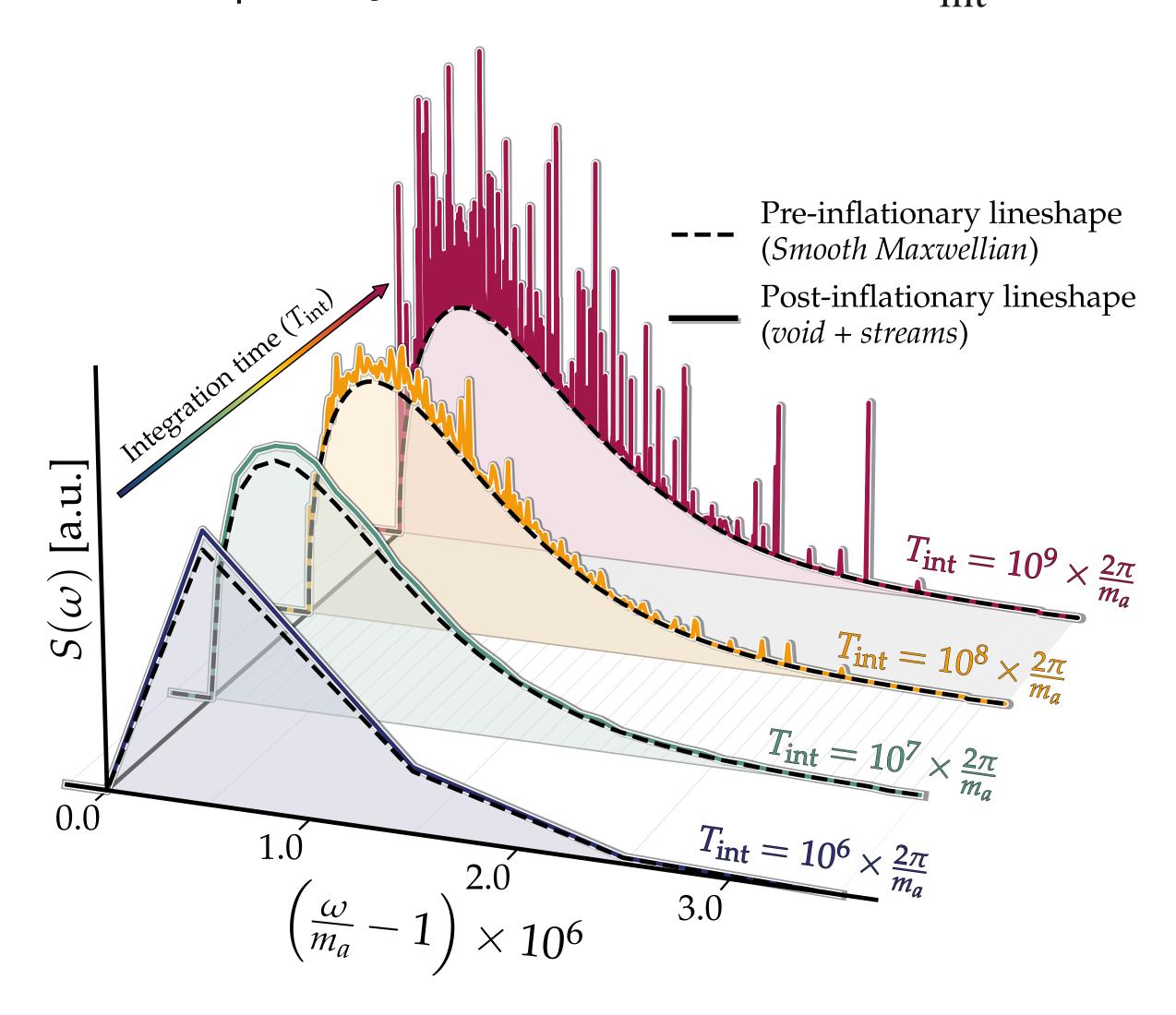
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Some important observations:

- Streams only enhance the signal by $\rho_{\rm str}/\rho_{\rm void} \sim 7$, but can enhance it by many orders of magnitude more in the *resolved* lineshape in certain bins
- Many streams are narrower than daily modulation in lab motion $v \sim 0.47 \; \text{km/s}$
- Streams persist in lineshape $\mathcal{O}(days-years)$ at a time



Summary

- Miniclusters, voids and streams are a *consequence* of the post-inflationary axion dark matter scenario so cannot be ignored
- Ignoring tidal disruption, the worst-case scenario is that we are in a minivoid which have only about ~10% of $\rho_{\rm DM}$ (suppression in $g_{a\gamma}$ by a factor of 3)
- Accounting for tidal disruption, phase space at Solar position re-filled by a factor of 6, to about 70% of $\rho_{\rm DM}$ (suppression in $g_{a\gamma}$ by a factor of 1.2)
- $\mathcal{O}(1000)$ ultra-cold tidal streams present in axion lineshape at any one time that persist for $\mathcal{O}(\text{days-years})$ at a time



